Superluminal light group velocity in tapered optical microfibers

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ABSTRACT

Superluminal light group velocity was formerly reported in anomalous dispersion, nonlinear amplification of light pulse, high-gain lasers’ cavities, and waveguides. Motivated by a recent observation of light acceleration in optical microfiber [1], the possibility of attaining the light group velocity exceeds its value in vacuum is investigated. The investigation of superluminal velocity is in tapered optical microfiber that has a radius decreases with propagation axis by a factor 10\(^3\). Our results show the possibility of attaining superluminal group velocity in this microfiber at length of about 1080µm. At this length the instantaneous acceleration of light is found to be 13×10\(^{19}\) m/s\(^2\) which its corresponding Unruh temperature is 0.527K.

Keywords: Tapered microfibers, Light velocity, Group velocity, Light acceleration, Unruh Temperature.

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1. Introduction

Famous historic experiments of measuring the speed of light \(c\) were indeed to determine its group velocity \(v_g\). The first who distinguished the group velocity \(v_g\) and the phase velocity \(v_p\) was Hamilton [2]. A few decades later Lord Rayleigh presented more clear recognition between \(v_g\) and \(v_p\) [3]. The theory of Special Relativity [4-7] presented the postulate that the speed of any moving object cannot exceed the speed of light in vacuum \((c = 299 792 458 \text{ m/s})\). This postulate, which represented the ultimate speed, was only several months after the first observation of superluminal light (light whose \(v_g > c\)) in the dispersion of Na-vapor [8]. This anomalous dispersion, in which \(v_g > c\), was theoretically explained in 1906 by Lorentz in the treatment of light dispersion and propagation in systems of molecules in his Theory of Electrons [9]. From the later theory, one can deduce the group velocity \(v_g = c / n_x\), where \(n_x = n + o d n / d \omega\). The latter expression of \(n_x\) explicitly explains how a slow or fast light is attainable. In certain media, the phenomenon of anomalous dispersion takes place near absorption lines, where \((dn/d\omega) < 0\), and so \(v_g\) can be superluminal when it exceeds \(c\). This phenomenon of attaining \(v_g > c\), near the absorption lines was also proposed and conducted in the works [6, 10-13]. The relativistic postulate of the ultimate speed of light seemed contradictory to the experimental findings [8] and its theoretical possibility [9]. This contradiction induced Sommerfeld and Brillouin to develop Lorentz treatment of light dispersion in spectral absorption regions, where anomalous dispersion takes place. In their works they made a special emphasize on the fundamental definitions of three velocities of light. These velocities were the group velocity, the signal velocity and the velocity of energy transfer. Their inspections showed that these three velocities are of the same value in non-absorbing media, but theses velocities notably differ in the spectral absorption regions [3]. Sommerfeld and Brillouin proved that the signal velocity is the velocity of the light which never exceeds the speed of light in vacuum \(c\) and even in the regions of anomalous dispersion [3]. Therefore, they concluded that superluminal group velocity, \(v_g > c\), never violates the relativistic postulate of light speed barrier. This was justified by proving that the group velocity is not the velocity of the pulse of light but it is merely the velocity of the peak of the pulse, where connection between \(v_g\) and \(c\) is through \(v_p\) via the formula.
The phenomenon of superluminal \( v_g \) is not only restricted to anomalous dispersion because predictions and observations firmed its possibility in nonlinear amplifications of light pulse [13], in photon tunneling [14] and in active and high-gain medium of lasers [15-26].

Recently a research team observed the acceleration of light in optical microfibers [1] in which the radius of the microfiber decreases gradually with the propagation axis \((z)\). They fabricated microfibers whose shapes depend on the relation between the decrement in the radius with the propagation the \( z \)-axis [1]

\[
\eta = \frac{\Delta r}{\Delta z} = \text{const.}
\]

The decrement in the radius with \( z \) produced a gradient in the optical fiber effective refractive index \( n_1 \) with \( z \). The decrement and variation in \( n_1 \) with \( z \) consequently led to the increment and variation in the group velocity of light in the microfiber via the definition [1]

\[
\eta = \frac{\Delta r}{\Delta z} = \text{const.}
\]

They experimentally determined the variation of \( n_1 \) with \( z \) by measuring the angle of the radiation penetrating the microfiber into its surroundings by measuring the angle of the radiation penetrating the microfiber into its surroundings. This penetrating radiation (leaky radiation) and the angle of penetration was expressed as [1]

\[
\frac{\omega}{k} \frac{d\omega}{dk} = c^2
\]

Where \( n_2 \) is the refractive index of the microfiber surroundings \((n_2 \) is constant). The acceleration of light velocity was obtained using the gradient in the effective refractive index of the optical fiber \( n_1 \) with \( z \) as given by [1]

\[
a = -c^2 \frac{d}{dz} \left( \frac{dn_1}{dz} \right)
\]

Their experimental finding of light acceleration was used for the evaluation of the Unruh temperature in tapered microfibers by [1]

\[
T_a = a \left( \frac{\hbar}{2\pi k_B c} \right)
\]

Where \( \hbar \) and \( k_B \) are respectively the universal constants of Planck and Boltzmann. The experimental results of [1], concerning the measurements of the angle of the leaky radiation, have proved the validity of the theoretical predictions for the variations in \( n_1 \), \( v_g \) and \( a \) with the propagation axis \((z)\). This motivated our concern to investigate the superluminal \( v_g \) and to determine the required length of the optical microfiber to reach that \( v_g \).

### 2. Calculations

The results of [1] were for microfibers whose lengths do not exceed 400µm and with \( \eta \) of order 10^{-3}. Therefore, we have to determine the domain of microfiber length we can search for superluminal group velocity by extrapolation. The table below shows the lengths of microfibers whose right end radius is 1µm and their left-end radii are ranging from 5µm to 1µm with shape factor \( \eta = 10^{-3} \) which are determined using equation (2).

<table>
<thead>
<tr>
<th>Microfiber length (µm)</th>
<th>Left-end Radius (µm)</th>
<th>Right-end Radius (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>3000</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>2000</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1000</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

The table exhibits the possibility to investigate superluminal \( v_g \) in microfibers for the range of lengths shown in the table. Our investigation is based on extrapolating the variation with \( z \) in the effective refractive index \( n_1 \), group velocity \( v_g \) and acceleration \( a \). The insets in our figures 1, 2 and 3 are the results appeared in Ref. [1], which we have extrapolated in this work.
3. Results and discussion

Fig. 1 shows the variation of $n_1$ with $z$ in the extrapolated region which exhibits similar behavior of the reduction of $n_1$ with $z$. It also shows that $n_1$ becomes zero when the length $z$ is about 1750μm and then it becomes negative beyond this length. The zero value of $n_1$ can be explained by equation (4) which indicates that the angle of leaky radiation becomes 90° while its negative value is merely due to the reflection of light inside the microfiber and propagating backward (from right to left). **Fig. 2** shows that the group velocity becomes superluminal ($v_g > c$) when $z \geq 1080μm$. From this figure one can directly deduce the instantaneous phase velocity $v_p$ at any point along the propagation axis $z$ via equation (1). **Fig. 3** shows that when $z = 1080μm$, $v_g$ is superluminal, the instantaneous acceleration which has a value of $13\times10^{19} \text{m/s}^2$. For this value of acceleration, Equation (6) gives $T_u = 0.527K$. 

![Fig. 1. The variation of the effective refractive index ($n_1$) with $z$](image1)

![Fig. 2. The variation of light group velocity ($v_g$) with $z$](image2)

![Fig. 3. The variation of light acceleration ($a$) with $z$](image3)
4. Conclusions

Tapered optical microfiber of radius varying with propagation axis by a factor of $10^3$ is investigated for superluminal light velocity. In this investigation the variation of the effective refractive index with the propagation axis is exhibited as well as the variation of the light group velocity. The possibility to attain superluminal light velocity in optical microfibers is verified. The backward propagation (the propagation of light inside the microfibers from right to left) due to the reduction of the microfiber radius with the propagation axis is also detected and determined. Finally, the variation of light acceleration with the propagation axis is also exhibited and the value of the Unruh temperature which corresponds to the superluminal light velocity is determined which has a value of 0.527K.

References

