Algorithm for computational and experimental determination of the acoustic characteristics of solid propulsion engine

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ABSTRACT

Combustion instability in rocket engines is accompanied by large-amplitude pressure fluctuations in the combustion chamber and (or) intense vibration of the load-bearing structural elements. Unstable operation leads to a decrease in thrust, specific impulse, a decrease in engine life, a violation of the normal operation of vibration-sensitive flight control system equipment. The study considers a method for determining the frequencies of the first and second tone natural resonances of the longitudinal mode of acoustic vibrations in the combustion chambers of solid propulsion engines. The gas path of the combustion chamber is divided into homogeneous sections, for which solutions of the wave equation are presented. To determine the natural frequencies and the distribution of vibrational pressures and velocities, the method of "stitching" acoustic fields at the boundaries of cavities was applied. In the course of the study, to determine the decrement from natural perturbations, the following methods were used: spectral, correlation, amplitude, and instantaneous period method. In the study, vibrations of a certain frequency in a given range were excited using an electrodynamic emitter. To obtain the acoustic characteristics of the camera at various moments of engine operation, experiments were carried out with several charge models having different sizes. The solution of the wave equation in the form of standing waves was considered. In addition, the study results present a description of the experimental setup, in particular, the distribution of the oscillatory pressure in the chamber gas cavity (according to the calculated data). In the course of the study, an algorithm was developed for calculating and experimentally determining the acoustic characteristics of the combustion chambers of solid propulsion engines.

Keywords: Acoustic vibrations, Longitudinal mode, Frequency, Damping decrement, Wave equation.

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1. Introduction

The tasks of ensuring the combustion stability in rocket engines are significant in the design and development of new engines [1-10]. Combustion instability in solid fuel engines is divided into two main types according to the frequency of pressure fluctuations: low-frequency instability (or z*-type instability) – the frequency of vibrations is significantly lower than the frequency of acoustic modes in the combustion chambers; acoustic instability – the frequency of vibrations coincides with the frequency of natural normal modes of acoustic vibrations in the combustion chamber. In most theoretical works, the analysis is carried out in a linear approximation. Its purpose is to determine the conditions for the spontaneous occurrence of instability, characterised by a gradual increase in the amplitude of small disturbances (soft excitation of vibrations). Taking into account nonlinear factors shows that a system that is resistant to small perturbations can become unstable when the amplitude of perturbations increases above a certain threshold limit. Disturbances having an amplitude below the threshold are damped [11-17]. Rigid excitation of vibrations is characteristic of nonlinear dynamical systems. The development of nonlinear theories is significantly complicated by mathematical...
difficulties. Linear theories provide a fairly complete understanding of instability, which is also useful for nonlinear applications.

According to modern concepts, acoustic instability in a rocket engine is related to the interaction between the combustion process and the acoustic vibrations in the combustion chamber. Pressure fluctuations are of decisive importance in this case, but in some cases, acoustic velocity fluctuations can also play an important role. When the combustion rate fluctuates, acoustic pressure fluctuations are excited in the combustion chambers. In turn, the effect of acoustic pressure fluctuations on the combustion process, in accordance with the Rayleigh criterion, causes fluctuations in the combustion rate. The presence of this feedback leads to an increase in pressure fluctuations in the combustion chamber due to heat and mass release [18-21]. When solid fuels burn, turbulent pulsations – combustion noises – are present in the grain channels. They are caused by different mechanisms of excitation and have a different physical nature. Acoustic noise is usually associated with gas-dynamic causes of the same nature as in turbulent gas flows. In combustion chambers, regular pressure fluctuations with a frequency close to the natural (acoustic) oscillation frequency of the gas column [22-26] can increase, and more often stabilize at a certain level due to acoustic losses.

Acoustic instability of combustion is an autowave process with feedback through the effect of sound waves on combustion [27-32]. The parameters of the wave process: frequency, amplitude, and shape of the oscillations are determined by the properties of the dynamic system itself. Sound noise in combustion chambers can also be considered as an auto-oscillatory process in which the energy source is heat release during combustion, and the feedback occurs due to the effect of sound waves on the combustion, while there are undamped pulsations of a stochastic nature, having a wide frequency band and random phases [33-39]. The gas pressure pulsations in the chamber are described by random functions, the spectral densities of which are broadband in comparison with the frequency response of the combustion chamber in the vicinity of the studied natural frequencies $\omega_{0\text{mk}}$ of normal oscillations, then the normal modes of acoustic oscillations of the reaction volume are denoted as follows – $\omega_0$. Then the pressure pulsations in the combustion chamber caused by these disturbances $\omega_0$ can be described by a system of linear equations [40-43], or in the form of a 2-order self-oscillatory system with the right part displayed by a random broadband noise effect [44-49]:

$$\frac{d^2 p_\nu(t)}{dt^2} + 2 \delta_\nu (\Pi, \lambda_\nu) \frac{dp_\nu(t)}{dt} + \omega_0^2 \nu_p(t) = \omega_0^2 \xi(t),$$

$$\frac{\omega_0}{\delta_\nu} > 1, \delta_\nu (\Pi) = (\delta_\nu^2 - \delta_\nu^1) > 0.$$  (1)

where: $p_\nu(t)$ – narrow-band pressure fluctuations of the $\nu$-th mode; $\omega_0$ – circular frequency of natural oscillations (without damping); $t$ – time; $\delta_\nu^1, \delta_\nu^2$ – coefficients of generation and loss of acoustic energy of the $\nu$-th mode of normal oscillations, which are functions of parameters $\Pi$ of the combustion chamber operation mode; $\xi(t)$ – stationary normal random broadband impact (noise of turbulent combustion).

Here one equation (1) is assumed for each mode of the normal acoustic oscillations of the inner chamber volume. An assessment of combustion stability in solid fuel engines can be performed by determining the balance of acoustic energy in the combustion chamber, taking into account the influx of acoustic energy (due to the interaction of acoustic vibrations with the combustion process) and the loss of acoustic energy during the oscillation period. The diagnostic indicator of the reserve of linear stability of the combustion process is the oscillation attenuation coefficient (decrement) [50-55]. The attenuation coefficient has a certain physical meaning:

$$\delta_\nu = \frac{E_2 - E_1}{2E_{\text{SUM}}},$$

where: $E_2$ – inflow of acoustic energy generated by the oscillating system during the oscillation period, $E_1$ – part of the energy dissipated by the oscillating system during the oscillation period, $E_{\text{SUM}}$ – acoustic energy stored by the system during the oscillation period [56-59].

2. Material and methods

The inflow of acoustic energy from the combustion process depends – for certain phase ($0 \div \pi/2$) relations between the pressure fluctuations and the combustion rate – on the amplitude of the pressure fluctuations,
increasing proportionally to the square of the latter. Experimental methods using a T-shaped chamber have been developed to determine the inflow of acoustic energy [60-67]. The loss of acoustic energy in the gas volume of the combustion chamber is essential if there are condensed particles in the combustion products, for example, in the case of fuels with aluminium additives. The calculation of losses at the boundaries of the combustion chamber, which has a complex configuration in the case of solid fuel engines, is associated with great difficulties. These losses can be approximately determined by conducting experiments on models of combustion chambers without gas flow. It follows from the above that the acoustic properties of the combustion chamber: the frequency of natural vibrations, the distribution of the amplitude of vibrations – can amplify the vibrations or dampen them, thereby affecting the stability of combustion in the engine [68].

Notably, the properties of the combustion chamber as an oscillatory system have differences with the properties of systems considered in acoustics. The differences are conditioned by the fact that the gas flow is superimposed on the fluctuations in the combustion chambers, the presence of a nozzle affects, there are temperature gradients, etc. However, despite these important differences, the shapes and frequencies of natural vibrations do not change significantly [69-74], which allows carrying out calculations and experiments to determine the properties of the combustion chamber as an oscillatory system in an acoustic approximation, i.e., without taking into account complicating factors. In practice, it is convenient to replace the attenuation coefficient with a dimensionless value obtained by multiplying by the period – the damping decrement $\delta T$.

The decrement of small oscillations for the $v$-th mode in the vicinity of the resonant frequency $f_v = 1/T_v$ of the reaction volume of the combustion chamber is equal to $d_v = \delta_v T_v$, where $T_v$ – the oscillation period. Here one equation (1) is assumed for each mode of the normal acoustic oscillations of the inner chamber volume [75].

To determine the decrement from natural perturbations, the following methods have been developed: spectral, correlation, amplitude, and instantaneous period method [76-80]. Currently, the amplitude method and the instantaneous period method are practically not used. The decrement by the spectral method is determined by the width of the "noise" power spectrum. With a linear mechanism of signal generation in the vicinity of the resonant frequency of the $v$-th mode of acoustic vibrations of the reaction volume of the combustion chamber, the width of the peak $\Delta f$ of the spectral density $S(f)$ of the signal $Y(t)$ at the level of 0.5 $S_{\text{max}}$ is proportional to the magnitude of the decrement of the $v$-th mode [81-85].

$$d_v = \delta_v T_v = \frac{\pi \Delta f_v}{f_v},$$

As a result of theoretical and experimental studies of the acoustic instability of combustion, a certain level of knowledge of physical processes at the present time has been established, which allows predicting the influence of changes in structural and regime factors on the instability areas. Acoustic vibrations of the longitudinal mode are characteristic of solid propulsion engines (SPE), transverse modes are not relevant. Such oscillations occur mainly in engines with a large value of the charge length-to-diameter ratio ($L/D\geq5$) [86-90].

3. Results and discussion

3.1 Natural oscillation frequencies and acoustic pressure fields in the combustion chamber

Acoustic waves propagating in the paths of various systems often have a wavelength comparable to the size of the channels. In this case it is appropriate to consider the solution of the wave equation as standing waves. Standing waves are formed as a result of the interaction of direct and reflected waves from "hard" walls. Next, the considers the superposition of two waves $F$ moving towards each other with constant phase velocities. Therefore, the solution of the wave equation is presented as a sum of partial solutions of the form [91-94].

$$\frac{\partial^2 F(x)}{\partial x^2} = -k^2 F(x); \frac{\partial^2 F(t)}{\partial t^2} = -k^2 c_0^2 F(t) = -\omega^2 F(t),$$

where: $\omega^2 = k^2 c_0^2$.

The system (5) is equivalent to the equations describing the elastic vibrations of a material point. The solutions of these equations represent harmonic oscillations with their phase shift:

$$F(x) = A \cos(kx + \phi_x); F(t) = B \cos(\omega t + \phi_t).$$
The solution of the wave equation is written as a product:

$$\psi = F(x)F(t) = C \cos(kx + \phi_x) \cos(\omega t + \phi_t). \quad (7)$$

Where: $C = AB$.

The function $F(x)$ describes the distribution of the oscillation amplitude, constant in time, and the function $F(t)$ shows that all points of the wave move synchronously. The oscillation does not propagate, the displacements of all points reach their maximum or minimum values at the same time points. The resulting partial solution is called a standing wave or a natural oscillation. To describe wave motion in an unlimited volume, it requires an infinite number of solutions (7) with a continuous frequency spectrum $\omega$ and $k = \omega/c_0$.

Where: $c_0$ – the speed of sound, and $k$ – the wave number. The general solution has the form of a Fourier integral. In the case of the implementation of a standing wave in a finite path with rigid walls, then both a general solution and each partial solution satisfying the boundary conditions are found $\xi_v(x,t) = 0; \frac{\partial \xi_v(x,t)}{\partial x} = 0$.

Each particular solution must satisfy the wave equation and describe the possible oscillation of the system [95-97]:

$$\psi_v(x,t) = A_v \cos(k_vx + \phi_{vx}) \cos(k_vc_0 t + \phi_{vt}), \quad (8)$$

where: $\psi_v(x,t)$ – velocity potential, and $\omega_v = k_v c_0$ – circular oscillation frequency.

If the potential function at the boundaries of the path is zero, then for any moment of time, the following must be performed:

$$A_v \cos \phi_{vx} \cos(\omega_v t + \phi_{vt}) = A_v \cos(\omega_v t + \phi_{vt}) \cos(k_v l + \phi_{vx}) = 0, \quad (9)$$

$$\phi_{vx} = \frac{\nu \pi}{2}, \quad k_v l = \nu \pi, \quad v = 1, 2, \ldots. \quad (10)$$

Each particular solution can be represented by the equation:

$$\psi_v(x,t) = \psi_v(x) \cos(\omega_v t + \phi_{vt}), \quad (11)$$

where:

$$\psi_v(x) = A_v \sin \frac{\nu \pi x}{l}, \quad \omega_v = k_v c_0 = \frac{\nu \pi c_0}{l}. \quad (12)$$

The function $\psi_v(x)$ describes the natural oscillations of the system in the absence of external forces and damping, and the frequencies $\omega_v$ are the natural circular frequencies. A more general solution can be obtained by adding up all the natural oscillations of the system with the corresponding amplitudes:

$$\psi_v(x,t) = \sum \psi_v(x) \cos(\omega_v t + \phi_{vt}) = \sum A_v \sin \frac{\nu \pi x}{l} \cos(\omega_v t + \phi_{vt}), \quad v = 1, 2, \ldots. \quad (13)$$

The solutions of the wave equation can be represented in the form of standing or travelling waves. Standing waves are a superposition of a direct and reflected travelling wave, for example, in a pipe closed on one side. A travelling wave is considered as a beating between standing waves. Travelling waves reflect the nature of unsteady processes, while standing waves characterise steady-state periodic processes. The solution in the form of standing waves in the pipe excited by some perturbation near the end of the pipe contains all the Fourier components of periodic natural oscillations with the same amplitudes and in phase. Next, the study considers a special case of plane wave propagation in a moving flow to find the relations for velocity pulsations and pressure perturbations. The general solution of the equation is represented in the form of two waves moving in the opposite direction [98-101]:

$$\psi = A e^{i \omega t - ik_1 x} + B e^{i \omega t - ik_2 x}, \quad (14)$$

where:
\begin{align*}
k_1 &= \frac{\omega}{c_0 + v_0} ; \quad \omega = k_1 (c_0 + v_0) ; \quad k_2 = \frac{\omega}{c_0 - v_0} ; \quad \omega = -k_2 (c_0 - v_0). \tag{15}
\end{align*}

Velocity pulsation:

\begin{align*}
v'_x &= -\frac{\partial \psi}{\partial x} = ik_1 A e^{-k_1 x + i\omega t} + ik_2 B e^{-k_2 x + i\omega t}. \tag{16}
\end{align*}

Pressure perturbations related to the velocity potential by the equation:

\begin{align*}
\delta p \frac{c_0^2}{\gamma} = \frac{p'}{\rho_0} = \frac{\partial \psi}{\partial t} + v_0 \frac{\partial \psi}{\partial x}.
\end{align*}

This equation is differentiated by time \(t\) and by the \(x\) coordinate:

\begin{align*}
p' &= \rho_0 \left( i \omega A e^{-k_1 x + i\omega t} + i \omega B e^{-k_2 x + i\omega t} - v_0 i k_1 A e^{-k_1 x + i\omega t} - v_0 i k_2 B e^{-k_2 x + i\omega t} \right). \tag{18}
\end{align*}

Next, new coefficients are introduced, then the expression for the pressure perturbation will be somewhat simplified:

\begin{align*}
p' &= Ce^{i\omega t - ik_1 x} + De^{i\omega t - ik_2 x}, \tag{19}
\end{align*}

where the coefficients are equal:

\begin{align*}
C &= (i \omega - v_0 i k_1) \rho_0 A = \frac{i \rho_0 c_0 \omega}{c_0 + v_0} A; \quad D = (i \omega - v_0 i k_2) \rho_0 B = \frac{i \rho_0 c_0 \omega}{c_0 - v_0} B. \tag{20}
\end{align*}

Taking into account the form of these coefficients, the expression for the velocity perturbation will be:

\begin{align*}
v'_x &= \frac{C}{\rho_0 c_0} e^{i\omega t - ik_1 x} - \frac{D}{\rho_0 c_0} e^{i\omega t - ik_2 x}. \tag{21}
\end{align*}

Next, the mechanical resistance of the medium is expressed in the form of the ratio of pressure perturbations to the velocity pulsation. For a wave travelling along the stream, this ratio is equal to \(\frac{p'}{v_x} = \rho_0 c_0\), and against the flow \(\frac{p'}{v_x} = -\rho_0 c_0\). The value \(z_c = \rho_0 c_0\), is the wave resistance of the medium. Thus, the sound pressure is equal to the propagation velocity of a plane wave multiplied by the value of the wave resistance and has a positive sign if the wave propagates in the positive direction of the coordinate axis. When the wave propagates in the opposite negative direction, the particle velocity is negative, and the sound pressure is positive. The wave resistance characterises the medium and is a constant value for it. The dispersion relation for a perturbation in the form of a plane entropy wave is defined as follows [102].

\begin{align*}
\omega - k_2 v_0 &= 0 \rightarrow v_0 = \frac{\omega}{k_2} \quad \text{from here} \quad \delta S = \delta S_H e^{i\omega t - ik_2 x}. \tag{22}
\end{align*}

Entropy waves propagate without dispersion, and vortex plane waves do not exist. Next, the study considers flat stationary waves in a cylindrical combustion chamber closed on one side. The solution of the wave equation is taken in the form:

\begin{align*}
\psi &= (A \cos k x + B \sin k x) e^{-i\omega t}. \tag{23}
\end{align*}

Then for \(x = 0\):

\begin{align*}
v'_x &= \frac{\partial \psi}{\partial x = 0} = (-Ak \sin k x + Bk \cos k x) e^{i\omega t} = 0. \tag{24}
\end{align*}
The identical equation (24) is fulfilled at $B = 0$, it follows that standing oscillations are excited, which are identical in phase at all points simultaneously [103-105]:

$$\psi = A \cos(kx) e^{iat}, \frac{\partial \psi}{\partial x} = (-Ak \sin kx) e^{iat}. \quad (25)$$

For $x = l$, the vibrational pressure is taken as zero ($\psi = 0$), then $\cos (kl) = 0$; hence, $kl = m\pi/2$, while $m$ – odd. Thus, for a pipe closed at one end and open at the other:

$$\psi = A \cos\left(\frac{m\pi}{2l} x\right) e^{iat}, \frac{\partial \psi}{\partial x} = \frac{m\pi}{2l} \left(Ak \sin \frac{m\pi}{2l} x\right) e^{iat}. \quad (26)$$

Where: $m = 1, 3, 5, \ldots$. In the case of a pipe closed on both sides, i.e., for $x = 0$ and $x = l$, it follows from the equation (24) that $B = 0$ and $\sin (kl) = 0$, hence, $m\pi = kl; m = 0, 1, 2, \ldots$:

$$\psi = A \cos\left(\frac{m\pi}{2l} x\right) e^{iat}. \quad (27)$$

The gas cavity of the combustion chambers of solid fuel engines has a more complex configuration. Figure 1 shows a gas cavity, which is formed if the charge is tightly connected to the chamber and combustion occurs on the inner surface of the charge channel and the ends.

![Figure 1. Diagram of the chamber gas cavity](image)

Note: a – constructive, b – calculated.

The propagation of vibrations in such a complex cavity is accompanied by diffraction phenomena. Considering that most of the sound wave approaching the nozzle is reflected from it into the chamber (due to the presence of a narrowing part of the nozzle and a change in the acoustic resistance of the medium $\rho c$), to simplify the considered oscillatory system, the nozzle is replaced with a rigid wall. A complex gas cavity is divided into three cylindrical cavities, each of which has a constant cross-sectional area. The solution of the wave equation is presented for each component part of a complex cavity. Further, for the natural frequencies and distribution of vibrational pressures and velocities, the method of "stitching" acoustic fields at the cavity boundaries is used [7-9], taking into account the continuity of the medium during the transition of a sound wave from one cavity to another. For example, the conditions of continuity of the medium during the transition from the first cavity to the second are determined by the equality of sound pressures (or velocity potentials) at $x = l_1$:

$$\psi_1(l_1) = \psi_2(l_1) \quad (28)$$

and the congruence of volumetric velocities:
where: $S_1$ and $S_2$ – cross-sectional areas of the first and second cavities.

The solution of the wave equation for a complex three-stage cavity should be written as a series, taking into account the fact that there is a standing wave in the cavity and diffraction reflections of vibrations during the transition from the cavity to the cavity (for $x = l_1$ and $x = l_2$). That is, for each cavity, taking into account zero boundary conditions for vibrational velocities at $x = 0$ and $x = l_1 + l_2 + l_3$. If, to simplify the problem, the diffraction of acoustic waves at the joints of cavities is neglected, then the solution of equation (27) for standing waves in each cavity can be written as [106]:

$$\psi_i = A_i [k(x - \alpha_i)] e^{iat},$$

where: $i$ – the number of the cavity, $\alpha_i$ – parameter that determines the conditions of the phase transition from the cavity to the cavity, $A_i$ – oscillation amplitude.

For the first cavity:

$$\psi_1 = A_1 [k(x - \alpha_1)] e^{iat},$$

From the zero-boundary condition for the vibrational velocity $v = 0, x = 0$, it follows that $\alpha_1 = 0$. From here:

$$\psi_1 = A_1 \cos(kx) e^{iat},$$

and for the second cavity:

$$\psi_2 = A_2 \cos[k(x - \alpha_2)] e^{iat}.$$  

Similarly, for the third cavity:

$$\psi_3 = A_3 \cos[k(x - \alpha_3)] e^{iat},$$

However, from the zero-boundary condition for the vibrational velocity: $v' = \frac{\partial \psi_3}{\partial x} = 0$ for $x = l_1 + l_2 + l_3$, it follows that $\alpha_3 = l_1 + l_2 + l_3$. It is possible to determine the unknown wave number $k$ and parameter $\alpha_2$ substituting the equations (32) and (33) in the conditions (28) and (29) characterising the continuity of the medium at $x = l_1$. Excluding the coefficients $A_1$ and $A_2$ from the system of equations, the relationship of unknown parameters is obtained:

$$tg kl_1 = \frac{s_2}{s_1} tg [k(l_1 - \alpha_2)].$$

The conditions for "stitching" acoustic fields at $x = l_1 + l_2$ are fulfilled similarly to conditions (28) and (29):

$$\psi_2(l_1 + l_2) = \psi_3(l_1 + l_2); S_2 \left(\frac{\partial \psi}{\partial x}\right)_2 = S_3 \left(\frac{\partial \psi}{\partial x}\right)_3.$$  

Excluding the amplitude coefficients $A_2$ and $A_3$, the second equation for determining the unknown $k$ and the parameter $\alpha_2$ is obtained:

$$\frac{s_3}{s_2} tg kl_3 + tg [k(l_1 + l_2 - \alpha_3)] = 0.$$  

Solving together the algebraic equations (35) and (37), a transcendental algebraic equation is obtained:

$$tg kl_2 + \frac{s_1}{s_2} tg kl_1 + \frac{s_2}{s_2} tg kl_3 - \frac{s_3 s_2}{s_2^2} tg kl_1 \cdot tg kl_2 \cdot tg kl_3 = 0.$$  

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The dependence (38) includes the geometric dimensions of the gas cavity and an unknown wave number \( k \). The distribution of the amplitude of pressure fluctuations (in relative values) along the length of the combustion chamber can be determined by the following equations: for the first cavity:

\[
\delta p_1 = \frac{p_1}{p_0} = \cos k x;
\]  

(39)

for the second cavity:

\[
\delta p_2 = \frac{p_2}{p_0} = \frac{\cos kl_1}{\cos[k(l_1 - \alpha_2)]} \cos[k(x - \alpha_2)],
\]  

(40)

where: \( \alpha_2 \) is calculated from equation (35).

\[
\alpha_2 = \frac{kl_1 - \arctg\left(\frac{S_{l_1} gk l_1}{k}\right)}{k},
\]  

(41)

For the third cavity:

\[
\delta p_3 = \frac{p_3}{p_0} = \frac{\cos kl_1 \cos[k(l_1 + l_2 - \alpha_3)]}{\cos kl_2 \cos[k(l_1 - \alpha_2)]} \cos[k(x - \alpha_3)].
\]  

(42)

where: \( p_0 \) – pressure amplitude at \( x = 0 \).

Figure 2 shows the graph \( p/p_0 = f(x) \) according to the calculated data. According to the described algorithm, approximate calculated dependences can be obtained for other configurations of the gas cavity of the combustion chambers. By comparing the calculated data found using approximate equations with the experimental data obtained on an acoustic installation, it is possible to find out the accuracy of the calculations. The tasks of experimental work should include the determination of the natural resonances of the acoustic longitudinal mode and the estimation of vibrational energy losses.

### 3.2 Description of the experimental setup

The experimental setup consists of a model chamber, an electrodynamic emitter, a sound generator that sets the excitation frequency, an amplifier and receiving and recording equipment. The model camera is detachable and consists of several sections. Charge layouts can be inserted inside the camera. The geometric dimensions of the acoustic cavity are shown in Figure 2. As a source of small vibrations, sound heads of the 10 DEG-5 type are used. The frequency range of these heads is in the range from 90 to 12000 Hz. The sound heads are powered by an UM-50A type amplifier. A sound generator of the GZ-18 type is used as a master generator. The receiving and recording equipment includes a pressure sensor and a computer. The sensing part of the sensor is a hollow piezoceramic ball having a diameter of 7 mm and a wall thickness of 0.25 mm. The dimensions of the sensor element are selected so that they are an order of magnitude smaller than the measured wavelengths, to avoid distortion of the sound field [101; 102].

Methods of conducting experiments. In operation, vibrations of a certain frequency in a given range are excited using an electrodynamic emitter. To obtain the acoustic characteristics of the camera at various moments of engine operation, experiments were carried out with several charge models of different sizes. For each variant of the gas cavity, the amplitude-frequency characteristic within the first two resonances of the longitudinal oscillation mode is removed by tuning the sound generator. The natural frequencies of the camera are determined by the amplitude-frequency response. At the same frequencies, the distribution of the oscillatory pressure amplitude for the longitudinal mode of oscillations is recorded using a sensor moved along the axis of the chamber. The sequence of calculations and processing of experimental data.

- Calculation of the acoustic characteristics of the model combustion chamber.

1. The lowest values of the wave number \( k \) are determined by the equation (38) (using auxiliary tables) and the natural frequencies of longitudinal vibrations are calculated by the equation:
When calculating the frequency of vibrations on an acoustic model installation, the speed of sound in the air is taken, and for a natural combustion chamber, the speed of sound in the combustion products should be calculated.

2. The amplitude of pressure fluctuations is calculated in relative values (related to the pressure amplitude $p_0$ at $x = 0$) according to the equations (39-42), (Figure 2).

3. Approximate determination of acoustic energy losses in the combustion chamber. The acoustic energy losses related to the unit of time, $dE/dt$, are proportional to the energy density $E$. Denoting the proportionality coefficient by $2\delta$, obtain $dE/dt = 2\delta E$, where after integration:

$$E = E_0 \exp(-2\delta t),$$

where: $E_0$ – energy density at time $t = 0$.

The pressure amplitude due to losses in the oscillatory system decreases exponentially $p' = p_0 e^{-\delta t}$. The value $\delta = \frac{dE}{2E dt}$ is the attenuation coefficient for the longitudinal mode and characterises the loss of acoustic energy per unit time. Approximately, the attenuation coefficient $\delta$ can be determined, having the amplitude-frequency characteristic of the oscillatory system, by the Q-factor value $Q$ [2; 6]. Notably, $Q$ is a characteristic of a linear system with concentrated parameters, but conditionally $Q$ can be used as a characteristic of nonlinear systems with distributed parameters. The value $Q$ represents the ratio of the average time available energy to the value of the energy loss over the period of oscillations [2; 6-8], therefore, $Q = \frac{\pi}{\delta \cdot T}$ or $Q = \frac{\omega}{2\delta}$, here $T = \frac{2\pi}{\omega}$ – the period of fluctuations. The Q-factor of an oscillatory system, having an amplitude-frequency characteristic, can be determined (Figure 3) by the width of the curve $(f_1, f_2)$ at a level equal to $\frac{\sqrt{2}}{2} p_{max}$, according to the equation:
From the above expressions for $Q$ the following is obtained:

$$\delta = \pi(f_2 - f_1).$$

(46)

If the combustion chamber is filled with gas with such characteristics that the ratio $\frac{(RT)k_k}{(RT)_{\text{mod}k_{\text{mod}}}} \approx 1$ is fulfilled, then in model experiments the oscillation frequencies and loss characteristics close to those occurring in the combustion chamber of a running engine will be simulated (considering the losses with the flow through the nozzle small).

- Construction of acoustic characteristics of a model combustion chamber based on experimental data.
  1. Graphs of the amplitude-frequency characteristics of the combustion chamber are constructed.
  2. A graph of the natural frequency of vibrations of the combustion chamber is plotted as a function of the ratio of the diameter of the charge channel to the diameter of the combustion chamber.
  3. Graphs $p/p_0 = f(x)$ are plotted for the first and second tones of the longitudinal mode.
  4. A graph of the attenuation coefficient is plotted as a function of the ratio of the diameter of the charge channel to the diameter of the combustion chamber [107].

4. Conclusion

Thus, the EU countries are one of the largest investors (the share 65-90% of all investments) in the polystructural space of international investment. We believe that European integration of the world countries will allow to secure a safe flow of foreign investments and investment attractiveness for the developing countries on the basis of: creation of consortia and alliances of domestic companies with leading European companies, taking into account the means of economic diplomacy; introduction of modern forms of international joint financing of strategic investment projects (we should note that in Ukraine, during 2016-2018 only 45-48% projects were implemented in the medium-term budgetary period); ensuring the investment needs of the manufacturing sector, taking into account the agricultural sector; increasing interaction and practical cooperation in the context of the EU 2020 Strategy; the introduction of monitoring pricing within transnational companies (TNCs), to prevent tax evasion and the territory of developing countries; formation of a system of mutual protection of investments, minimization of geopolitical, macroeconomic, as well as military threats.

In the context of deepening cooperation and realizing the unique capabilities of the states in shaping the global investment climate, it is necessary to ensure a high level of employment of the population by creating new jobs, updating the transfer and introducing the latest technologies, solving social problems at the general level; to carry out an investment modernization of the economy to increase the fixed assets of enterprises; to implement a more effective investment policy.
References


Steel Skin and B4C + (Cr, Fe)7C3 + Al Filler,” *Metallofizika i Novoishie Tekhnologii*, vol. 42, no. 9, pp. 1265-1282, 2020.


