

## Maximum intensity determination of beam waist in BBO optical nonlinear crystal using single harmonic generation

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### ABSTRACT

Nonlinear optics is a branch of optics that explains the nonlinear response of atoms and molecules to a field of light radiation. There is a difficulty in cutting crystals in shapes commensurate with their optical properties, and thus it was significant to determine the descriptive shape of crystal properties based on its optic properties to be ideal in use. The problem of determining the length of the crystal to give the best output power is still under study and research and is one of the most important topics. In our research, a computational method based on mathematical relationships was used to estimate the length of the BBO crystal of I-type, which gives the best output power, by manipulating the location of the optical light scattering to be in the middle of the crystal. It was found that the best length of a crystal with a width of 3mm and a height of 3mm is 7mm. the comparison of these results with the previous literatures gave a great match, which confirms the correctness of the computations and the reliability of used method.

**Keywords:** Nonlinear optics, Beam waist, SHG, Nonlinear crystal, BBO crystal

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### 1. Introduction

The nonlinearity in optics was appeared by E. Schrödinger in 1943. A very strong intensity is required to achieve this nonlinear response to the electromagnetic field that is coming in. The real observation of the coherent nonlinear optical effect happened only with the invention of the Ruby laser at 347.2nm in the quartz crystal when Franken reported the second harmonic generation (SHG). Bass's discovery of total frequency generation (TFG) in triglyceride sulfate in 1962 sparked research into nonlinear optical processes [1]. Artyom T in 2019 proposed a formula that has been simplified for the nonlinear regime (NR) to calculate the efficiency of SHG for I-type synchronization. The original formula for NR contains an ellipsoidal Jacobian sine, which makes calculations more difficult. There is no Jacobian ellipsoid sine in the proposed simpler formula, and it takes several seconds to calculate an individual graph even in a program that does mathematical operations in the simplest way possible. In levels of higher radiative spacing ( $\varphi$ ) and crystal length (L), The SHG efficiency graphs derived using the initial and proposed formulas are very similar, with a variation of roughly 10% or less in their values. As a result, it is found that the fulfilling waist of light inside the crystal due to double refractions and It's still difficult to match phase conditions across various wavelengths [2]. Duarte E used the knowledge of the characteristics of the optical beam propagation through various media, the analysis of those characteristics begins with the study of Gaussian beams propagation, while permits the introduction of some characteristic parameters, whose meaning is extended to the analysis of any other type of optical beam. The Gaussian beam propagation analysis in homogeneous linear media can in two ways: paraxial approximation, and Fresnel diffraction integral, the analysis results gave a good documentation for the properties of light in different media [3]. Wang J. used micro-resonators on the basis of a photonic chip for enhancing the effects of nonlinear optics, which looked promising for the development of scalable, high-efficiency frequency converters. He presented a feasible scheme for degenerate sum-frequency



conversion that only the two-mode phase matching criterion is required. An excellent on-chip photon-number conversion efficiency of up to 42% was achieved when the drive and the signal are both approaching resonance to the same telecom mode, demonstrating a tuning bandwidth of more than 250GHz [4]. Wang X. proved that due to beam divergence, phase mismatching occurs when a concentrated Gaussian beam is doubled in frequency. Integration of the second-harmonic (SH) field along the energy flow propagation channel was used to calculate the conversion efficiency. The resulted distribution of SH spatial field showed that the blue light in critically type-I phase-matched  $\text{LiB}_3\text{O}_2$  generates a short and intense electric field is a single-pass crystal arrangement with a sufficiently high conversion efficiency. Results showed the achievement of high transduced power requires continuous wave applications with a resonant boosting cavity that matches the optimum waist of the beam inside the crystal [5]. John R. studied the waist establishment in the design of photonic systems after optimizing the laser beam settings for a particular crystal in order to achieve optimal conversion efficiency. Optical geometry, Beam form, and various possibilities for the size, orientation, and chemical composition of the nonlinear crystal employed for greater conversion are among these characteristics. Ultraviolet lasers can be created using nonlinear optical properties (NLO) of nonlinear materials such as barium beta borate (BBO), lithium triborate (LBO), and periodically polarized lithium niobate (PPLN) [6].

Previous literature indicated that it was difficult to determine the dimensions of the crystals used in dioptric optical systems, which is a problem faced most workers in non-linear crystal manufacturing laboratories that includes how to choose the appropriate length of this crystal that makes the light constrict approximately in the middle of the crystal that ensure the best output power. Our contribution in this research is to use the mathematical relationships that describe the passage of light inside a crystal of BBO type for calculating the best length of the crystal that gives the best output power.

## 2. Research method

The adopted method is an assemblage one that has been combined from theoretical concepts published in the previous literature. It is mentioned in this literature, when the light impacts the nonlinear crystal, it starts with linear optics and then moves on to nonlinear optics. From near-infrared to deep ultraviolet, barium borate (BBO) is a flexible nonlinear crystal that can be used in harmonic generation (HG) processes, optical parametric oscillators, and photovoltaic applications [7]. SHG is based on the second term for polarization expansion and is a coherent optical process of radiation dipoles in matter. As shown in Figure 1, the dipoles oscillate with a frequency ( $\omega$ ) electric field and radiate an electric field of ( $2\omega$ ) plus ( $1\omega$ ). As a result, the near-infrared input light is emitted near ultraviolet light [8]. The beam waist is the location along the direction of propagation where the beam radius is minimum. The waist radius is the radius of the beam at that location [9].

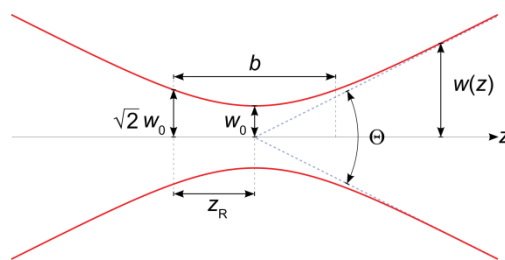


Figure 1. Waist is occurred at smallest radius of light path

In the present work, it is assumed we have a BBO nonlinear crystal of I-type of 3mm width, and 3mm height with undetermined length ( $L$ ) and phase angle of  $56^\circ$  as shown in Figure 2. It is intended to trace the light path inside this crystal to localize the waist ( $w_0$ ) of the light and determine the length ( $L$ ) that gives maximum output power. In addition, the determinations include the output intensity, location of the waist ( $w_0$ ), and efficiency, which passing through determining the shapes of the incoming and outgoing lights [9].

The SHG process is simulated in the used nonlinear crystal using the mathematical relationships that govern the path of light inside the crystal. Figure 3 shows that the cross section of the crystal is so greater than that of the incoming beam of laser when any reflections on the crystal surfaces are ignored. The incoming waveguide is directed in the direction of the beam axis ( $Z$ -axis), which is normal to the crystal surface. The crystal's insertion surface is at  $z=0$ . In this case, the optical axis is assumed to be in the  $X$ - $Y$  plane. Between the input

fundamental beam and the second harmonic (SH) output beam,  $m$  is the phase matching angle and is the spatial exit angle. As a result, the SHG occurs when the input field is polarized along Y, and the output SH field is polarized along X [10].

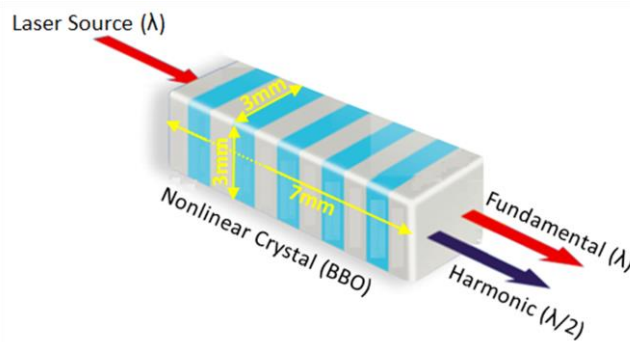


Figure 2. Used SHG setup using BBO to achieve the waist of light

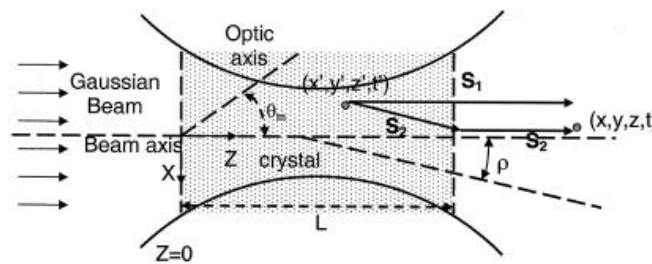


Figure 3. Light paths scheme [10]

Since BBO is a negative crystal, the ordinary (fundamental) refractive index ( $n_o$ ) is greater than the extraordinary (harmonic) index of refraction ( $n_e$ ), so  $n_e$  can vary as a function  $\theta$  of a constant frequency  $\omega$ , but the normal index of refractive index is constant [7].

$$n_{e\omega}(\theta) = n_{o\omega} \quad \dots (1)$$

$$n_e^2 = 2.3730 + \frac{0.0128}{\lambda^2 - 0.0156} - 0.0044\lambda^2, \quad \text{for BBO} \quad \dots (2)$$

$$n_o^2 = 2.7405 + \frac{0.0184}{\lambda^2 - 0.0179} - 0.0155\lambda^2, \quad \text{for BBO} \quad \dots (3)$$

When the fundamental wavelength  $\lambda$  is  $532nm$ , then  $n_e=1.5555$ ,  $n_o=1.6749$ . The single-pass transformation efficiency ( $\eta$ ) is a factor that enables us to predict the magnitude of the second harmonic force ( $P_{2\omega}$ ) generated with respect to the value of the fundamental beam power ( $P_\omega$ ) passing through the nonlinear crystal only once. Energy conversion is not linear and has a form [10].

$$P_{2\omega} = \eta P_\omega \quad \dots (4)$$

To maximize  $\eta$ , we must have an effective focus of the laser inside the crystalline medium. The circular radius is above  $\eta$ . Thus, it first considers the ray to be a perfect circular ray, and then it will list the effects of the ellipsoidal modulation caused by the discontinuity phenomenon. To achieve a desired power output from the UV laser at  $266\text{ nm}$ , one can focus a fundamental beam at  $532nm$  that must be optimized as a Gaussian beam. The focus related parameters are the radial waist or electric field radius  $w_o$  at  $Z=0$ , Rayleigh range  $R(z)$ , crystal length ( $L$ ), and confocal parameter ( $b$ ). The beam radius  $w(z)$  and wavefront curvature  $R(z)$  can be written as [3]:

$$W^2(z) = W_o^2 \left[ 1 + \left( \frac{z}{z_R} \right)^2 \right] \quad \dots (5)$$

$$R(z) = z \left[ 1 + \left( \frac{z_R}{z} \right)^2 \right] \quad \dots (6)$$

$$b = 2z_R = W_o^2 k_\omega \quad \dots (7)$$

Where,  $k_w$  is the wave vector as the fundamental beam  $532nm$ . So the efficiency can be written as [4]:

$$\eta = \frac{P_2\omega}{P_0^2} = \left[ \frac{2\omega^2 d_{eff}^2 k_\omega L}{\pi \epsilon_0 n_{e2\omega} c^3} \right] h(B, \xi) \quad \dots (8)$$

where,  $\omega$  is the angular frequency,  $d_{eff}$  nonlinear coefficient which is equal to  $1.606 pw/v$  for the BBO at beam wavelength  $532nm$ .  $c$  is speed of light and the factor  $h(B, \xi)$  is equal  $0.637$  when  $L=b$ , in which  $B$  is the walk off parameter and  $\xi$  is the focusing parameter. For the nonlinear crystal BBO the  $\epsilon_0=1$  [7]. Later, the relationship between efficiency and beam waist can be given as [9].

$$\eta = \sqrt{\pi} \gamma P_\omega \frac{w_o L}{b |\theta_m|} \quad \dots (9)$$

Where  $\gamma$  is given as:

$$\gamma = \frac{4 k^2 n_2}{\lambda_o c \epsilon_0 n_1} = \frac{4 \omega_1^2 d_{eff}^2}{n_1 n_2 \lambda_o c^3 \epsilon_0} \quad \dots (10)$$

Therefore, the optimal length (L) of the crystal equals the confocal parameter ( $b=2\pi n_o w_o^2/\lambda$ ) focusing given as [4]:

$$L = 2Z_R = \frac{2\pi w_o^2}{(\lambda/n)} \quad \dots (11)$$

Then, the final beam diffraction intensity does not remain constant throughout the crystal, it can be determined as follows [4]:

$$I \sim e^{\left(\frac{-2 r^2}{w_o^2}\right)} \quad \dots (12)$$

### 3. Results and Discussion

From the observation of Figure 4, it is found that the efficiency increases with the increase in the length of used nonlinear crystal, where the efficiency reaches its highest level at the length of  $7mm$  to be about  $36.31$ , which is a good value compared to published literatures that used same nonlinear crystal. Because the length of the path traveled by light inside the crystal, there are ordinary and extraordinary refractions occurred by splitting of light into original light (fundamental) and another (harmonic) one, and the longer the crystal was, the more accurate the splitting values.

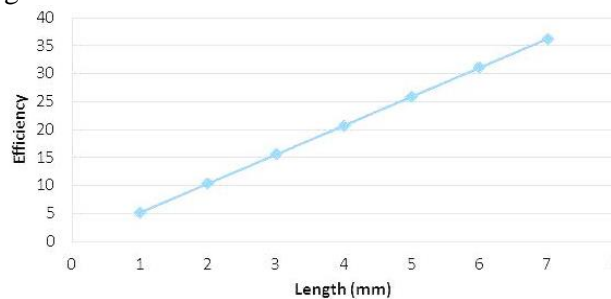


Figure 4. Computed efficiency with the variation of crystal length

Also, by observing Figure 5, it becomes clear that the value of the efficiency depends on the input and output power, it is shown that the increase in the output power when the input power is set at  $100mW$  along the length of the crystal. This relationship is considered reasonable due to the efficiency is directly proportional to the output power and can reach its highest value when the output power is about  $0.37mW$  at the length of  $7mm$ .

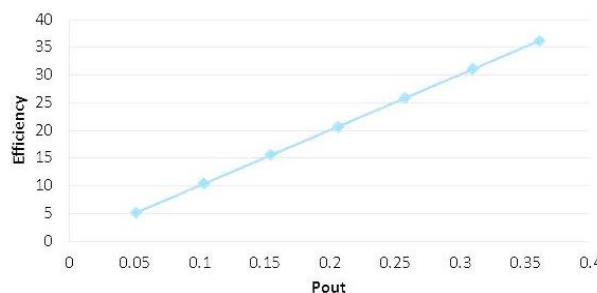


Figure 5. Computed Efficiency with the variation of output power

Figure 6 depicts the location of the waist ( $w_o$ ) by changing the length of the crystal, as the increase in length leads to the waist of the light being closer to the middle of the crystal length, and this is a very important result that can theoretically be proved through the calculations, as the amount of the waist ( $w_o$ ) was within (0) at a length of  $L=1mm$ , the amount of the waist ( $w_o$ ) was within (0.90) at a length of  $L=2mm$ , while this waist is 3.373 at  $L=7mm$ , which indicates that the waist is approach to be equal the half of crystal length ( $L/2=3.5mm$ ). In a higher waist region, the process of double refraction occurred, and the wavelength of the fundamental ( $532nm$ ) and harmonic ( $266nm$ ) wavelength values are as accurate as possible within the SHG process. And the value of the waist ( $w_o$ ) can be obtained by calculating it from the Eq.9 when the light reaches a crystal length of  $3.37mm$ , which is a value closer to the ideal value that is equal to the middle of the crystal length  $3.5mm$ .

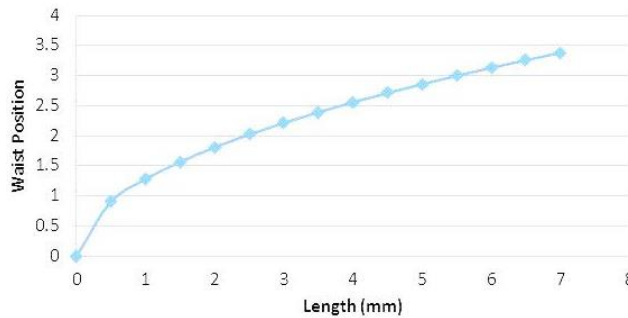


Figure 6. Computed waist position versus crystal length

In addition, Figure 7 indicates that the calculation of the light intensity was depending on crystal length and the waist position. It is necessary to calibrate the crystal length ( $7mm$ ) during the calculations to be within the range ( $L=-3.5-3.5mm$ ) instead of ( $L=0-7mm$ ). It was found that the highest value of the intensity reaches  $12W/m$  at the middle of the crystal, which is an acceptable but the very tight focusing is not a good case; it makes the efficiency to be lower, the crystal may be damage in the focus, and the phase matching appear poor and high divergence of the output of the light. It is noticeable that the Gaussian behavior of the intensity occurs in non-linear crystals when light entering the crystal is in the form of high diffraction and then confined to waist ( $w_o$ ) region and then returns to be diverged again after leaving a higher scattering region that gives the highest intensity due to the gathering of light uniformly with higher quality.

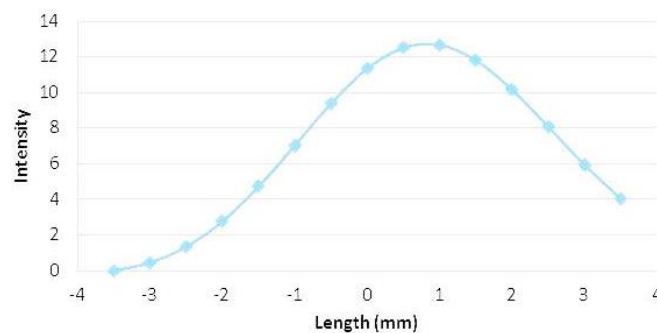


Figure 7. Computed intensity versus crystal length

When the length of the crystal is set at  $7mm$ , after making sure that it gives the best efficiency value and shortens the center of the crystal with a higher intensity, the intensity of the input power is changed within the range ( $P_{in}=10-100mW$ ), where it is noticeable that the change of the output power ( $P_{out}$ ) is all linear with the input power and that the efficiency value is remains at 36.3% as shown in Figure (8), which is a good value compared to previous research related to the linear crystal used in SHG. Therefore, the final value of the ideal case for calculating the intensity at the site of the waist as well as the efficiency when the length of the crystal is  $7mm$  and the input power is  $100mW$ , a shortening value of  $3.73nm$  was obtained with an efficiency of 36.3% and an intensity of about  $12W/m$ , which are values that can be considered very good for a nonlinear BBO type crystal.

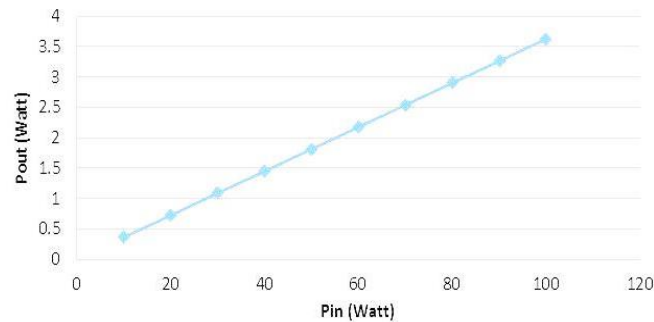


Figure 8. Computed output power versus input power

#### 4. Conclusions

From observing the results of the passage of light in used BBO crystal, it was found that there is an appropriate length of this crystal of  $7\text{mm}$  in which the light is reduced inside the crystal in the middle region, and it was found that the efficiency of the output power is directly proportional to both the output power and the length, and its highest value reaches the waistline area in the middle of the crystal. While, the conductance of the waist with the change in length is an increasing nonlinear behavior and has its highest value at the middle of the crystal, where the light intensity is concentrated and is at its highest value. The recommendations we recommend are the use of other crystals, and observing the behavior of the waist, then determining its location in terms of the best crystal length that gives the best output power, and comparing it with experimental results.

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