Periodicals of Engineering and Natural Sciences ISSN 2303-4521
Vol. 7, No. 1, June 2019, pp.242-248
Available online at: http://pen.ius.edu.ba

Trajectory optimization for mobile robots using model predictive control

Nicolae Pop[1], Luige Vlădăreanu [*1], Marcel Migdalovici[1], Adrian I. Pop [2] and Mihai Radulescu[1]
[1] Department of Mechatronics, Romanian Academy - Institute of Solid Mechanics, Bucharest, Romania
[2] Technical University of Cluj-Napoca - North University Centre of Baia Mare, Romania

Article Info

Received Dec 18, 2018

Model predictive control
Real time robot control
Mobile robot control
Constrained optimization problem
Linear quadratic optimal control

ABSTRACT

The goal of this article is trajectory generation for biped robots based on Model Predictive Control (MPC) and the receding-horizon principle. Specifically, we want to minimize the error between the desired CoM and ZMP trajectory and the actual one and the cancellation of the shock gradient of the CoM and ZMP movements. Model predictive control (MPC) consist in a finite horizon optimal control scheme which uses a prediction model to predict vehicle response and future states, thus minimizing the current error and optimizing the future trajectory within the prediction horizon. The proposed algorithm will provide a trajectory of control inputs which will optimize the system states utilizing a quadratic form cost function similar to standard linear quadratic tracking. Specific to finite horizon control, the cost is summed over the finite prediction horizon of time length, rather than over an infinite time horizon. Many techniques have been proposed, developed, and applied to solve this constrained optimization problem for the mobile robots. With our aproach we try to investigate how is the MPC framework is applicable to trajectory generation for point-to-point problems with a fixed final time and to find a set of assumptions and methods that allow for real-time solutions.

Corresponding Author:

Luige Vladareanu,
Romanian Academy,
Institute of Solid Mechanics,
15 Constantin Mille street, 010141 Bucharest, Romania.
Email: luige.vladareanu@vipro.edu.ro

1. Introduction

Mobile robots, unlike other types of robots such as those with wheels or tracks, use similar devices for moving on the field like human or animal feet. Several researchers used switching techniques between the control laws needed at certain times in motion of the robot. An adaptive method to switch between different gain values used in tracking control on a motion trajectory for serial manipulators (Ouyang, et al., 2006).
Compliant movement control which is essentially the default force control based on position was suggested by Lawrence, Stoughton (1987) and Kazerooni, Waibel, Kim (1990). Salisbury (1980) presented a method for active control of the end-effector apparent stiffness of the robot in Cartesian space. In this method the reference position is used to control the contact force and no force reference points are used. Khalil's method (Khalil et al., 1983) method stands out among them, for the advantages they offer.

The first one allows inverse kinematics problems to be solved regardless of the values of robot geometric characteristics, for robots with six degrees of mobility which have three rotational kinematic couplings on concurrent axis or three translation kinematic couplings.

Because of the flexibility and that it has a solution for the inverse kinematics problem, this "decoupled" structure with three rotation couplings and concurrent axis is found in most robot models on the market. The position of the three axes intersection point is uniquely determined only by the q1, q2, q3 variables. Another advantage of the decoupled structure is allowing splitting and separate negotiating of the positioning and orientation. Paul's method as well as Lee and Elgazaar’s treat each case separately without generalizations.

Moustris and Tzafestas (Moustris &Tzafestas, 2010) used a fuzzy switch, outside the control loop, for switching from a reference trajectory to another for the motion control of a mobile robot. Nicolás and Sagüés (Nicolás &Sagüés et. al., 2008) a switching control based on the epipolar geometry presented, which has the purpose to switch between different captured images by a mobile robot for to compute its trajectory up to target.

These switching techniques and many others were used mainly to control between reference values (Vladareanu V. et al., 2013, 2014) or between constant values for a certain control law (Wang H.B. et al., 2015, Sandru O.I. et al., 2013).

In many robotic problems, it is desired to achieve a certain state of the robot at a given time. This will lead to a problem of tracking the reference signal when the desired state is specified as a function of time during the entire course (Pop &Vladareanu et al., 2018). If the desired state is discontinuous in time, we need to make a prediction between points, that is, a continuous transfer of the robot, from an initial state to the next state, and the robot movement must meet certain imposed restrictions. This behavior will be specified by an objective function.

Model predictive control is proposed to address this problem, a model designed to predict the behavior of the system by minimizing predicted tracking errors and control effort used to achieve restrictions on control inputs and state variables in a finite time horizon.

At each time sample, an optimal control input sequence is generated after solving the minimization problem. The first element of this control input sequence is applied to the system. Then the problem is resolved again at the next sampling time with the updated measurements and a shifted horizon.

In this MPC formulation, the following cost is minimized to determine the optimal sequence of commands uk in the prediction horizon length. A trajectory generator must be able to determine the action ((u(x,t)), that satisfies a set of state constraints ((xC(t)) or minimize a cost function ((F(x)), or both subject to initial state constraints ((x(t0)) and the predictive motion model $\dot{x}$(x,u,t).

For this, an objective function, a system model (differential equation of the motion), state restrictions, and a time horizon are required. Our goal is to use MPC as a point-to-point trajectory generation problem and to track the reference signal (desired trajectory). Figures 1.1 and 1.2 depicts the basic principle of MPC.

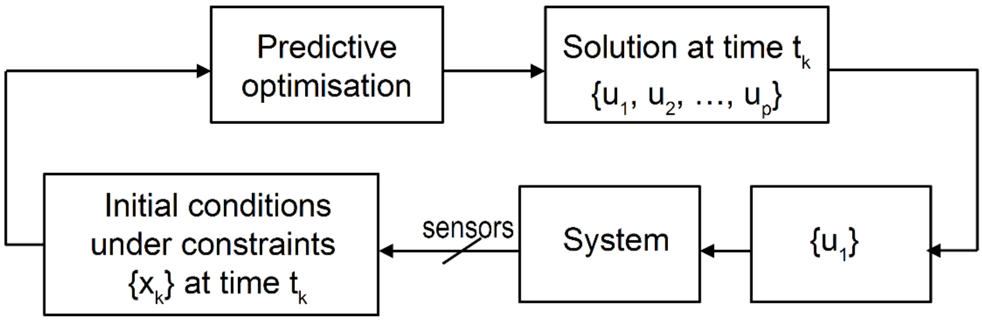

Figure 1.1. Principle of the Model predictive control (MPC) - block diagram

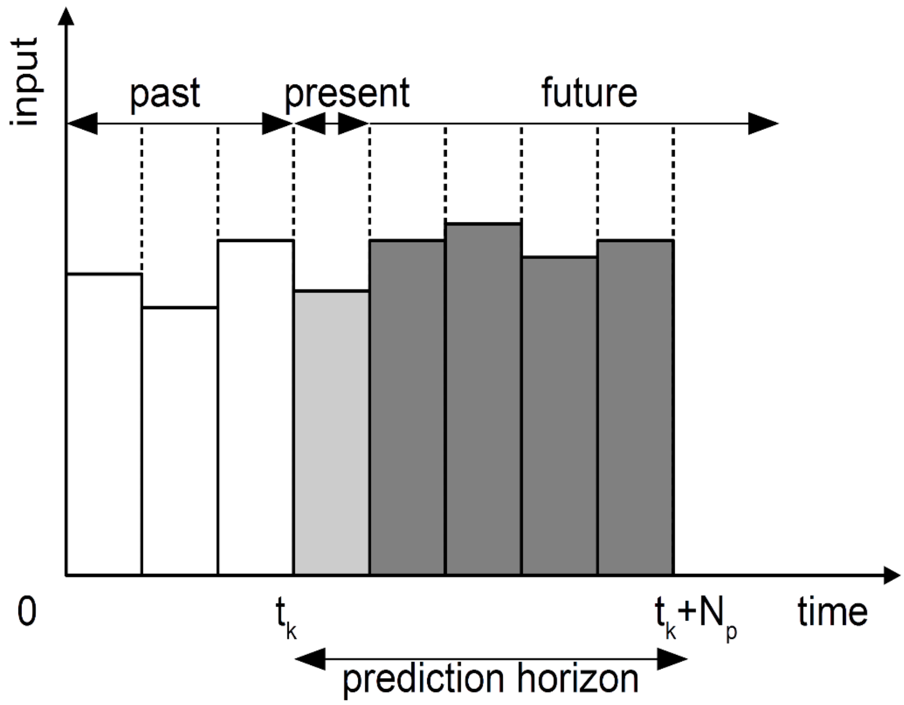

Figure 1.2. Principle of the model predictive control

2. Mathematical Formulation of the MPC

Starting from information (measurements) at time *t*, the controller predicts the dynamic behavior of the system on a predictive horizon $T_p=N_p\ xT_c$, where N_p is number of the pre-calculated optimal inputs $U=\{u_1, \ldots, u_{Np}\}$ and T_c is the control sampling time. We enter only the first input u_1 in the system, and at the next sampling period, the measurements are repeated. If there are disturbances than the whole process is repeated (this case is often encountered).

Let the following system of non-linear differential equations describing the dynamic system:

$$\dot{x}(t) = f(x(t), u(t)), \qquad x(0) = x_0 \tag{2.1}$$

Subject to the next constraints:

$$\begin{aligned} x(t) &\in X, \quad \forall t \geq 0 \\ u(t) &\in U, \quad \forall t \geq 0 \end{aligned} \tag{2.2}$$

Where $x(t) \in R^n$ and $u(t) \in R^m$ denote the vector of state and inputs, respectively, and $U = \{u \in R^m \mid u_{\min} \leq u \leq u_{\max}\}$ is the set of input constraints and $X = \{x \in R^n \mid x_{\min} \leq x \leq x_{\max}\}$ the set of state vector restrictions, with constant vectors $u_{\min}, u_{\max}$ and $x_{\min}, x_{\max}$.

The optimal control problem with the control input applied to the system is achieved by solving the optimal control problem over a finite time horizon at each time sample:

Find $$\hat{u}(.) = \min_{u(.)} J(x(t), u(.)) \tag{2.3}$$

subject to:

$$\begin{aligned} &\hat{u}(\tau) \in U, \forall \tau \in [t, t+T_p], \\ &x(\tau) \in X, \forall \tau \in [t, t+T_p]. \end{aligned} \tag{2.4}$$

With an additive constraint, due to the sampling control

Where
$$\begin{aligned} &\hat{u}_k(\tau) = \hat{u}(t+(k+1)T_c), \forall \tau \in [t+kT_c, t+(k+1)T_c], \\ &\forall k \in \{0, \ldots, N_p - 1\}. \end{aligned} \tag{2.5}$$

For the sequential case, let $\hat{u}_k : [t_0, t+T_p] \to U$ be the control sequences from the current time $t_0 \; to \; t+T_p$. The MPC is then defined as a minimization of the objective function J to find the optimal control sequence

$$\hat{u}_k^* = \min J(x_k(t), \hat{u}_k(.)) = \int_t^{t+T_p} F(x_k(\tau), \hat{u}_k(\tau)) d\tau \approx g \tag{2.6}$$

Subject to (2.1), (2.4) and (2.5).

3. Problem Formulation

3.1. Dynamic model of walking robot approximated by 3D LIPM

The complete dynamic model of a human robot is non-linear and complex. In order to generate the movement of the biped robot, it was considered that the approximation of the non-linear dynamic model with the linear pendulum 3D model (3D-LIPM) is good enough and additionally, the cost of calculation is low and convenient for tracking the online trajectory. Assuming that the robot's CoM (center of mass) is restricted to move on a horizontal plane with a altitude h constant, a set of decoupled equations that governs the CoM and ZMP (zero moment point) will result.

.

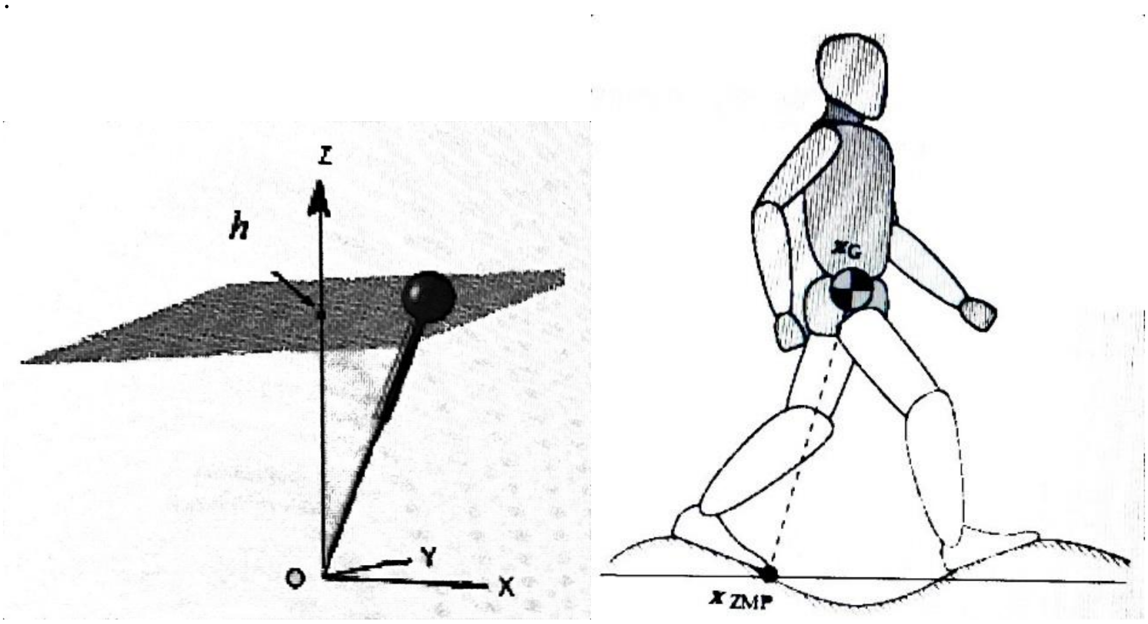

Figure 3.1. Biped legged system

Equations describing the trajectory of CoM(x, y, h), in the case of the 3D linear pendulum model, are given in (3.1):

$$\ddot{x} = \frac{g}{h}x + \frac{1}{mh}\tau_y; \qquad \ddot{y} = \frac{g}{h}y + \frac{1}{mh}\tau_x \tag{3.1}$$

where g is acceleration due to gravity, m is the mass of the pendulum, τ_x and τ_y are the moments around the x and y axes respectively. Considering 3D-LIPM with altitude h constant, the ZMP equations become (3.2):

$$z_x = -\frac{\tau_y}{mg}; \qquad z_y = -\frac{\tau_x}{mg} \tag{3.2}$$

where (z_x, z_y) are the coordinates of the ZMP on the flat floor. Substituting equations (3.2) in (3.1) we get the decoupled movement equations of ZMP:

$$z_x = x - \frac{h}{g}\ddot{x} \qquad z_y = y - \frac{h}{g}\ddot{y} \tag{3.3}$$

3.2. The dynamic system with discrete time of walking robot representing the movement in the x direction of CoM and ZMP (similar for the y direction) for 3D-LIPM

We will consider the shock in the direction x and y respectively as the input controller. The two trajectories of the CoM and the ZMP are discretized with cubic polynomials on portions, depending on the shocks $\dddot{x}$ and $\dddot{y}$, assumed constant on the samples of length T:

$$X_{k+1} = AX_k + B\Delta U_k \tag{3.4}$$

$$Z_k = CX_k \tag{3.5}$$

where:

$$A = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix} \quad \Delta U_k = \dddot{x}_k - \dddot{x}_{k-1} \quad C = \begin{bmatrix} 1 & 0 & -\frac{h}{g} \end{bmatrix} \quad k = 0,1,\cdots,N-1$$

$$X_k = \begin{bmatrix} x(kT) & \dot{x}(kT) & \ddot{x}(kT) \end{bmatrix}^T, \ddot{x}_k = \ddot{x}(kT), z_k \equiv z_x(kT)$$
$$Z_k = \begin{bmatrix} z(kT) & \dot{z}(kT) & \ddot{z}(kT) \end{bmatrix}^T$$

and T is the length of the time sample. Similarly, in the y direction.

3.3. Linear Quadratic optimal control for Model Predictive Control

We will define a square cost function on a finite N-step horizon:

$$J_0(X_0, \Delta U_0) = X_N^T P X_N + \sum_{k=j}^{N-1} (X_k - X_k^d)^T Q (X_k - X_k^d) + \Delta U_k^T R \Delta U_k \tag{3.6}$$

where X_k is the state vector at the time k, obtained from the state variable X_0=X(0) and from (3.4) we will consider the matrix system:

$$X_{k+1} = AX_k + B\Delta U_k \tag{3.7}$$

where $U_0=[U_0, ...,U_{N-1}]^T$ is the input sequence. We consider the problem of optimal control

$$J_0^*(X(0)) = \min_{\substack{U_0 \\ X_{k+1}=AX_k+B\Delta U_k,\ k=0,1,\cdots,N-1 \\ X_0=X(0)}} J_0(X(0), \Delta U_0) \tag{3.8}$$

Here we assume that the weighting penalizing matrices of the state variables are positively defined, $Q = Q^T \geq 0$, $P = P^T \geq 0$ and also the weight matrix of penalizing the inputs is positively defined $R = R^T$. By solving the optimal control problem (3.8), it means that we can minimize the error between the desired trajectory of the CoM and the ZMP and the real one. Through the penalty matrix Q we minimize the errors between the real and desired trajectory $\{X_k^d\}$. By means of the penalty matrix P, the robot is rebalanced in the final step N, and with the penalty matrix R it is desired cancellation the gradient of the shock in the direction x,

$\Delta U_k = 0 \Rightarrow \dddot{x}_k - \dddot{x}_{k-1} = const.$

We mention that the problem (3.8) can be solved globally or recursively using dynamic programming. Although our problem is linear, there are two reasons for addressing the second option. The first reason is that in the first version, the work arrays are of large size and the computational effort is larger and the second reason is that in the trajectory tracking problem, the system may be subject to unpredictable perturbations in the model so that the feedback law would be more accurately calculated because at each step of the time the observed state variable X(k) is used to determine the control action before the predicted variable X_k at time t=0.

4. The difference between MPC and LQR

The problem of predictive control formulated as a problem of modeling the future trajectory for dynamic systems with continuous or discrete time is similarly solved. Both types of problems are related to the classic linear quadratic regulator (LQR) when using a sufficiently long prediction horizon. The basic difference between predictive control and LQR is that in the case of predictive control the optimization problem is solved using a moving time horizon window, while in the case of LQR the same problem is solved in a fixed time window. The advantage of using a moving time window is the ability to perform real-time optimization with constraints on certain system variables.

One of the well-known issues in classic predictive control is a numerical instability problem when the prediction horizon is big because the model used contains an integration process. To overcome this shortcoming, the design pattern must be asymptotically stable.

5. Conclusions

The development of control technology and software offers great possibilities for implementing advanced control algorithms. MPC is one of the most popular advanced techniques for industrial process applications, being able to handle a wide variety of control constraints and can be used at different levels of the process control structure. In this article we used MPC for to minimize the error between the desired CoM and ZMP trajectory and the actual one and the cancellation of the shock of the CoM and ZMP movements.

Acknowledgements

This work was supported by a grant of the Romanian Ministry of Research and Innovation, CCCDI-UEFISCDI, MULTIMOND2 project number PN-III-P1-1.2-PCCDI2017-0637/33PCCDI/01.03.2018, and by KEYTHROB project, number PN-III-P3-3.1-PM-RO-CN-2018-0144 / 2 BM ⁄ 2018, within PNCDI III, and by the European Commission Marie Skłodowska-Curie SMOOTH project, Smart Robots for Fire-Fighting, H2020-MSCA-RISE-2016-734875. The authors gratefully acknowledge the support of the Robotics and Mechatronics Department, Insitute of Solid Mechanics of the Romanian Academy.

References

[1] Khalil T.R., Levinson D.A., “The use of Kane’s dynamic equations in robotics”, *International Journal of Robotic Research*, no.2, 1983

[2] López-Nicolás G., Sagüés C., et al., „Switching visual control based on epipoles for mobile robots”, *Robotics and Autonomous Systems*, Vol. 56, Issue 7, pp. 592-603, doi:10.1016/j.robot.2007.10.005 ISSN 0921-8890, 2008

[3] Moustris G., Tzafestas S.G., (2010), „Switching fuzzy tracking control for mobile robots under curvature constraints”, *Control Engineering Practice*, Vol. 19, Issue 1, pp. 45-53, doi:10.1016/j.conengprac.2010.08.008, ISSN 0967-0661, 2010

[4] Ouyang P.R., Zhang W.J., Gupta M.M, „An adaptive switching learning control method for trajectory tracking of robot manipulators”, *Mechatronics*, Volume 16, (1), pp. 51-61, doi:10.1016/j.mechatronics.2005.08.002, ISSN 0957-4158, 2006

[5] Pop N., Vladareanu L., Wang H., Ungureanu M., Migdalovici M., Vladareanu V., Feng Y., Lin M., Mastan EP., Emary I El., “The Walking Robot Equilibrium Recovery Applied on the NAO Robot”, *Emerging Technologies for Health and Medicine: Virtual Reality, Augmented Reality, Artificial Intelligence, Internet of Things, Robotics, Industry 4.0*, pp. 179-189, John Wiley & Sons, Inc. Hoboken, NJ, USA, 2018

[6] Şandru O., Vlădareanu L., Şchiopu P., Vlădareanu, V., Şandru A., „Multidimensional Extenics Theory”, *75* (1), pp. 3-12., U.P.B. Sci. Bull., Series A, ISSN 1223-7027, 2013

[7] Vladareanu Victor, Schiopu Paul, Cang Shuang, Yu Hongnyan, Deng Mingcong, „Enhanced Extenics Controller for Real Time Control of Rescue Robot Actuators”, *UKACC International Conference On Control (CONTROL)*, Loughborough, UK, pp. 725-730, 2014

[8] Vladareanu Victor, Deng Mingcong, Schiopu Paul, “Robots Extension Control using Fuzzy Smoothing”, *Proceedings of the 2013 International Conference on Advanced Mechatronic Systems*, ISBN 978-0-9555293-9-9, pp. 511-516, Luoyang, China, https://ieeexplore.ieee.org/document/6681698, 2013

[9] Wang Hongbo , Dong Zhang, Hao Lu, Feng Yongfei, Xu Peng, Mihai Razvan Viorel, Vladareanu Luige, „Active Training Research of a Lower Limb Rehabilitation Robot Based on Constrained Trajectory”, *Proceedings of the 2015 International Conference on Advanced Mechatronic Systems*, Beijing, China, pp. 24-29, https://ieeexplore.ieee.org/document/7287123 , 2015