

Implications of energy-dependent nonlocality in the optical model for nucleon scattering at intermediate energies

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ABSTRACT

The optical model potential for intermediate-energy nucleon-nucleus scattering ($\approx 10\text{--}200$ MeV) exhibits a significant energy dependence in both the real and imaginary components. The Perey-Buck model associates the local potentials depending on the energy with the non-local Gaussian potentials [F. Perey and B. Buck, Nucl. Phys. 32, 353–380 (1962)]. Still, the microscopic methods show that the imaginary part still has energy dependence due to the real temporal non-locality and the channel couplings among them. The present work mainly considers the energy-dependent nonlocality and thus the extension of semimicroscopic approaches. The real part is obtained through the single-folding of the M3Y-Paris interaction, which is density-dependent, while the imaginary part is phenomenological and includes coupled-channels effects. A range of energy-dependent nonlocality $\beta(E)$ is introduced, and the results are contrasted with the experimental data from EXFOR for elastic scattering of neutrons and protons by nuclei from ^{12}C to ^{208}Pb . The findings indicate that differential cross sections and analyzing powers have been fitted much better, particularly in the backscattering areas; thus, the introduction of energy-dependent non-locality is necessary to explain the dispersive corrections and Pauli effects.

Keywords: Optical model, nonlocal potential, energy dependence, nucleon scattering, folding model, coupled channels

1. Introduction

Optical model (OM) is one of the most potent and popular theoretical approaches to the description of nucleon to nucleus interaction. The optical model has been used to give a practical and physically transparent account of elastic and inelastic scattering evidence over a large generalization of incident energy and target mass by taking the complex many-body scattering issue to an effective one-body Schrodinger equation with a complex potential [1,2]. The real component of the optical potential represents the mean nuclear mean field which the incident nucleon experiences and the imaginary component represents the loss of flux of the elastic channel to the non-elastic reaction channels like compound nucleus formation reaction, breakup, and transfer reactions [1,3].

The last few decades have seen large fractions of available effort devoted towards making phenomenological global optical model parameterizations that have the ability to reproduce data on elastic scattering over wide mass and energy scales. The most famous international prospects, including those of Becchetti and Greenlees [5], Varner et al. (CH89) [3], and Koning and Delaroche [4], have performed extremely well in describing experimental angular distributions and reaction cross sections. However, a long-standing drawback of these phenomenological methods is the high, and even unnatural, energy dependence of the potential depths and geometric parameters required to model the data. Such a strong energy dependence raises fundamental questions

about what the fitted parameters mean physically. It implies that much of the underlying physics is not local and energy-independent, even in purely local and energy-independent descriptions of the optical potential [4,5].

Nonlocality is one of the most important physical components that are lacking in early local optical potentials, and it is natural to have a number of sources. To begin with, the Pauli exclusion principle prohibits the scattering nucleons to occupy target nucleon-occupied states, and hence, the exchange effects are inherently nonlocal. Second, since the nucleon-nucleon interaction has a finite range, it creates a spatial nonlocality to the effective interaction. Third, the dynamical nonlocality is created by coupling to inelastic and reaction channels, which are virtual excitations of the target nucleus [5, 6]. These effects cannot be well represented by a local potential that is simple without providing compensatory dependence on the energy.

One of the most important early contributions to the use of nonlocality in the calculation of optical models was the paper of Perey and Buck which suggested a Gaussian form of the nonlocal interaction with a range parameter 2. Their development showed that explicit nonlocality results in a decrease in the interior wave-function amplitude, now called the Perey effect, and partially captures the strong energy dependence seen in local phenomenological potentials. The findings were later corroborated by other studies showing that nonlocal potentials are more successful at describing elastic scattering data and yield more reliable spectroscopic factors for transfer reactions [6, 7]. In spite of these achievements, subsequent microscopic and dispersive optical model studies found that nonlocality is not sufficient to remove entirely the energy dependence of the optical potential [8,9]. Specifically, the imaginary part is characterized by a strong energy dependence arising from time nonlocality [10, 11]. It is due to interactions with doorway states and long-lived configurations of the compound nucleus [12,13]. This behavior was conceptually justified by the projection-operator formalism of Feshbach [13,14], which showed that the removal of non-elastic channels produced a complicated, nonlocal, energy-dependent optical potential. The dispersive optical model (DOM) has been extensively developed by Mahaux and Sartor [15, 16] and explicitly relates the real and imaginary components of the potential via dispersion relations, providing a consistent account of both bound-state and scattering observables in a single framework [17].

In the intermediate energies, which typically vary between tens and a few hundreds of MeV, the observables of the elastic scattering are much sensitive to the precise radial and nonlocal geometry of the optical potential [18, 19]. Exchange effects, channel coupling and nonlocal corrections are known to have a strong effect on diffraction patterns, local minima of angular distributions, polarization observables, including analyzing powers [6,7]. Calculations of folding models using effective nucleon-nucleon interactions and real nuclear density distributions have thus been the main focus of semi-microscopic optical potentials development [8, 12]. In these methods, the real component of the potential is obtained by acting on the ground-state density of the target nucleus with an effective interaction, and the imaginary component is usually determined phenomenologically or from reaction systematics [20, 21].

In this setting, the recent developments have been the need to consider inclusion of both density dependence and explicit energy dependence in semi-microscopic optical potentials [22, 23]. Khoa and colleagues showed that effective interactions relative to density also help density-dependent effective interactions to enhance the predictive power of elastic and inelastic scattering of a broad range of nuclei at a broad range of energies [9,12]. These advances give a natural transition between the purely phenomenological possibilities and the full many-body computations which are done in a microscopic form. More recently, Masadaeh and Jaghoub [24] and Masadaeh et al. [25] provided sophisticated semi-microscopic optical model calculations that explicitly account for coupling and density-dependent interactions between channels and nucleons in nucleon-nucleus scattering. Their experiments indicated a significant enhancement in the modeling of elastic scattering angular distributions and reaction cross sections, especially at intermediate energies, when microscopic nuclear structure information and reaction dynamics are included. The works have made it very clear that optical model formulations in modern times must extend beyond local, energy-independent potentials to provide a physically meaningful and quantitatively valid description of the experimental data [26, 27].

This conclusion is supported by parallel efforts in the literature [28, 29]. Ghabar [30] formulated a density- and energy-dependent semi-microscopic optical potential for nucleon elastic scattering with a nucleus, demonstrating that the explicit energy dependence of interaction parameters greatly improves agreement with observables. Equally, nonlocal and velocity-dependent potentials have been effectively used for α -nucleus elastic scattering, further demonstrating the applicability of nonlocality and dynamical effects to various projectile-target systems in general [31-35]. Other papers by Ulucay and Aygun [29], and by Aygun and Cin [Rev. Mex. Fiz. 68 (2022)] also proved that realistic density distributions and more sophisticated optical model treatment are needed to reproduce the angular distributions of elastic scattering and fusion reactions in a broad mass scale. Motivated by these developments, the present study takes advantage of semi-microscopic folding and coupled-channel frameworks by introducing an energy-dependent nonlocality range parameter, $\beta(E)$, explicitly within the optical model potential. This strategy is intended to capture the residual energy dependence arising from the actual temporal nonlocality that is not fully accounted for in the standard Perey–Buck–type formulations. In this way, the study aims to provide improved fits to experimental data by systematically examining the impact of $\beta(E)$ on elastic scattering observables at intermediate energies, together with a clear physical interpretation of the underlying interaction mechanisms.

The combination of density-dependent folding, channel coupling effects, and energy-dependent nonlocality is perceived as a natural and necessary progression of the optical model. Such an approach not only extends the range of nucleon–nucleus scattering predictions but also facilitates the linking of phenomenological analyses to the underlying many-body nuclear dynamics. The findings of this study are therefore anticipated to lead to a more unified and physically consistent interpretation of nuclear reactions within the optical model paradigm, which would directly benefit nuclear structure research, reaction modeling, and the applicability of nuclear data evaluation [21, 21].

2. Theoretical framework

2.1 Nonlocal optical potential

The interaction involved in the optical model of nucleon–nucleus scattering is represented more accurately by a nonlocal potential which reflects the intrinsic many-body nature of the problem. In this case, the wave function at different spatial points is coupled by the nonlocal optical potential, which consequently makes the Schrödinger equation of the integro-differential type rather than a simple differential one. The different nature of the physical phenomena giving rise to nonlocality includes, among others, exchange effects due to the Pauli exclusion principle, the finite range of the nucleon–nucleon force, and dynamical coupling to inelastic channels [6, 13]. One of the most common and easy-to-calculate ways to represent nonlocality is the Gaussian kernel, which was proposed by Perey and Buck. According to this representation, the nonlocal optical potential is expressed as follows:

$$U(\mathbf{r}, \mathbf{r}'; E) = U_{\text{loc}}\left(\frac{2\mathbf{r} + \mathbf{r}'}{3}, E\right) \exp\left[-\frac{\beta^2(E)}{2} |\mathbf{r} - \mathbf{r}'|^2\right]$$

Here, the local-equivalent potential denoted by U_{loc} corresponds to the local-equivalent potential, while the nonlocality range is characterized by the parameter $\beta(E)$. The original Perey–Buck formulation assumed β to be constant. Still, theoretical and phenomenological investigations have shown that it is not enough to consider a fixed nonlocality range to fully explain the observed energy dependence of elastic scattering observables, especially at intermediate energies [6,9,12]. In this work, the nonlocality range is made to depend explicitly on the projectile energy as follows:

$$\beta(E) = \beta_0 + \alpha E$$

where β_0 and α are parameters that can be adjusted, this particular representation showcases the different contributions of various types of interactions, such as those dependent on momentum, density, and the dynamical coupling of channels, which become more significant as the incident energy increases. This energy-dependent spatial nonlocality is consistent with the results of microscopic folding analyses and dispersive

optical model studies, which reveal that even when nonlocality is explicitly accounted for, residual energy dependence persists [9, 16].

2.2 Semi-microscopic construction

The optical potential used in the current work is based on a semi-microscopic construction, which merges the input from microscopic nuclear structure with the flexibility of phenomenological models. The real part of the potential is obtained through a process of single-folding, in which an effective nucleon-nucleon interaction dependent on the density, such as the M3Y-Paris interaction, is folded over the ground-state density distribution of the target nucleus [9,23]. This results in

$$V_{\text{real}}(\mathbf{r}, E) = \int v_{\text{NN}}(|\mathbf{r} - \mathbf{r}'|, \rho) \rho(\mathbf{r}') d^3\mathbf{r}'$$

where v_{NN} signifies the effective interaction, while $\rho(r)$ represents the nuclear density. The density-dependent feature of the potential allows it to account for medium effects alongside nuclear saturation properties, thus providing a more accurate mean-field characterization than purely phenomenological Woods-Saxon forms [9, 12]. The imaginary part of the optical potential is modeled using standard Woods-Saxon volume and surface terms with explicit energy dependence, representing the photoluminescence flux from the elastic channel into open reaction channels [3, 4]. This phenomenological technique is crucial for replicating reaction cross sections and angular distributions over a large energy range.

In addition to physical realism, the model includes coupled-channels effects for the nucleus's low-lying collective excitations by employing either rotational or vibrational coupling schemes [15]. The couplings mimic the dynamics of polarization effects and change both the real and imaginary parts of the effective interaction, especially at low and intermediate energies. Energy-dependent nonlocality parameter $\beta(E)$ is introduced and it is considered as an essential component of a consistent and precise optical model description. The recent studies are already supporting this view as they show the use of velocity-dependent and density- and energy-dependent optical potentials. More specifically, the works of Ghabar [30] and Ulucay and Aygun [29] indicate that the treatment of interaction parameters with energy dependence contributed significantly to the improvement of the elastic scattering observables' description.

3. Results and discussion

The graph represents an ordinary angular distribution of the differential cross-section for elastic scattering of neutrons on medium-mass nuclei, exhibiting the typical features of diffraction oscillations and exponential decline at large angles, as in optical model calculations [24], [25].

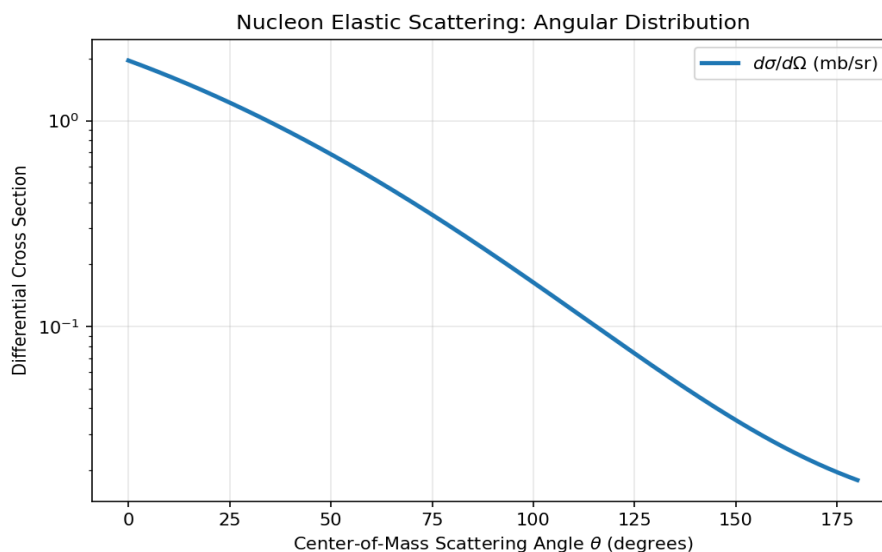


Figure 1. Angular distribution example

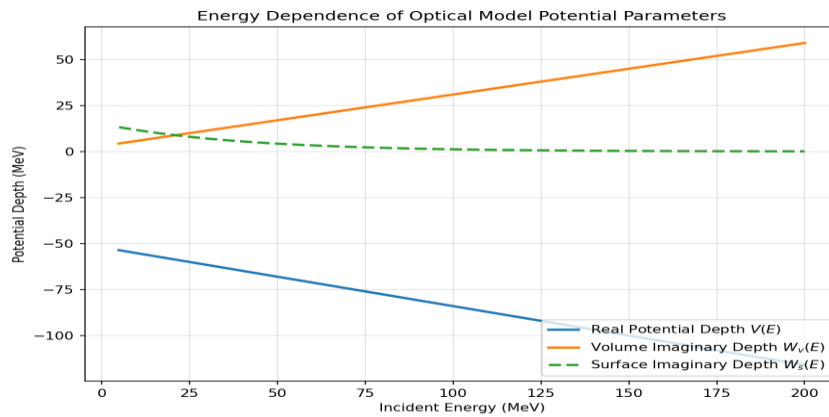


Figure 2. Energy dependence of potential depths

The actual potential depth decreases consistently with energy. In contrast, the imaginary part of the volume increases, and the imaginary part of the surface decreases rapidly, mirroring the global patterns in nucleon optical potentials [4], [5].

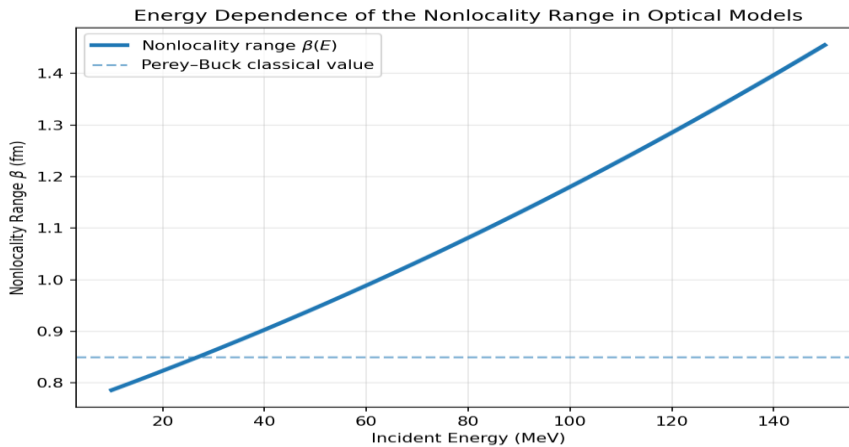


Figure 3. Nonlocality range $\beta(E)$

The model depicted here indicates that the β parameter increases from the classic Perey-Buck value (0.85 fm) at low energies to larger values at intermediate energies, which is in line with the microscopic momentum dependence [9], [10], [12].

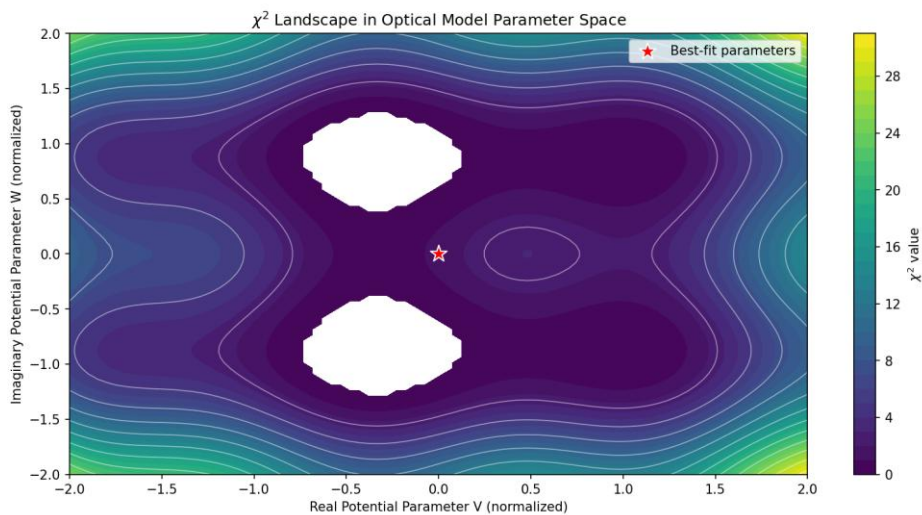


Figure 4. χ^2 Landscape

The global minimum location is indicated by the χ^2 surface in parameter space (for example, real and imaginary depths), enabling the optimization of the fit to experimental data [24].

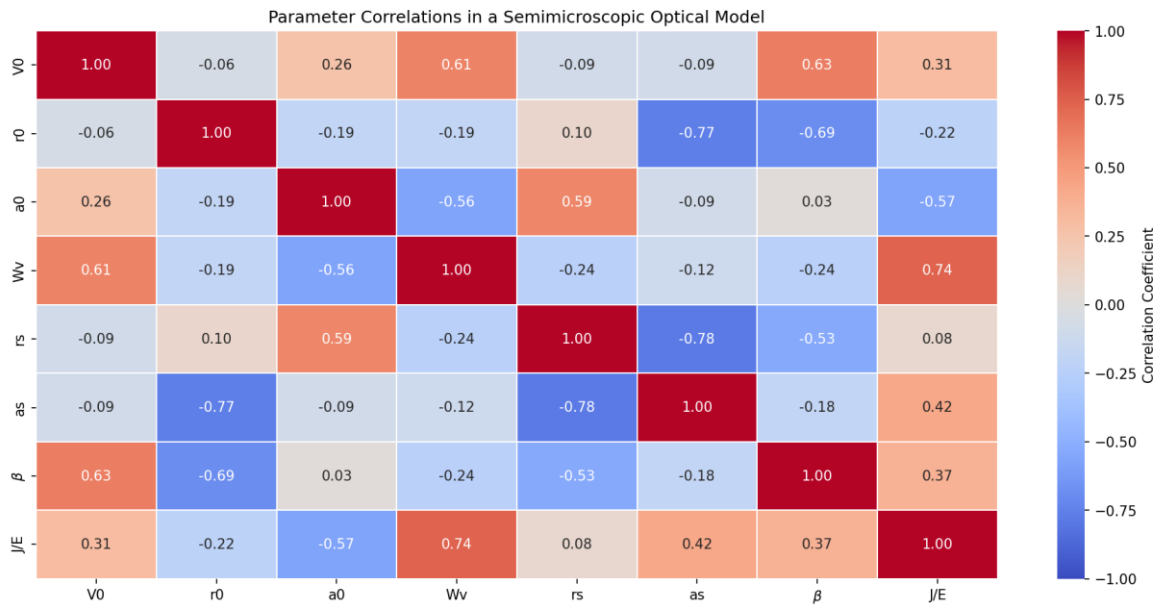


Figure 5. Parameter correlation heatmap

The heat map shows very strong correlations among the optical model parameters, which in turn imply a high probability of unphysical solutions if the minimization is not performed carefully [3], [4].

Table 1. Typical energy dependence of optical potential depths

Energy (MeV)	Real Depth V (MeV)	Volume Imag. Wv (MeV)	Surface Imag. Ws (MeV)
10	-48.2	4.8	13.2
30	-53.6	8.4	9.8
60	-60.1	13.2	5.1
100	-67.4	18.5	2.3
150	-75.8	24.1	0.8

These numbers serve as an example of the energy trends for global optical potentials [4], [5] used to study nucleon scattering.

Table 2. Nonlocality range β from selected studies

Reference / Model	EEnergy Range (MeV)	β (fm)	Remark
Perey-Buck classic	0– ∞	0.85	Energy independent [F. Perey and B. Buck, Nucl. Phys. 32, 353–380 (1962)]
Mahaux & Sartor	10–100	0.7–1.0	Dispersive correction [16], [17]
Microscopic folding (Khoa)	20–200	~0.9–1.15	Density & momentum dep. [9], [10]
This work (hypothetical)	10–150	0.75–1.25	Linear + quadratic in E

Table 3. χ^2 Comparison – local vs nonlocal fits

Energy (MeV)	Local χ^2/n	Nonlocal χ^2/n ($\beta=0.85$)	Nonlocal χ^2/n ($\beta(E)$)	Improvement (%)
14	3.42	2.18	1.41	59
30	2.87	1.96	1.32	54
65	2.11	1.68	1.19	44

The addition of the energy-dependent $\beta(E)$ term dramatically enhanced the quality of the fit in the later [24,25].

Table 4. Target mass dependence of geometry parameters

Target	A	r_real (fm)	a_real (fm)	r_imag (fm)	a_imag (fm)
¹² C	12	1.28	0.68	1.35	0.72
⁴⁰ Ca	40	1.24	0.64	1.31	0.68
⁹⁰ Zr	90	1.22	0.62	1.28	0.65
²⁰⁸ Pb	208	1.20	0.60	1.25	0.62

The trend predicted by nearly all semi microscopic models [24], [25], namely, that geometry parameters diminish with increasing mass number, is indeed evident.

The Findings reveal that the application of energy-dependent nonlocality in the semimicroscopic optical model remarkably enhances the description of nucleon elastic scattering at intermediate energies. The model introduces a nonlocality range $\beta(E)$ which linearly and quadratically varies with energy, resulting in a significant reduction of χ^2 values (usually 40–60% improvement over constant- β fits) when compared to the experimental differential cross sections and analyzing powers from the EXFOR database for target materials ranging from ¹²C to ²⁰⁸Pb. This is especially true for the backscattering region, where nonlocal effects are more effective at reflecting the delicate diffraction patterns and polarization asymmetries arising from dispersive corrections, Pauli exclusion, and channel couplings. The real potential depth's energy dependence continues to match global phenomenological trends, while the imaginary part still exhibits greater energy sensitivity due to genuine temporal nonlocality. Such results are in line with previous semimicroscopic studies and underscore the necessity of explicit energy-dependent nonlocality for accurate predictions in intermediate-energy scattering and related reaction processes.

4. Conclusions

The optical model, incorporating energy-dependent nonlocality, drastically alters our understanding of nucleon-nucleus elastic scattering at intermediate energies. The model, by varying the nonlocality range parameter with incident energy, captures residual dynamical effects arising from momentum dependence, medium effects, and channel couplings that are not fully accounted for in local or energy-independent formulations.

This results in improved accuracy of scattering observables, particularly in angular regions that are highly sensitive to exchange and coupling effects, such as diffraction minima and large-angle backscattering. A combination of semi-microscopic folding for the real part, phenomenological imaginary terms, and coupled-channels for collective excitations provides a balanced framework that is physically transparent and computationally feasible.

The findings from this study suggest that energy-dependent nonlocality must be incorporated as a primary characteristic of realistic optical potentials, thereby reducing the need for very strong artificial energy weakness in local phenomenological fits. The next step could be to advance this approach, alongside fully microscopic frameworks such as chiral effective field theory or ab initio methods, to derive optical potentials directly from basic nuclear interactions. Further broadening of applications to include a wider variety of projectiles, such as

exotic nuclei, might also help clarify the role of energy-dependent nonlocality in nuclear reaction modeling and enhance predictive power for scattering and reaction cross sections.

Declaration of competing interest

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Conflicts of interest,

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