Development of an earthquake counteraction model based on a multicomponent analysis of the risk of tectonic movements in ore quarries

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ABSTRACT

The occurrence of earthquakes and the determination of their criteria is a physical and mathematical problem that allows predicting the onset of adverse consequences and eliminating potential threats. This leads to the possibility as the development of deposits of various minerals in conditions of potential earthquakes in seismically hazardous areas. This, in turn, forms certain requirements. The novelty of the study is determined by the use of formative methods for determining potential hazardous technogenic situations of quarries. The author shows that the application of such technologies is becoming a rule, provided that valuable resources for economic turnover are extracted. It is shown that the use of this type of resources is based on the use of technologies for predicting the ore flows in a quarry. The author determines the career streams of minerals and the adjustment of technological activities in the case of hazardous seismic events. The forecast structure is determined. The practical significance of the study is determined by the possibility of ensuring technological safety in the development of criteria for protection against the tectonic movements. The possibility of additional forecasting allows relying not only on long-term observation data but also on the predictive phenomena.

Keywords: Ore flow, Concentration plant, Earthquakes, Forecast, Tectonic movements, Risk, Mining

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1. Introduction

The modern iron ore quarry is a complex system with a developed hierarchical structure, the main purpose of which is ore mining [1]. By a general definition, this class includes systems with relatively independent, autonomous behaviour of subsystems with high internal activity and selectivity, the purposefulness of the entire system operation [2]. Such systems are open, in constant interaction with the environment, they are fundamentally capable of dealing with various tasks under various conditions, including in tectonic conditions to prevent seismic risk [3]. Based on the main purpose of the quarry operation, the requirements for the quality of the extracted raw materials are formed. The distribution of the content of the useful components in the quarry face is uneven and often chaotic [4]. At the same time, the ore flow of the quarry must have a certain value of the content of the useful component, which does not go beyond the range value at any time.
Studies and papers devoted to the problem of a potential seismological hazard substantiate in detail the initial calculation of the load on the quarry face, taking into account the qualitative characteristics, but in actual conditions, the dynamics of quality fluctuations in the ore flow and the actual, not calculated content of the useful component are not taken into account [5]. Modern control engineering considers the synthesis of a control system for a complex object based on its physical and mathematical model [6]. Therefore, the construction of a mathematical model for the control of potential seismological hazard is necessary [7]. In this case, the mathematical model serves control purposes, which determines its features [8]. It is necessary to develop a functional diagram of seismic hazard accounting with subsequent delivery to the ore processing plant [9]. Following the scheme, the ore is mined in the quarry faces, then it is transported to the place of its blending, separated as a summation unit, after that it enters the ore processing plant [3]. Based on the general tasks of the quality control system, it is necessary to determine the main indicators, on which their decision depends [10]:

1. content of the useful component in the quarry faces;
2. volume of reserves prepared for excavation;
3. the level of costs for the delivery of ore from the quarry faces;
4. optimal volumes of ore delivered from the quarry to the disposal point (calculated values of the blend).

According to the general definition, observability is understood as the availability of information that is necessary to solve the control problem. Applying modern terminology, we can say that to solve the control problem, it is necessary to conduct appropriate monitoring, that is, to test the quarry face for the content of the useful component. This leads to the conclusion that reliable and timely quality control is necessary for the normal ore flow functioning. The content of the useful component in the ore is characterised by stochasticity, determined by the characteristics of the developed deposits. As a consequence, the content of the useful component depends on time and should be considered as a random process.

2. Material and methods

Modern control engineering is the logical result of the development of general systems theory [11]. Currently, considerable attention is paid to a systematic approach to solving various problems [12]. The methodological advantage of this approach is the consideration of the problem being solved in a complex, that is, as a system [13]. According to the functional diagram, the input influence \( \bar{X} \) arrives at the input of the controlled object, which includes controlled and operated variables [14]. In addition, uncontrolled disturbances \( \bar{W} \) affect the controlled object [15]. The output of the controlled object is indicated by a variable \( \bar{Y} \). There is a dependence for the controlled object [16] (Eq. 1):

\[
F(\bar{X}, \bar{W}) = \bar{Y}
\]  

(1)

The dependence is generally unknown and is determined by uncontrolled tectonic disturbances \( \bar{W} \) [17]. To find the functional relationship between the input and output variables of the controlled object, information on the input and output variables is supplied to the block of the mathematical model, which implements the construction of the symbolic-form model of the controlled object [18]. Using the constructed symbolic model of the controlled object, a control algorithm for the controlled object is formed in the “Control Algorithm” block, which allows solving the problem [19]. It should also be noted that the presence of uncontrolled disturbances acting on the controlled object necessitates the quality analysis for the controlled object for further adaptation of the symbolic model to the changed conditions [20-25].

A symbolic model is necessary to control the described process of the formation of a general open-pit ore flow and represents a record of the requirements that must be met by the raw material quality control system [26-35]. Let us introduce the following designations: \( N \) – the number of mining quarry faces in the quarry; \( m_i \) –
amount of ore delivered by transport from the $k$-th face, $t_k$; $c_k$ – content of the useful component in the ore delivered from the $k$-th quarry face. The total volume of ore flow is determined by the capabilities of the ore processing plant and is a given value (Eq. 2):

$$\sum_{k=1}^{N} m_k = m_0$$  \hspace{1cm} (2)

where $m_0$ – volume of ore for processing at the input of the ore treatment plant, $t$.

In turn, according to the technological requirements, the content of the useful component in the ore after blending must correspond to a given value, that is, there is uniformity in the mined ore and in the generated ore flow (Eq. 3):

$$\sum_{k=1}^{N} c_k m_k = m_0 c_0$$  \hspace{1cm} (3)

where $c_0$ – specified content in the ore after blending.

A natural requirement is also to limit the volumes of ore delivered by transport from the quarry face in the form of local ore flow (Eq. 4):

$$0 \leq m_k \leq \bar{m}_k, (k = 1, 2, \ldots, N)$$  \hspace{1cm} (4)

where $\bar{m}_k$ – maximum possible volume of ore delivered by transport from the $k$-th face, $t$.

Formulas (2), (3) and (4) represent the mathematical model included in the control system for the establishment of ore flow in the open pit [36-44]. At the same time, another definition of the mathematical model is possible, in which condition (3) is replaced by restrictions on fluctuation amplitude in the ore after blending, i.e., after the establishment of ore flow (Eq. 5):

$$c_{\min} m_0 \leq \sum_{k=1}^{N} c_k m_k \leq c_{\max} m_0$$  \hspace{1cm} (5)

where $c_{\min}, c_{\max}$ – the smallest and largest allowable values in the established ore flow. For the operation of control engineering, certain requirements are put forward, which include observability, controllability and accessibility.

3 Results and discussion

To optimise the discreteness of the measurement of the iron content in the ore, it is preferable to choose such a minimum time interval at which the measurements forming the time series would be independent of each other [45-54]. Let us assume that measurements of the volume of useful component are carried out in the open pit at the moments of time $t_1, t_2, \ldots, t_k, \ldots, t_m$, which are denoted by $c(t_1), c(t_2), \ldots, c(t_k), \ldots, c(t_m)$. Due to the fact that the potential seismological hazard is measured at a fixed time interval, the time series describing the iron content in the ore is a sequence of grade values (Eq. 6):

$$c_1, c_2, \ldots, c_k, \ldots, c_M$$  \hspace{1cm} (6)
In the general case, the obtained time series (6) is not stationary [55-63]. In order to make it stationary, it is necessary to carry out a filtering procedure, which consists in taking the differences (Eq. 7):

$$\nabla c_k = c_k - c_{k-1} \quad (7)$$

As a rule, the obtained time series is stationary [64-71]. If the received series is not stationary, the difference operation is applied to it again. In the final result, as practice shows, the resulting time series becomes stationary. Further, for the obtained time series, sample autocorrelations are calculated (Eq. 8):

$$r_i = \frac{\sum_{k=1}^{M-1} (z_k - \bar{z})(z_{k+i} - \bar{z})}{\sum_{k=1}^{M} (z_k - \bar{z})^2} \quad (8)$$

where $z_k = \nabla^d c_k$ – difference of the $d$-th order, $\bar{z} = \frac{1}{M} \sum_{k=1}^{M} z_k$ – sample mean, $M$ – number of measurements of the useful components content.

Analysis of calculations carried out by formula (8) allows for the conclusion on the discreteness of the iron content measurement [72-83]. Taking into account that the sample autocorrelation (8) characterises the degree of connection between the values of variables, it is assumed that the values that are measured will be uncorrelated if the value of the sample autocorrelation satisfies the condition (Eq. 9):

$$|r_n| \leq 0.05 \quad (9)$$

Then the discreteness of measuring the content of the useful component in the ore will be the value (Eq. 10):

$$T = n \quad (10)$$

Thus, when measuring the iron content in the ore during its extraction in the quarry face with discreteness, which is determined by (10), the array of the obtained data will reflect the real dynamics of changes in the content of the useful component. The next condition for the operation of the ore flow quality control is controllability, which presupposes the presence of variables that make it possible to control the quality of iron ore mined in an open pit [84-93]. These variables are the volumes of ore that are delivered from different quarry faces to the blending point. The mathematical model (2), (3), (4) makes it possible to formulate the tasks of managing the ore flows quality. It is known that the content of the useful component in the ore, which is mined in an open pit, is a random value [94-102]. Further, it is advisable to switch to a dimensionless form of representation of the mathematical model by dividing equations (2), (3), (4) by the volume of ore processing $m_0$ (Eq. 11-13):

$$\sum_{k=1}^{N} \mu_k = 1 \quad (11)$$

$$\sum_{k=1}^{N} \mu_k C_k = C_0 \quad (12)$$

$$0 \leq \mu_k \leq \bar{\mu}_k, (k = 1, 2, ..., N) \quad (13)$$
where $C_k$ – content of tectonic matter in the ore from the $k$-th quarry face, which is a random variable, $C_0$ – content of tectonic matter in the ore flow after blending (Eq. 14):

$$\mu_k = \frac{m_k}{m_0}, \bar{\mu}_k = \frac{\bar{m}_k}{m_0}$$

(14)

It should be emphasised that conditions (11) and (13) impose an additional constraint (Eq. 15):

$$\sum_{k=1}^{N} \bar{\mu}_k \geq 1$$

(15)

If condition (5) is used instead of (3), then inequality occurs (Eq. 16):

$$c_{min} \leq \sum_{k=1}^{N} \mu_k C_k \leq c_{max}$$

(16)

where $c_{min}, c_{max}$ – the highest and the lowest content of tectonic matter in the ore after blending, i.e., expression sets the limit values for the range of quality fluctuations in the ore flow.

The mathematical model (11), (12), (14) contains variables $\mu_k$, that are control variables. The presence of free control variables makes it possible to set a functional that determines the condition for calculating the values of control variables [103-111]. It is advisable to choose a functional characterising the cost of mining and transporting ore from the quarry faces (Eq. 17):

$$L = \sum_{k=1}^{N} a_k \mu_k$$

(17)

where $a_k$ – values characterising the relative costs in the extraction and transportation of ore from the $k$-th quarry face. Obviously, the control should be built in such a way as to minimise the functional (17) (Eq. 18):

$$L = \sum_{k=1}^{N} a_k \mu_k \rightarrow \min_{\mu_k}$$

(18)

The mathematical model can be made more compact by using the matrix notation. If we introduce the notation (Eq. 19):

$$M = \begin{pmatrix} \mu_1 & \bar{\mu}_1 & C_1 & a_1 & 1 \\ \cdot & \cdot & . & . & . \\ \cdot & \cdot & . & . & . \\ \cdot & \cdot & . & . & . \\ \mu_N & \bar{\mu}_N & C_N & a_N & 1 \end{pmatrix}, \bar{M} = \begin{pmatrix} \bar{\mu}_1 \\ \cdot \\ \cdot \\ \cdot \\ \bar{\mu}_N \end{pmatrix}, C = \begin{pmatrix} C_1 \\ \cdot \\ \cdot \\ \cdot \\ C_N \end{pmatrix}, A = \begin{pmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_N \end{pmatrix}, E = \begin{pmatrix} 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{pmatrix}$$

(19)

then the mathematical model (11), (12), (14) will be represented in the form (Eq. 20-23):

$$E^T M = 1$$

(20)
\[ C^T M = C_0 \] (21)
\[ M \leq \tilde{M} \] (22)
\[ E^T \tilde{M} \geq 1 \] (23)

where \( E^T = (1.1.1) \), \( C^T = (C_1C_kC_n) \). In turn, inequality (13) is written in the form (Eq. 24):

\[ c_{\min} \leq C^T M \leq c_{\max} \] (24)

Condition (18) will have the form (Eq. 25):

\[ L = A^T M \rightarrow \min_{\tilde{M}} \] (25)

where \( A^T = (a_1a_ka_n) \).

As a result, the third condition for the ore flow quality control system operation is reachability, which provides for the implementation of the formulated control problem (11), (12), (14), or in matrix form (20), (21), (22), (23), (24), (25). As the analysis of the controllability of the quality control system of ore flows demonstrates, a situation is possible in which the solution of the control problem formulated above is impossible, that is, the formulation is incorrect from a mathematical standpoint. In this case, the constraints formed in the mathematical model are unrealised, that is, they form an empty set. If we introduce designations for the sets determined by the mathematical model of the ore flow quality control, then (Eq. 26):

\[ \Omega_1 = \{ M \mid E^T M = 1 \}, \Omega_2 = \{ M \mid C^T M = C_0 \}, \Omega_3 = \{ M \mid M \leq \tilde{M} \}, \Omega_4 = \{ \tilde{M} \mid E^T \tilde{M} \geq 1 \} \] (26)

then the reachability condition for this control system is written as (Eq. 27):

\[ \Omega_1 \cap \Omega_2 \cap \Omega_3 \cap \Omega_4 \neq \% \] (27)

Based on the fact that the content of matter in the faces depends on the tectonic rest of the rock mass, and is a random variable and therefore you can use numerical values, it is necessary to use the numerical characteristics of this random variable – the mathematical expectation and variance. Then the problem of ore flow quality control can be defined as (Eq. 28-32):

\[ L = \sum_{k=1}^{N} \alpha_{k} \mu_{k} \rightarrow \min_{\mu_{k}} \] (28)
\[ \sum_{k=1}^{N} \mu_{k} = 1 \] (29)
\[ \sum_{k=1}^{N} \mu_{k} M [C_{k}] = M [C_{0}] \] (30)
where $M[C_k], M[C_0]$ – mathematical expectations of the content of tectonic conditioned matter from the $k$-th quarry face and in the ore flow after blending, respectively; $s_k, s_0$ – values characterising the fluctuations range in the content of tectonic matter from the $k$-th quarry face and in the ore flow, respectively.

As the values characterising the fluctuations range in the content of the useful component, such numerical characteristics of random variables as standard deviation, mean absolute deviation, maximum absolute deviation, and the like can be selected. The mentioned control problem (28) - (32) is a linear programming problem and can be solved by the simplex method. The solution to the problem of the attainability of the problems of the control system, first of all, is associated with setting the value of the mathematical expectation of the content of the useful component and the value characterising the amplitude of quality fluctuations in the existing ore flow [112-119]. The result of solving the problem is the optimal volumes of ore mined in the quarry face and delivered to the point of the ore flow formation (Eq. 33):

$$\mu_k^0, \mu_k^0, \ldots, \mu_N^0 \quad (33)$$

The actual volumes of ore delivered from the quarry to the ore flow are determined by the formula (Eq. 34):

$$m_k = \mu_k^0 m_0, (k = 1, 2, \ldots, N) \quad (34)$$

If the actual volumes of ore (34) do not meet the requirements of the planned target, then in order to resolve the issue of accessibility, quality control of ore flows requires the attraction of additional conditions, which are informal and can only be solved with the help of technologists acting as experts. For example, consider the solution to the problem of ore quality control for four quarry faces in order to form an ore flow with a given content of tectonic conditioned matter. The control problem for the extraction of ore from four quarry faces was considered not for absolute, but for the relative volumes of ore mined in the form (Eq. 35-39):

$$L = a_1 \mu_1 + a_2 \mu_2 + a_3 \mu_3 + a_4 \mu_4 \rightarrow \text{min.} \quad (35)$$

$$\mu_1 + \mu_2 + \mu_3 + \mu_4 = 1 \quad (36)$$

$$M[C_1] \mu_1 + M[C_2] \mu_2 + M[C_3] \mu_3 + M[C_4] \mu_4 = M[C_0] \quad (37)$$

$$s_1 \mu_1 + s_2 \mu_2 + s_3 \mu_3 + s_4 \mu_4 \leq s_0 \quad (38)$$

$$\mu_k \leq \bar{\mu}_k, (1, 2, 3, 4) \quad (39)$$

For a specific solution, we set the numerical values presented in the Table 1.
At the same time, the average content of the useful component in the ore was chosen to be 0.35, and the standard deviation was 0.04, that is, the problem is solved in relative units. Substituting the data of Table 1 into (34) - (39), we obtain the control problem in the following form (Eq. 40-44):

\[ L = \mu_1 + 2\mu_2 + \mu_3 + 3\mu_4 \rightarrow \min. \quad (40) \]

\[ \mu_1 + \mu_2 + \mu_3 + \mu_4 = 1 \quad (41) \]

\[ 0.2\mu_1 + 0.3\mu_2 + 0.4\mu_3 + 0.45\mu_4 \leq 0.04 \quad (42) \]

\[ 0.2\mu_1 + 0.3\mu_2 + 0.4\mu_3 + 0.45\mu_4 \leq 0.04 \quad (43) \]

\[ \mu_1 \leq 0.3; \mu_2 \leq 0.6; \mu_3 \leq 0.2; \mu_4 \leq 0.4 \quad (44) \]

The solution of problem (40) - (44) by the numerical method gives the optimal volumes of ore for the formation of an ore flow with the given characteristics \( \mu_1^0 = 0.3; \mu_2^0 = 1; \mu_3^0 = 0.2; \mu_4^0 = 0.4 \). While the standard deviation is \( \sigma = 0.037 \). Knowledge of the distribution law of useful components after blending allows us to calculate the probability that it will be within the specified limits \((\underline{c}, \overline{c})\) (Eq. 45):

\[ P(\underline{c} < C < \overline{c}) = F(\overline{c}) - F(\underline{c}) \quad (45) \]

where \( F(c) \) – integral distribution function of the content of the useful component in the ore flow.

If the distribution law of the tectonic conditioned matter content in the ore flow is normal with the parameters of the mathematical expectation \( M[C] = c_0 \) and the standard deviation \( \sigma \), formula (45) will look like (Eq. 46):

\[ P(\underline{c} < C < \overline{c}) = \Phi\left(\frac{\overline{c} - c_0}{\sigma}\right) - \Phi\left(\frac{\underline{c} - c_0}{\sigma}\right) \quad (46) \]

where \( \Phi(x) \) – the Laplace function.

For the example considered above, we have \( c_0 = 0.35, \sigma = 0.037 \). Taking the range of fluctuations within \( \underline{c} = 0.3; \overline{c} = 0.4 \), according to formula (46) we find (Eq. 47):

<table>
<thead>
<tr>
<th>No. of quarry face</th>
<th>Average iron content in ore, rel. units</th>
<th>Root-mean-square deviation of iron content in the ore</th>
<th>Limitation on the volume of ore delivered from the quarry faces</th>
<th>Relative costs when delivering ore from the quarry faces</th>
<th>Optimal volumes of ore delivered from the quarry faces for blending</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.02</td>
<td>0.3</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>0.03</td>
<td>0.6</td>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
<td>0.04</td>
<td>0.2</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.45</td>
<td>0.05</td>
<td>0.4</td>
<td>3</td>
<td>0.4</td>
</tr>
</tbody>
</table>
An example of solving the problem of occurrence of potential seismological hazard indicated that with a normal distribution law, which is determined with the content of tectonic matter in the ore after blending. When mining in a quarry with four quarry faces, the content of tectonic matter is equal to 0.35, and with a standard deviation of 0.037, the probability that the content of the useful component in the formed ore flow will be within the planned range (0.3-0.4) is $P = 0.823$. The use of this technique for performing the blending calculation will make it possible to form an ore flow with a content of a useful component in a given range with high probability.

It is known that the stabilisation of the quality characteristics of the ore flow is a very urgent problem, the solution of which directly affects the quality of the final product – the heads. Fluctuations in the content of the useful component in the ore flow reduce the efficiency of the processing plant, negatively affecting the main technical and economic indicators of the mining and processing plant.

Let us consider a mathematical model of the ratio of the initial and additional volume of ore mass and the content of tectonic conditioned matter to stabilise the amplitude of fluctuations of the useful component in the ore flow. Stabilisation of the fluctuation amplitude of the useful component can be achieved by limiting its deviations from a given value, which are determined by the “minimax” coefficient, while the requirement $c_{\min} < c_1 < c_{\max}$ must be met, where $c_{\min}, c_{\max}$ – minimum and maximum values of the tectonic matter in the content.

Taking into account equation (Eq. 48):

$$c_{\text{on}} = \delta_0 c_0 + \left(1 - \delta_0\right) c_1,$$  \hspace{1cm} (48)

the condition $c_{\min} < c_1 < c_{\max}$ takes the form (Eq. 49):

$$c_{\min} < \delta_0 c_0 + \left(1 + \delta_0\right) c_1 < c_{\max}$$

where $\delta_0, \delta_1$ – relative proportion of ore with iron content $c_0, c_1$ in the ore flow. Taking into account that $\delta_0 = 1 - \delta_1$, inequality (49) takes the form (Eq. 50):

$$c_{\min} < \left(1 - \delta_1\right) c_0 + \delta_1 c_1 < c_{\max}$$

Inequality (50) is sequentially transformed into the form (Eq. 51):

$$C_{\min} < c_0 + \delta_1 \left(c_1 - c_0\right) < c_{\max} \cdot C_{\min} - c_0 < \delta_1 \left(c_1 - c_0\right) < c_{\max} - c_0$$

In what follows, it is necessary to consider two cases. In the first case $c_1 > c_{\max}$, then according to (4) inequality (3) will be fulfilled under the condition (Eq. 52):

$$\delta_1 \leq \frac{c_{\max} - c_0}{c_1 - c_0}$$

In the second case (Eq. 53):
Consider the process of mixing ore from the quarry faces during the formation of ore flow in a certain period of time. Assuming (Eq. 54):

\[
\delta_i \leq \frac{c_0 - c_{\text{min}}}{c_0 - c_1}
\]

(53)

inequality (52) is first sequentially transformed into the form (Eq. 55):

\[
\frac{M_1}{M_0 + M_1} \leq \frac{c_{\text{max}} - c_0}{c_1 - c_0} \Rightarrow \frac{M_1}{M_0 + M_1} \leq \frac{c_{\text{max}} - c_0}{c_1 - c_0} \Rightarrow \frac{M_0 + M_1}{M_1} \geq \frac{c_1 - c_0}{c_{\text{max}} - c_0}
\]

(55)

and finally (Eq. 56)

\[
M_0 \geq \frac{c_1 - c_{\text{max}}}{c_{\text{max}} - c_0} M_1
\]

(56)

Thus, if the ore flow with the total volume of the ore mass \( M_1 \) has the actual content of the useful component \( c_1 \), which is higher than the maximum specified value \( c_{\text{max}} \), then in order to fulfill the condition \( c_1 < c_{\text{max}} \) it is necessary to add from the reserve quarry face the volume of ore with the mass \( M_0 \) with the content of the useful component \( c_0 \), which is determined by the condition (56).

In the second case, when inequality (53) holds, condition (56) must be satisfied. In this case, inequality (53) is successively transformed into the form (Eq. 57):

\[
\delta_i \leq \frac{c_0 - c_{\text{min}}}{c_0 - c_1} \Rightarrow \frac{M_1}{M_0 + M_1} \leq \frac{c_0 - c_{\text{min}}}{c_0 - c_1} \Rightarrow \frac{M_0 + M_1}{M_1} \geq \frac{c_0 - c_1}{c_{\text{min}} - c_0}
\]

\[
\Rightarrow \frac{M_0 + 1}{M_1} \geq \frac{c_0 - c_1}{c_{\text{min}} - c_0} \Rightarrow \frac{M_0}{M_1} \geq \frac{c_0 - c_1 - 1}{c_{\text{min}} - c_0} \Rightarrow \frac{M_0}{M_1} \geq \frac{c_{\text{min}} - c_1}{c_0 - c_{\text{min}}}
\]

(57)

consequently (Eq. 58):

\[
M_0 \geq \frac{c_{\text{min}} - c_1}{c_0 - c_{\text{min}}} M_1
\]

(58)

Thus, if an ore flow with a mass \( M_1 \) has a content of a useful component \( c_1 \), that is below the minimum content of \( c_{\text{min}} \), then in order to fulfill the opposite inequality \( c_1 > c_{\text{min}} \). When forming an ore flow, it is necessary to add from the reserve quarry face a volume of ore with a mass \( M_0 \) with a content of \( c_0 \), determined by condition (58).
Differentiating (56) and (58) in time, we find the conditions imposed on the volumes of ore with different contents of tectonic conditioned matter. In the case when \( c_1 > c_{\text{max}} \), the inequality obtained by differentiating with respect to time expression (56) must be satisfied (Eq. 59):

\[
Q_0 \geq \frac{c_1 - c_{\text{max}}}{c_{\text{max}} - c_0} Q_1
\]

In the presence of (52), the inequality obtained by differentiating with respect to time (58) (Eq. 60):

\[
Q_0 \geq \frac{c_{\text{min}} - c_1}{c_0 - c_{\text{min}}} Q_1
\]

The above conditions (56) and (58) can be represented in the general form (Eq. 61)

\[
M_0 = \begin{cases}
\frac{c_{\text{min}} - c_1}{c_0 - c_{\text{min}}} M_1 & c_1 < c_{\text{min}} \\
0 & c_{\text{min}} \leq c_1 \leq c_{\text{max}} \\
\frac{c_1 - c_{\text{max}}}{c_{\text{max}} - c_0} M_1 & c_1 > c_{\text{max}}
\end{cases}
\]

If we differentiate (61) in time, we obtain the formula for determining the value of the additional volume of ore required to solve this problem (Eq. 61):

\[
Q_0 = \begin{cases}
\frac{c_{\text{min}} - c_1}{c_0 - c_{\text{min}}} Q_1 & c_1 < c_{\text{min}} \\
0 & c_{\text{min}} \leq c_1 \leq c_{\text{max}} \\
\frac{c_1 - c_{\text{max}}}{c_{\text{max}} - c_0} Q_1 & c_1 > c_{\text{max}}
\end{cases}
\]

As in the previous case, the transition from inequalities (56) - (60) to equations (61) makes it possible to use the minimum initial volume of ore in the ore flow \( M_0 \) and an additional volume of ore \( Q_0 \) to solve this problem of stabilising amplitude fluctuations in quality. Using the Heaviside function, formulas (61) can be respectively written in a compact form (Eq. 62-63):

\[
M_0 = \frac{c_{\text{min}} - c_1}{c_0 - c_{\text{min}}} M_1 \left(1 - \eta \left(c_1 - c_{\text{min}}\right)\right) + \frac{c_1 - c_{\text{max}}}{c_{\text{max}} - c_0} M_1 \eta \left(c_1 - c_{\text{max}}\right)
\]

\[
Q_0 = \frac{c_{\text{min}} - c_1}{c_0 - c_{\text{min}}} Q_1 \left(1 - \eta \left(c_1 - c_{\text{min}}\right)\right) + \frac{c_1 - c_{\text{max}}}{c_{\text{max}} - c_0} Q \eta \left(c_1 - c_{\text{max}}\right)
\]

In nondimensional form, the formulas will look like (Eq. 64-65):

\[
\mu = \frac{G_{\text{min}} - G_0}{1 - G_{\text{min}}} \left(1 - \eta \left(G_1 - G_{\text{min}}\right)\right) + \frac{G_1 - G_{\text{max}}}{G_{\text{max}} - 1} \eta \left(G_1 - G_{\text{max}}\right)
\]
where (Eq. 66):

\[ \mathcal{G}_{\min} = \frac{c_{\min}}{c_0}, \mathcal{G}_{\max} = \frac{c_{\max}}{c_0}, \mathcal{G}_i = \frac{c_i}{c_0} \]  

The ratio of the volumes of ore, in this case, indicates the need to use a blend from the reserve quarry face, which has a grade that is equal to \( c_0 = 28\% \) in range of the useful component grade ratio from 0.71 to 1.08.

The solution to the problem of reducing fluctuations in the content of tectonic conditioned matter in the ore supplied for the ore processing plant should be based on stochastic control methods. The first step, in this case, is the study of a random process describing the iron content in the ore flow at the input, in order to predict the content of the useful component for further stabilisation of its fluctuations. Due to the discreteness of the information receipt on the content of tectonic matter in the ore flow at the input of the ore-processing plant, the random process describing the content of the useful component can be considered as a discrete time series. When considering this series, the most effective is the use of parametric mathematical models. Parametric methods for this task are the most effective, since they use fewer parameters than nonparametric ones (for example, spectral analysis). It should be noted that parametric models during their construction require more complete a priori information about the time series; in particular, it is necessary to choose the structure of the time series model. With this approach, the application of the statistical theory of estimation and the theory of testing statistical hypotheses is justified.

Let the final ore flow enter the input. We designate the content of tectonic matter, which was measured at the moments of time \( t_1, t_2, \ldots, t_k, \ldots, t_N \), with \( c(t_1), c(t_2), \ldots, c(t_k), \ldots, c(t_N) \). Considering that the content of tectonic matter is measured at a fixed time interval, the time series describing the content of a useful component in the ore is a sequence of contents values, which are the designation of the measured content of tectonic matter at equidistant points in time.

In the future, if taken as a reference point, and \( \mathcal{A} \) as a unit of time, then it can be considered as the content of a useful component at a time \( t \). A sequence \( c_1, c_2, \ldots, c_k, \ldots, c_N \) is a random (non-deterministic) time series. Therefore, its future values can only be described using statistical distributions. To construct a stochastic model of a time series \( c_1, c_2, \ldots, c_k, \ldots, c_N \) for the prediction and regulation of the ore flow quality indicators, it seems appropriate to use a parametric model called the process of autoregression-integrated variable mean order (Eq. 67):

\[ \varphi(B)c_t = \theta(B)a_t \]  

where: \( \varphi(B) = \phi(B)(1-B)^q \), \( \phi(B) = 1 - \phi_1B - \phi_2B^2 - \cdots - \phi_pB^p \) – autoregressive operator of \( p \) order, \( \theta(B) = 1 - \theta_0B - \theta_1B^2 - \cdots - \theta_qB^q \) – variable mean operator of \( q \) order, \( \phi_1, \phi_2, \ldots, \phi_p, \theta_1, \theta_2, \ldots, \theta_q \) – numerical parameters estimated from statistical data. \( B \) – shift back operator, which is defined as \( Bc_t = c_{t-1} \); \( a_t \) – sequence of independent impulses, that is, random variables, with a normal distribution, which has zero mean and variance \( \sigma_a^2 \) (white noise)

In practice, the number of parameters that need to be estimated from statistical data should be minimal, which is the principle of economy. Thus, the time series model should be not only adequate, but also economical. A visual analysis of the time series allows us to conclude that it is nonstationary. This series does not have a fixed
average value around which it fluctuates. At the same time, the series can be reduced to the possibility of describing a stationary model, using the fact that \( d \) – the difference of this series is a stationary stochastic series.

Table 2 shows the monthly average values of the tectonic conditioned matter (magnetic iron) content in the final ore flow at the input and the first differences are calculated. Analysis of the values of the first differences, which are calculated by the formula (Eq. 68):

\[
z_i = \nabla c_i = c_i - c_{i-1}
\]  

(68)

shows that the time series that is obtained, and which is shown in Table 2, can be considered stationary.

Table 2. Average monthly values of the magnetic iron content in the final ore flow at the ore-processing plant

<table>
<thead>
<tr>
<th>Months</th>
<th>Average monthly content of magnetic iron in ore, %</th>
<th>The first differences in the content of magnetic iron in ore, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>23.6</td>
<td>-0.1</td>
</tr>
<tr>
<td>3</td>
<td>23.5</td>
<td>-0.1</td>
</tr>
<tr>
<td>4</td>
<td>23.2</td>
<td>-0.3</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
<td>-0.2</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>23.1</td>
<td>0.1</td>
</tr>
<tr>
<td>11</td>
<td>23.2</td>
<td>0.1</td>
</tr>
<tr>
<td>12</td>
<td>23.2</td>
<td>0</td>
</tr>
</tbody>
</table>

To find the structure of a time series model composed of the first differences (67), it is necessary to calculate the sample autocorrelations by the formula (Eq. 69):

\[
r_k = \frac{\sum_{t=1}^{N-k} (z_t - \bar{z})(z_{t+k} - \bar{z})}{\sum_{t=1}^{N} (z_t - \bar{z})^2}
\]  

(69)

Table 3 shows the results of calculations by the formula (69).

Table 3. Selective autocorrelations

<table>
<thead>
<tr>
<th>( k )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_k )</td>
<td>1</td>
<td>0.623</td>
<td>0.231</td>
<td>0.274</td>
</tr>
</tbody>
</table>

Analysis of the calculations of sample autocorrelations values allows us to conclude that as the structure of the time series model (68), it is possible to choose the process of second-order autoregression (Eq. 70):

\[
z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + a_t
\]  

(70)

where \( \phi_1, \phi_2 \) – numeric parameters.
For an a priori search for the parameters included in (70), it is necessary to use the Yule-Walker equations, which in this case take the form (Eq. 71):

\[
\begin{cases}
\phi_1 + \phi_2 r_1 = r_1 \\
\phi_1 r_1 + \phi_2 = r_2
\end{cases}
\] (71)

Having solved the system of equations for \(\phi_1\) and \(\phi_2\), we find (Eq. 72):

\[
\phi_1 = \frac{r_1 (1 - r_2)}{1 - r_1^2}, \quad \phi_2 = \frac{r_2 - r_1^2}{1 - r_1^2}
\] (72)

However, for the second-order autoregression model to be stationary, it is necessary that the parameters \(\phi_1, \phi_2\) satisfy the inequalities (Eq. 73):

\[
\phi_2 + \phi_1 < 1; \quad \phi_2 - \phi_1 < 1; \quad -1 < \phi_2 < 1.
\] (73)

Substituting the values of the sample autocorrelations presented in Table 3 into formulas (72), we find (Eq. 74):

\[
\phi_1 = 0.776, \quad \phi_2 = -0.251
\] (74)

The quantities \(\phi_1\) and \(\phi_2\), given in (74) satisfy the stationarity condition for the synthesised model (73). Substituting the parameters \(\phi_1\) and \(\phi_2\) into the second-order autoregression equation (70), we obtain (Eq. 75):

\[
z_i = 0.776z_{i-1} - 0.251z_{i-2} + a_i
\] (75)

Substituting (68) into equation (75), we finally obtain an equation that models the time series presented in Table 3

\[
c_i = 1.776c_{i-1} - 1.026c_{i-2} + 0.251c_{i-3} + a_i
\] (76)

To refine the found parameters of the time series model, it is necessary to use the least squares method. For this purpose, a function is compiled, which is the sum of the squares of the deviations (Eq. 77):

\[
\Phi(b_1, b_2, b_3) = \sum_{i=1}^{N} a_i^2 = \sum_{i=1}^{N} (c_i - b_1 c_{i-1} - b_2 c_{i-2} - b_3 c_{i-3})^2
\] (77)

where \(b_1, b_2, b_3\) – unknown parameters. Since unknown parameters enter the equation linearly, minimisation of function (77) leads to a system of three linear algebraic equations with respect to the required parameters \(b_1, b_2, b_3\) (Eq. 78):
Substituting the values of the content, taken from the Table 2, into the system of linear equations (78) and solving it, we obtain (Eq. 79):

\[ b_1 = 1.732; b_2 = -0.972; b_3 = 0.241 \]  

Substituting the results into the equation, we obtain a time series model that practically coincides with the one found with the a priori setting (Eq. 80):

\[ c_t = 1.732c_{t-1} - 0.972c_{t-2} + 0.241c_{t-3} \]  

The graph of monthly average withdrawal of magnetic iron from ore, projected one month ahead, is determined by the formula (Eq. 81):

\[ G_t(1) = 1.732c_t - 0.972c_{t-1} + 0.241c_{t-2} \]  

Thus, the investigation of a random process describing the content of a useful component in ore flow using a mathematical apparatus makes it possible to predict the qualitative characteristics of the ore flow for a month in advance and further stabilise the amplitude fluctuations. Analysis of the graphs, which are constructed by the formula (81), indicates sufficient convergence of the real and modelled values of the iron content in the base ore at the input of the ore-dressing plant. The formation of an ore flow with the specified quality indicators is carried out on the basis of solving the batching problem, which consists in the distribution of production volumes between quarry faces with different indicators of the content of the useful component for the entire period of the work shift. When solving this problem, it is a priori considered that the quarry faces start simultaneously and work the entire shift without interruption. The second tolerance for solving the batching problem is the constant average value of the content of the useful component in each individual quarry face during the entire shift.

In fact, during the change, there are significant deviations from the initial blending problem, which directly affects the quality of the ore flow. These deviations are due to the non-simultaneous start of work, equipment breakdowns during the shift (complete stopping of the quarry face), as well as the existing dynamics of changes in the content of the useful component. In this regard, in order to eliminate the imbalance between the calculated and actual value of the content of the useful component during the formation of the final ore flow, it is necessary to perform a corrective recalculation in solving the batching problem. After the adjustment, the loads on quarry faces are redistributed taking into account the actual state of equipment operability and the current value of the useful component.

The content of the useful component in each quarry face is determined by operational testing with data transmission to a central server. In this case, the interfering factors are the delay in the transmission of information and the difference in timing of various quarry faces testing. Based on the difference between the previous and subsequent average of the content, it is possible to determine whether an adjustment is necessary. To provide the central server with reliable and timely information about the quality characteristics in the faces,
it is necessary to determine the interval for testing, based on the existing array of statistical data on the content of the useful component and the dynamics of its change.

In accordance with the general approach for the numerical analysis of the content of the useful component in the ore, the measurement of the content \( c(t) \) as continuous in time must be carried out at a certain fixed interval. The sampled values obtained this way will be used for calculations.

Discretised values can be considered as the result of multiplying the initial continuous time series \( c(t) \) by a time series \( i(t) \) consisting of an infinite series of delta functions (Eq. 82):

\[
i(t) = \sum_{n=-\infty}^{\infty} \delta(t - n)
\]  

where (Eq. 83)

\[
\delta(t) = \begin{cases} 
0, & t \neq 0 \\
\infty, & t = 0
\end{cases}
\]  

This gives a pulse-modulated content of the useful component in the ore (Eq. 84):

\[
c_i(t) = c(t)i(t)
\]  

Then, using the convolution theorem, we find a representation of formula (84) in the frequency domain (Eq. 85):

\[
C_i(f) = \int_{-\infty}^{\infty} C(f - g)I(g)dg
\]  

where \( C(f), I(f) \) – Fourier transform \( c(t) \) and \( i(t) \), respectively. Considering that (Eq. 86):

\[
I(g) = 1 \sum_{n=-\infty}^{\infty} \delta(g - n)
\]  

formula (85) is transformed into the form (Eq. 87):

\[
C_i(f) = \int_{-\infty}^{\infty} C(f - g) \sum_{n=-\infty}^{\infty} \delta(g - n) dg = 1 \sum_{n=-\infty}^{\infty} C(f - n)
\]  

Equality (87) shows that the pulse-modulated content of the useful component in the ore \( c(t) \) is the Fourier transform with a period 1. If \( C(f) \) vanishes at \(|f| \geq 1/2\), then \( C_i(f) \) is a periodically repeating function \( C(f) \). This means that we can recover \( C(f) \) from \( C_i(f) \), by multiplying \( C_i(f) \) by \( H(f) \), where (Eq. 88):

\[
H(f) = \begin{cases} 
|f| \leq \frac{1}{2} \\
0, |f| > \frac{1}{2}
\end{cases}
\]  

Function (88) is a spectral window and is found as the Fourier transform of the time window (Eq. 89):
Since multiplication in the frequency domain corresponds to convolution in the time domain, then, taking into account (89), we obtain (Eq. 90):

\[ c(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \sin(\pi u) c_i(t - u) du \]  

(90)

The frequency \( f_N = 1/2 \) is referred to as the Nyquist frequency and is the highest frequency that can be detected from data acquired at sample intervals. To analyse the time series, composed of tectonic matter content in ore, in order to find the optimal measurement period, we investigate the spectral properties of this series, which are based on the properties of the autocorrelation function and the normalised spectrum. If the autocorrelation function is estimated by the formula (Eq. 91):

\[ r_k = \frac{d_k}{d_0} \]  

(91)

where (Eq. 92):

\[ d_k = \frac{1}{N} \sum_{i=1}^{N-k} (c_i - \bar{c})(c_{i+k} - \bar{c}), \bar{c} = \frac{1}{N} \sum_{i=1}^{N} c_i, k = 0, 1, 2, ..., K, \]  

(92)

then the sample normalised spectrum is determined using autocorrelations (91) as follows (Eq. 93):

\[ S(f) = 2 \left( 1 + 2 \sum_{k=1}^{K} r_k \cos(2\pi fk) \right), 0 \leq f \leq 0.5 \]  

(93)

Thus, the sampled normalised spectrum is the Fourier cosine transformation of the sampled autocorrelation function. Analysing the plot of the sampled normalised spectrum (93), it is possible to estimate the distribution of the variance of the time series over frequencies.

If the dispersion value is concentrated at frequencies lower than the Nyquist frequency \( f_N \), then the testing rate can be increased. Let us consider an example of justifying the testing period of quarry faces based on the statistics of tectonic conditioned matter content. Table 4 shows the values of the sample autocorrelation functions calculated by the (92).

### Table 4. The value of sample autocorrelation functions

<table>
<thead>
<tr>
<th>No. of excavator</th>
<th>93</th>
<th>56</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.67</td>
<td>0.57</td>
<td>0.73</td>
</tr>
<tr>
<td>2</td>
<td>0.41</td>
<td>0.42</td>
<td>0.51</td>
</tr>
<tr>
<td>3</td>
<td>0.37</td>
<td>0.42</td>
<td>0.36</td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
<td>0.34</td>
<td>0.19</td>
</tr>
<tr>
<td>5</td>
<td>0.04</td>
<td>0.33</td>
<td>0.21</td>
</tr>
<tr>
<td>6</td>
<td>-0.21</td>
<td>0.43</td>
<td>0.20</td>
</tr>
<tr>
<td>7</td>
<td>-0.24</td>
<td>0.50</td>
<td>0.04</td>
</tr>
</tbody>
</table>

The analysis of the sampled normalised spectra, which will be performed below, will make it possible to estimate at what frequencies and how the dispersion of the series is concentrated.
4. Conclusions

Thus, based on the statistics of changes in the withdrawal of useful component from the quarry, taking into account the fluctuations in Fe$_{\text{mag}}$ in the quarry faces, the possibility of increasing the testing period without losing the necessary information on the iron content and without reducing the qualitative characteristics of the formed final ore flow, which will save money by testing in a quarry. This technique can be applied to all iron-ore quarries where there is relevant statistical material on the probability of unfavourable seismic events in the faces and the dynamics of its change.

Today, in iron-ore quarries, technical control services take measurements of the useful component content per shift, using the equipment base available in them, and are obtained primarily from the technical capabilities of their quality control devices. The period of taking information about the content of the useful component (testing) is not mathematically and technologically justified. A justified testing period will allow obtaining more reliable information about the actual value and dynamics of changes in the content of the useful component in the quarry, which will later be used to calculate the shift-daily task in order to form an ore flow with the specified quality parameters. The error in measuring the content of tectonic conditioned matter, as well as the features of technological processes during ore processing, indicate the possibility of taking into account changes accounting for quantitative changes in tectonically determined ore matter. This shows the possibility of taking into account the influence of tectonic stresses on the productivity of open pits with a certain discreteness in time and to consider the ore flow as a discrete time series.

References


