

Algebraic topology: On results of quotient for topological modules

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ABSTRACT

In this paper, we have the principal goal is to study a topology property of important algebraic construction namely the quotient module. We use a new tool with a quotient module which is a tensor product of modules. Therefore all topological submodules in this notion are a tensor product. The meaning of the tensor module introduced in this notion and the important fact of this article is to explain the quotient module when all submodules are tensor. Finally, several results have been obtained about the tensor product of the finite quotient module.

Keywords: Topological module, Topological submodule, the tensor product of Topological submodule, Quotient Topological module

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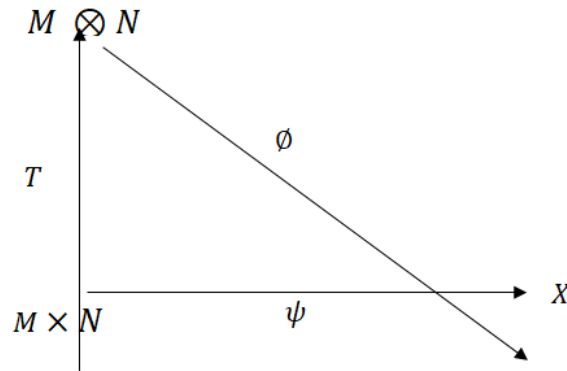
1. Introduction

Arithmetical geography is one of the significant parts of math and epitomizes the connections between variable based math and geography. The certified beginning of the examination of arithmetical geology during the 1920s through the examination of Topological social occasion. The meaning of the module overall can discover it in [1]. Numerous creators contemplated topography gathering [2].

Consequently, the scientists needed to consider the Topological module. In 1955, Cabaske presented the meaning of the Topological module. A definition and a few properties of the Topological module and Topological submodule can discover it in [1]. To contemplate the remainder module we need to present a simple meaning of shape similarity Topological module:

Let $M: E \rightarrow E_1$ be a mapping between two topography modules. μ is called a shape similarity Topological module if M is a shape similarity and continuous [3][4].

A tensor module of Topological R-modules M and N have been defined in and more details about a tensor concept in [5]. A tensor product of Topological R-module M, N is a Topological R-module denoted by $M \otimes N$ together with R-bilinear mapping $T: M \times N \rightarrow M \otimes N$ such that for every R-bilinear mapping $\psi: M \times N \rightarrow X$. There exists a unique linear mapping $\phi: M \otimes N \rightarrow X$ such that the diagram:



Commutates that is $\psi = \phi \sigma$.

Leave E alone a left Topological R -module. A subset M of E is called a Topological submodule if:

1. M is a submodule of E
2. M is a Topological subspace of Topological space E .

Let E, E' be a Topological module. The mapping $f: E \rightarrow E'$ is called homeomorphism Topological module if

1. f is a shape similarity module.
2. f is a continuous map

And f is called homeomorphism topology module if

1. f is isomorphism module
2. f is shape similarity topology.

Now The mapping $f: E \rightarrow E'$ is called regular embedding if it's the embedding mapping and $f(E)$ is open of E' where E, E' be a Topological module.

2. Material and methods

2.1 Quotient of topological module

In this section, we introduce new results about the quotient of the Topological module. We use a tensor product of modules to satisfy our goal.

3. Results

Definition (2.1):

Let $M_1 \otimes M_2$ be a Topological submodule of a Topological R -module E . The family F of subsets of $E/(M_1 \otimes M_2)$ is a topology on $E/(M_1 \otimes M_2)$ and denoted quotient of the Topological module where q^{-1} is open in E ($q: E \rightarrow E/(M_1 \otimes M_2)$).

Example (2.2).

The $\mathbb{Z}/a \otimes \mathbb{Z}/b = 0$ is a tensor module.

Using the Euclidean-ness of \mathbb{Z} , let $r, s \in \mathbb{Z}$ such that $1 = ra + sb$

Proposition (2.3):

Let $M_1 \otimes M_2$ be a Topological submodule of a Topological R -module E . Then the quotient of the Topological module $E/(M_1 \otimes M_2)$ is a Topological module.

Proof:

We must prove that the mapping

$$(\lambda, x + \mu \otimes M_2) \rightarrow \lambda(x + \mu \otimes M_2) = \lambda x + M_1 \otimes M_2$$

from $R \times E / M_1 \otimes M_2 \rightarrow E / M_1 \otimes M_2$ is continuous.

Since Topological group $R \times E / (1 \leq \alpha \leq n \otimes M_\alpha)$ is a homeomorphism with a Topological group $R \times E / (\{0\} \times M_\alpha)$.

We need simply that to demonstrate the planning

$$(\lambda, x) \rightarrow \lambda(x + 1 \leq \alpha \leq n \otimes M_\alpha)$$

from $R \times E$ into $E / (M_1 \otimes M_2)$ is continuous and this satisfied from the synthesis of two continuous functions $(\lambda, x) \rightarrow \lambda x$ and $x \rightarrow x + 1 \leq \alpha \leq n \leq n \otimes M_\alpha$ such that λ in R and $x \in E$.

Proposition: (2.4)

If $M_1 \otimes M_2$ is a Topological submodule of a Topological module E then

$q: E \rightarrow \frac{E}{M_1 \otimes M_2}$ is constant and open, and a subset F from $\frac{E}{M_1 \otimes M_2}$ is closed if and only if $q^{-1}(F)$ be closed.

Proof:

Let $p = p_1 \otimes p_2$ be an open set of E . Then $q^{-1}(q(p))$ equal the union of each coset of the subgroup $M_1 \otimes M_2$. But $M_1 \otimes M_2$ is an intersection with p (i.e. $q^{-1}(q(p)) = p(M_1 \otimes M_2) = (p_1 \otimes p_2)(M_1 \otimes M_2)$). Also, $q(p) = q(p_1 \otimes p_2)$ is an open of $\frac{E}{M_1 \otimes M_2}$ (see definition of (2.1)).

Let F be a closed set of $\frac{E}{M_1 \otimes M_2}$. Thus $q^{-1}(F)$ is a closed set of E .

Since q is continuous, and if $q^{-1}(F)$ is a closed set of E , then $\frac{E}{q'(F)}$ is open of E . So $q\left(\frac{E}{q'(F)}\right)$ is an open of $\frac{E}{M_1 \otimes M_2}$.

But $q\left(\frac{E}{q^{-1}(F)}\right) = \frac{E}{F}$. Thus F is closed off $\frac{E}{M_1 \otimes M_2}$.

Proposition (2.5):

Let $M_1 \otimes M_2$ be a Topological submodule of a Topological module E . Then $1-E / M_1 \otimes M_2$ is a Hausdorff if and only if $M_1 \otimes M_2$ is closed.

$2-E / (M_1 \otimes M_2)$ is discrete if and only if $M_1 \otimes M_2$ is open.

In the next Theorem, we use two Topological submodules as a tensor product to get a Topological quotient module.

Theorem (2.6):

Let $M_1 \otimes M_2, N_1 \otimes N_2$ are two Topological submodules of a Topological module E such that $M_1 \otimes M_2 \subset N_1 \otimes N_2$.

The Topological quotient module $(N_1 \otimes N_2) / (M_1 \otimes M_2)$ is congruent with a topology generated on subspace $\frac{N_1 \otimes N_2}{M_1 \otimes M_2}$ of $E / (M_1 \otimes M_2)$ by quotient Topological module $E / (M_1 \otimes M_2)$.

Proof:

Let $q: E \rightarrow E / M_1 \otimes M_2$ be a canonical map and $q': N_1 \otimes N_2 \rightarrow \frac{N_1 \otimes N_2}{M_1 \otimes M_2}$ be a quotient mapping.

Suppose that p is an open subset of $N_1 \otimes N_2$. Then $p = (N_1 \otimes N_2) \cap B$ such that B is open of E .

Since q is open, so $q(B)$ is open of $E / (M_1 \otimes M_2)$. Hence $q(N_1 \otimes N_2) \cap q(B)$ open in $q(N_1 \otimes N_2)$. Since N is a union of a coset of $M_1 \otimes M_2$, we obtain $qB \cap qN_1 \otimes N_2 = qB \cap N_1 \otimes N_2 = qp$.

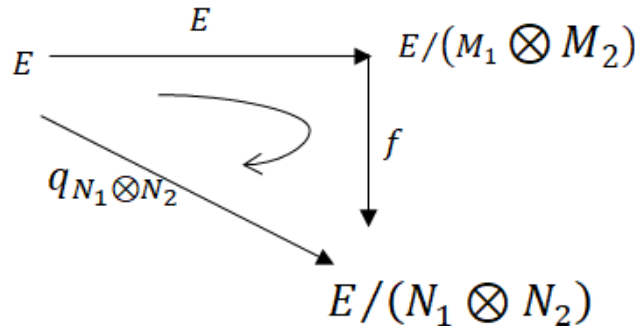
Thus $q(p)$ is open of $q(N_1 \otimes N_2)$

Proposition (2.7):

Let $M_1 \otimes M_2, N_1 \otimes N_2$ be a Topological submodule of Topological module E such that $M_1 \otimes M_2 \subset N_1 \otimes N_2$. Then surjective map $f: E/M_1 \otimes M_2 \rightarrow E/N_1 \otimes N_2$ which defined by $f(x + M_1 \otimes M_2) = x + N_1 \otimes N_2 \forall x \in E$ is continuous and open.

Proof:

By the diagram:



Since $f \circ q_{M_1 \otimes M_2} = q_{N_1 \otimes N_2}$ and $q_{N_1 \otimes N_2}$ is continuous, so f is continuous and hence f is open.

Proposition (2.8):

Let $M_1 \otimes M_2$ be a Topological submodule of a Topological module E . If V is a neighbourhood of zero in E , then $q(V)$ is a neighbourhood of zero in $E/(M_1 \otimes M_2)$ and if F is a system of principle neighbourhood of the zero in E , then of $q(F)$ is a system of a neighbourhood of the zero in $E/(M_1 \otimes M_2)$.

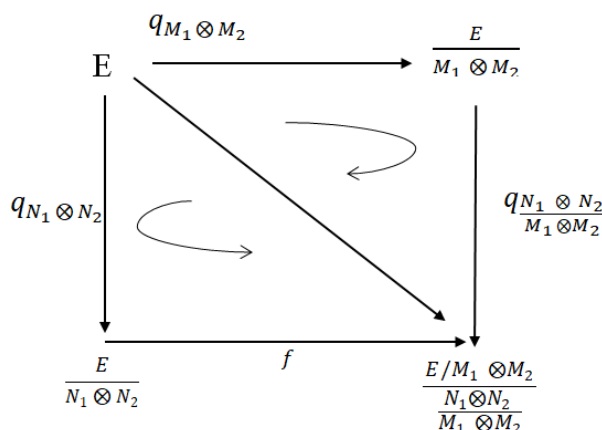
Proposition (2.9):

Let $M_1 \otimes M_2$ be a Topological submodule of E and let $N_1 \otimes N_2$ be another Topological submodule containing $M_1 \otimes M_2$. Then the canonical map

$$f: \frac{E/M_1 \otimes M_2}{N_1 \otimes N_2/M_1 \otimes M_2} \rightarrow \frac{E}{N_1 \otimes N_2}$$

is homeomorphism.

Proof:



Since: $f \circ q_{\frac{N_1 \otimes N_2}{M_1 \otimes M_2}} \circ q_{M_1 \otimes M_2} = q_{N_1 \otimes N_2}$ And since $q_{\frac{N_1 \otimes N_2}{M_1 \otimes M_2}} \circ q_{M_1 \otimes M_2}$ and $q_{N_1 \otimes N_2}$ are open continuous and onto, then f is open continuous and hence f is homeomorphism.

Definition (2.10):

Let E, E' be a Topological module. The mapping $f: E \rightarrow E'$ is called regular embedding if it's the embedding mapping and $f(E)$ is open of E' .

Remark (2.11):

1. If $M_1 \otimes M_2$ is a Topological submodule of a Topological module E . Then the mapping $i: M_1 \otimes M_2 \rightarrow E$ is one to one shape similarity Topological module.
2. A mapping $f: E \rightarrow E/(M_1 \otimes M_2)$ is a shape similarity topography module.
3. In (2.7) a map $f: E/(M_1 \otimes M_2) \rightarrow E/N_1 \otimes N_2$ is one to one shape similarity Topological module.
4. In (2.9) a mapping $f: \frac{E/M_1 \otimes M_2}{N_1 \otimes N_2/M_1 \otimes M_2} \rightarrow \frac{E}{N \otimes N_2}$ is a homeomorphic Topological module.

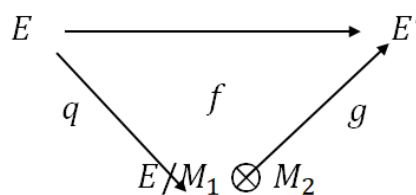
Proposition (2.12):

Let E, E' be a top. Module and let f is a shape similarity module from E to E' . If $M_1 \otimes M_2$ is a submodule of E contained in $\text{Ker}(f)$ and if $g: E/(M_1 \otimes M_2) \rightarrow E'$ is a shape similarity module such that $g \circ q = f$, then

1. g is continuous if and only if f is continuous.
2. g is open if and only if f is open.
3. g is a homeomorphism module if and only if f is an onto continuous shape similarity module and open.

Proposition: (2.13).

Let E, E' are two Topological modules, $M_1 \otimes M_2$ be a Topological submodule of E and let f continuous shape similarity from E to E' such that $M_1 \otimes M_2 \subseteq \text{Ker } f$. Then, at that point, there exists an exceptional



Proof:

In algebraic the homeomorphism $g: E/(M_1 \otimes M_2) \rightarrow E'$ exists and a unique and from Proposition: (2.12), $g(1)$ is continuous and hence it is a continuous shape similarity.

Remark (2.14):

- 1) In Proposition: (2.13), if $f: E \rightarrow E'$ is onto shape similarity module and continuous, then $g: E/(M_1 \otimes M_2) \rightarrow E'$ is homeomorphism Topological if $\text{Ker } f = M_1 \otimes M_2$.
- 2) The Intersection of two tensor products is also a tensor product.

Proposition (2. 15):

Let $M_1 \otimes M_2, N_1 \otimes N_2$ are a Topological submodule of Topological module E . Then the homeomorphism

$$g: \frac{(M_1 \otimes M_2) + (N_1 \otimes N_2)}{N_1 \otimes N_2} \rightarrow \frac{M_1 \otimes M_2}{(M_1 \otimes M_2) \cap (N_1 \otimes N_2)}$$

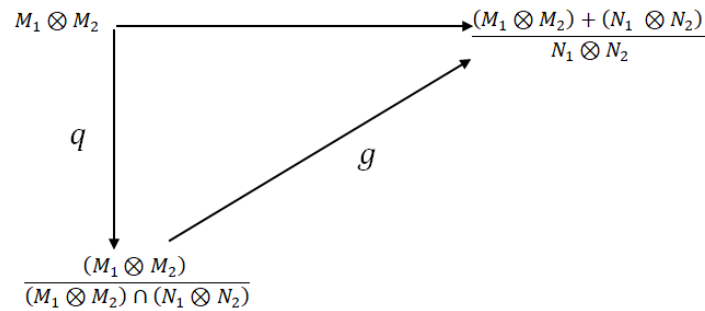
is continuous.

Proof: Assume that the following shape similarity module

$$V: M_1 \otimes M_2 + N_1 \otimes N_2 \rightarrow \frac{M_1 \otimes M_2 + N_1 \otimes N_2}{N_1 \otimes N_2}$$

is onto shape similarity module and $\text{Ker } V = N_1 \otimes N_2$

A restriction of this shape similarity on $M_1 \otimes M_2$ $\alpha = V: M \rightarrow \frac{M_1 \otimes M_2 + N_1 \otimes N_2}{N_1 \otimes N_2}$ and is a continuous homeomorphism module $\text{Ker } \alpha = M_1 \otimes M_2 \cap N_1 \otimes N_2$.



Also, α is continuous. Hence from Proposition (2.12), $g(1)$ is continuous.

4. Conclusion

In this paper we study a topological module and study a quotient of topological module, we use a new tool with quotient which is tensor product, several results have been obtained about tensor product of finite quotient module was written in the form of new propositions.

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