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Quantum fuzzy genetic algorithm with Turing to solve DE

Ahmed Hamid[1], Elias Igor Klebanov [2], Salah A. Albermany[3]
1 School of Electronic Engineering and Computer Science, South Ural State University, Russia.
2 School of Electronic Engineering and Computer Science, South Ural State University, Russia.
3 Computer Science department, Faculty of Computer Science and Mathematics, Kufa University, Iraq

ABSTRACT

In this study, we create the quantum fuzzy Turing machine (QFTM) approach for solving fuzzy differential equations under Seikkala differentiability by combining it with a differential equation and a genetic algorithm. A theoretical model of computation called a quantum fuzzy Turing machine (QFTM) incorporates aspects of fuzzy logic and quantum physics.

Keywords: Quantum, Fuzzy genetic, algorithm, Turing DE

Corresponding Author:

Ahmed Hamid
School of Electronic Engineering and Computer Science,
South Ural State University, Russia
E-mail: ahmediteco@gmail.com

1. Introduction

A genetic algorithm (GA) is a metaheuristic computational method [1]. It is possible to employ genetic algorithms (GAs), which have been around for a while, to carry out artificial intelligence (AI) tasks like learning, categorization, and optimization [2]. GAs have been employed in a variety of wireless network scenarios and are famous for their outstanding generality and adaptability [3]. To do computations in ways that are not conceivable with regular computers, quantum computers use the principles of quantum physics. The best-known illustration of quantum algorithms [4]. There aren't many examples of these exponential speedups that are known, and those that do (such utilizing quantum computers to simulate other quantum systems [5]) have only occasionally been used in domains other quantum mechanics. This letter introduces a quantum method for determining a substance's features. In recent years, fuzzy differential equations have assumed a significant significance. in several models in the sciences of biology [8], engineering [9], physics, and other [10]. There are numerous applications for the first order fuzzy differential equations. It is conceivable to combine a fuzzy quantum genetic algorithm, and Turing machine, to solve differential equations.

A quantum fuzzy genetic algorithm is a type of optimization technique that blends elements of quantum computing with genetic algorithms. By modeling different answers as quantum bits (qubits) and employing quantum-inspired operations to find the optimal one, it is able to progress from one solution to a better one [11]. A theoretical computer model, often known as by following predetermined rules, a Turing machine may perform operations by reading and writing symbols on a tape. It can be applied to solve problems that can be explained by a set of rules or logical steps [12]. A learning machine is an artificial intelligence system that is designed to learn from data. It is trainable on a dataset, and with time it will become more effective.

These three components could be combined to solve differential equations by transforming the equations into a set of rules that the Turing machine can comprehend, optimizing the solution using the fuzzy quantum genetic algorithm, and gradually increasing the accuracy of the solution using the learning machine.
In this paper, we offer a hybrid method that combines fuzzy differential equations with a Genetic Turing machine to solve nth-order fuzzy differential equations under the Seikkala differentiability principle [13]. The fitness function was used to compute the results of the Genetic Turing Machine in order to calculate the discrepancies between the exact and approximative answers. The approximation findings produced using the

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Genetic Turing Machine show, after contrasting the approximate answer provided by the GTM approach with the exact,

2. Materials and methods

They describe a hybrid method for employing fuzzy differential equations in [14] to solve nth order fuzzy differential equations under the Seikkala differentiability notion. The Genetic Turing Machine generated its results after using the fitness function to compute the errors between the precise and approximative solutions. After comparing the approximate solution offered by the GTM method with the precise solution, the approximation findings utilizing Genetic Turing Machine demonstrate the usefulness of hybrid approaches for solving fuzzy differential equations (FDE). In [15] They presented the trapezoidal rule for the numerical solution of first-order fuzzy differential equations and hybrid fuzzy differential equations. Both the method's stability and convergence are examined. They demonstrated how their approach differs from the Midpoint rule by using a few instances.

For systems of linear ordinary differential equations with constant coefficients, they offer a quantum algorithm in [17]. At a predetermined end time, the software generates a quantum state proportional to the response. When compared to earlier quantum solutions to this issue, the approach has polynomial complexity in the inverse error logarithm.

The method for solving second-order ODEs that proposes a polynomial and use an evolutionary process to determine the coefficients of the suggested polynomial has been modified in [18]. By applying the Genetic algorithm to identify the coefficients of the polynomial, Evolution Strategies (ESs) offer a polynomial to solve the order nth ordinary differential equations (ODEs) and order two partial differential equations (PDEs). This is because ESs represent the answer to a problem using a string.

2.1. Fuzzy differential equation by quantum genetic algorithms Turing machine

In general, the optimal solution |f in the range RRn to the approximate solution of the vector function || are distinct from the fitness function E for a quantum fuzzy differential equation (QFDE). The best approximation solution among those that are accessible is selected using the fitness function. typically, defined by

$$|E\rangle = (|f\rangle - |\phi\rangle)^2 \tag{1}$$

may It is defined in the ODE.

$$|E\rangle = \left(\left|f(y^{(n-1)}, y^{(n-2)}, \dots, y', y, x)\right\rangle - |\phi(x)\rangle\right)^2 \tag{2}$$

where n denotes the order of the ODE, Then there's the fuzzy differential equation (FDE):

$$|y^{(n)}(x) >= |f\left(y^{(n-1)}, y^{(n-2)}, \dots, y', y, x\right) >, \tag{3}$$

$$|y^{(i)}(a) >) = |k_{i+1} > \ , i = 0,1, \dots, n-1 \tag{4}$$

where the function $f: [a, b] \times S^n \ \rightarrow S$ is a fuzzy process with fuzzy initial values,
and $S^n = S \times S \times \dots S$ is the set of all fuzzy sets on $\mathbb{R}^n$that are both compact and convex.

2.2. Fuzzy differential equation of first order with QGTM

Let's say you're solving a first-order fuzzy differential equation:

$$|y'(x) >= |f(x, y) >, \quad |y(x_0) > = |y_0 > \tag{5}$$

Since the fuzzy differential equation of (1) may be solved using a decent method provided by the solution of the FDE of (1), where function f:[x 0,b] We can utilize initial values to solve the FDE if |y(x 0)>)=|y 0> is a continuous fuzzy mapping and S S is a continuous fuzzy mapping (1). Therefore, let us

$$|[y(x)]^{\alpha} > = \langle u_{\alpha}(x) | v_{\alpha}(x)\rangle \tag{6}$$

and

$$|[f(x,y(x))]^{\alpha}>= \langle h_{\alpha}(x,u_{\alpha}(x),v_{\alpha}(x))|g_{\alpha}(x,u_{\alpha}(x),v_{\alpha}(x))\rangle \quad (7)$$

When a fuzzy set's membership functions, u and v in S (where S is a nonempty set), $\langle u|$ and $\langle v|$, denoted to $u : S \longrightarrow [0,1]$, and $v : S \longrightarrow [0,1]$ with respectively. The degree to which an element x is a member of the fuzzy set u and v for each $x \in S$ is thus represented by u(x) and v(x).

2.3. FDE with QGTM algorithm

For each generation, the chromosomes produce a certain set of expressions. If an expression satisfies the initial condition and minimizes the fitness function E to zero or very nearly zero, the procedure may be finished; otherwise, the GP technique must be performed.

Example (1):

Algorithm (1): **FDE with QGTM algorithm**
Input: FDE eq(3) Input of the inputs of population Technique for TM

Output: optimal solution of (3)
1. Apply (6) and (7).
2. Determining the starting circumstances the ki is a fuzzy triangular number with a support interval.
3. For TM, use a population technique algorithm.
4. Determine the eq(2) by evaluating the chromosome's fitness.
5. If E < ε then stop
6. Create a new population using genetic manipulations.
7.Goto step 2

Suppose the fuzzy differential equation is

$$|y'(x)>= k|y(x)>, \quad |y(0)>=|y_0> \quad (8)$$

where k is a constant, and y0 is a symmetric triangular fuzzy number with a support range of [a,b]. As in

$$[y_0]^{\alpha}>= \langle a|(1-\alpha)|b(1-\alpha)\rangle \quad (9)$$

The resulting fuzzy differential system will look like the one presented above.

$$|u'_{\alpha}(x)>=k|v_{\alpha}(x)>, \quad |u(0)>=a|(1-\alpha)>$$

$$|v'_{\alpha}(x)>=k|u_{\alpha}(x)>, \quad |v(0)>=b|(1-\alpha)>$$

Using a Quantum Genetic Turing Machine (QGTM), the solution system may be solved as follows:

$$|u_{\alpha}(x)>=a|(1-\alpha)|e^{kx}>,$$

$$|v_{\alpha}(x)>=b|(1-\alpha)|e^{kx}>,$$

As a result of completing (8), we find that the level sets exist in the fuzzy function y(x).

$$|y(x)>=\langle a|(1-\alpha)|e^{kx}, b|1-\alpha|e^{kx}\rangle$$

$$=(1-\alpha)\big|e^{kx}\big|<a,b>>$$

And then we try it out in symmetric triangular fuzzy numbers with x=0 using the QGTM algorithm to approximate α.

$$|\alpha >= \begin{cases} |1 > -(\frac{1}{a})|y|e^{-kx} > & y \in \{a, \left(\frac{a+b}{2}\right)) \\ |1 > -(\frac{1}{b})|y|e^{-kx} > & y \in \{\left(\frac{a+b}{2}\right), b] \end{cases}$$

For example, a=3, b=11, and k=25, That are illustrated by the graph (1):

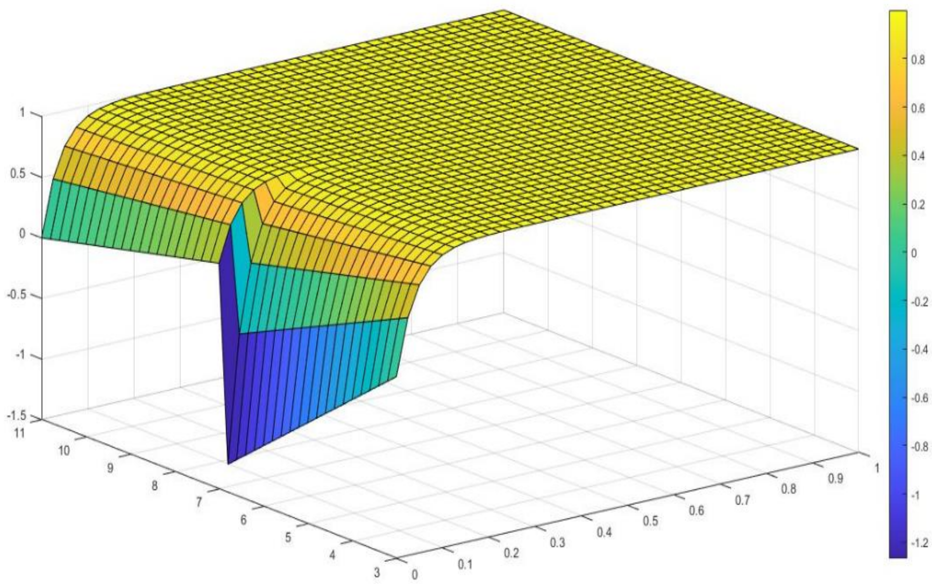

Figure 1. Symmetric triangular fuzzy number for example (1)

2.4. Quantum fuzzy genetic Turing machine algorithms

The outcome of utilizing a Fuzzy Turing machine (FTM) to solve differential equations largely depends on the probability that influences the appearance of particular functions.

The fuzzy Turing machine reduces the amount of time needed to solve the differential equation (FTM). The answer, which is unknown at one point but has intervals that are uniquely specified, is found via the fuzzy Turing machine by reducing some functions while increasing others.

2.5. Quantum fuzzy Turing machine

If there are more than two tapes that are important for grammatical analysis, it can be difficult to arbitrarily select one acceptable recording for one TM transition. The precision of the random tape selection for each cassette is not yet known. Finding the maximum or minimum of a function f| under linear constraints is another issue. Each tape has a pi probability of being randomly chosen, where n is the number of tapes in a single TM transition, 1in-1, and the issues are figuring out how to choose each tape's unknown probabilities and how to get these tapes to follow the GA fitness function., $1 \leq i \leq n-1$,

$$|p_0 > +|p_1 > + \cdots + |p_{n-1} >= |1 >$$

The above problem has the structure.

$$Max/Min \quad < f|$$

Subject to $a_i \leq p_i \leq b_i$, $1 \leq i \leq n-1$, $\sum_{i=1}^{n-1} |p_i >= |1 >$

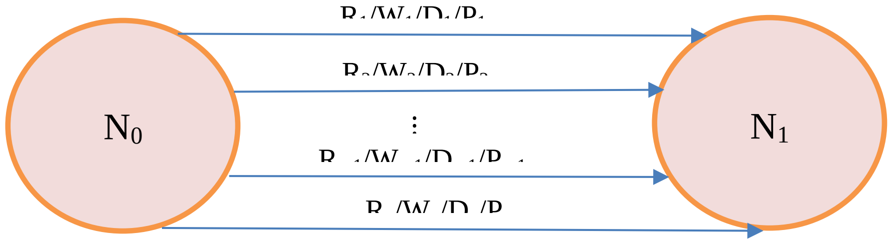

Figure 2. Label (read/write/direction/ probability)

where the label Ri/Wi/Di/Pi (read/write/direction/ probability), between the states N0, and N1, i=1, 2,.., n, and $\sum_{i=1}^{n-1}|p_i>=|1>$, shown graphically figure (2)**.**

Since p_i uncertain exactly value belong to the interval $[a_i,b_i]$, where $0 \le a_i, b_i \le 1$ and the intervals $[a_i,b_i]$ provided α-cuts of fuzzy numbers (triangle, shape triangle,...), the structure above is known as the *Optimal Selection Model* (OSM) of a Fuzzy Turing Machine (FTM). It follows that if is a linear function of the, then the Optimal Selection Model may be used to provide numerical solutions to the OSM issue. There are three ways to solve OSM, which is a linear programming problem:

1) Graphically for n=2 or 3.
2) calculus or numerical analysis,
3) the Simplex technique for Minimizing or maximizing a linear form subject to linear inequality restrictions and solved it.

It is obvious that the above method requires a lot of time and results in an approximate answer, but employing the Quantum Fuzzy technique to resolve the OSM problem using the -cuts of an FTM results in a speedy and precise solution. The formula below demonstrated the presence of a resolution to the Optimal Selection Model (OSM) problem in FTM without using the aforementioned method: There is an OSM solution.

$$|p_i>=|a_i>+\alpha|b_i>-\alpha|a_i>=|1>,\ p_i \in [a_i,b_i]$$

where,

$$\alpha=\frac{1-\sum a_i}{\sum(b_i-a_i)}$$

Such that $0<a_i<b_i$, $\sum a_i \le 1, and\ \sum b_i<1$.

Example(2):

a_i	0.19	0.23	0.28	0.1	0.09	0.04	0.01	0.008	0.005
b_i	0.28	0.34	0.4	0..12	0.11	0.05	0.07	0.020	0.009

$\sum_{i=1}^{9} a_i=0.953$,and $\sum_{i=1}^{9} b_i=1.279$, $\alpha=\frac{1-\sum_{i=1}^{9} a_i}{\sum_{i=1}^{9} b_i-\sum_{i=1}^{9} a_i}=\frac{1-0.953}{1.279-0.953}=0.144172$

$$\boldsymbol{p_i=a_i+\alpha\,(b_i-a_i)}$$

$\boldsymbol{p_i}$	0.1996	0.241	0.29264	0.10210	0.09210	0.04105	0.01632	0.009265	0.0054

2.6. Quantum fuzzy probability of TM.

Let the set $X=\{x_1,x_2,\dots,x_n\}$, representing tapes or pathways with Quantum fuzzy probabilities $\bar{p}_i$, where $p_i \in [a_i,b_i]$, and $\sum_{i=1}^{n-1}|p_i \ge |1>$, the problem to determine p_i, $1 \le i \le n-1$, with

$$\boldsymbol{min/\,max\ \sum_j |p_{i_j}}$$

If we let $p_{i_1}, p_{i_2},\dots,p_{i_n}$ denote $\{x_{i_1},x_{i_2},\dots,x_{i_n}\}$, where it's represents the set A such that $A \subseteq X$, then the Quantum Fuzzy Probability Distribution

$$|p(\{x_i\}) > = |a_i >$$

since $A = \{x_1, x_2, \dots, x_k\}$, with 1≤ k<n, then we can define the quantum fuzzy probability distribution on the set A by

$$|\bar{p}(A)|[\alpha] > = \left\{ \sum_{i=1}^{k} |\boldsymbol{p_i} > \ \text{such that}\ \boldsymbol{p_i} \in \bar{p}_i[\alpha_i], \qquad 1 \leq i \leq n, \sum_{i=1}^{n} |\boldsymbol{p_i} > = |1 > \right\}$$

Where $|\bar{p}_i[\alpha_i] >$ it displays fuzzy numbers with α-cuts, for example, the triangle function may be represented as

$$|\bar{p}_i[\alpha_i] > = < \ |a_i > +\alpha_i|b_i > -\alpha_i|a_i >, |c_i > +\alpha_i|b_i > -\alpha_i|c_i > >$$

The problem is to determine the maximum of $|\bar{P}(A)|[\alpha] >$, where

$$\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n), \qquad 0 \leq \alpha_i \leq 1, \qquad i = 1,2, \dots, n$$

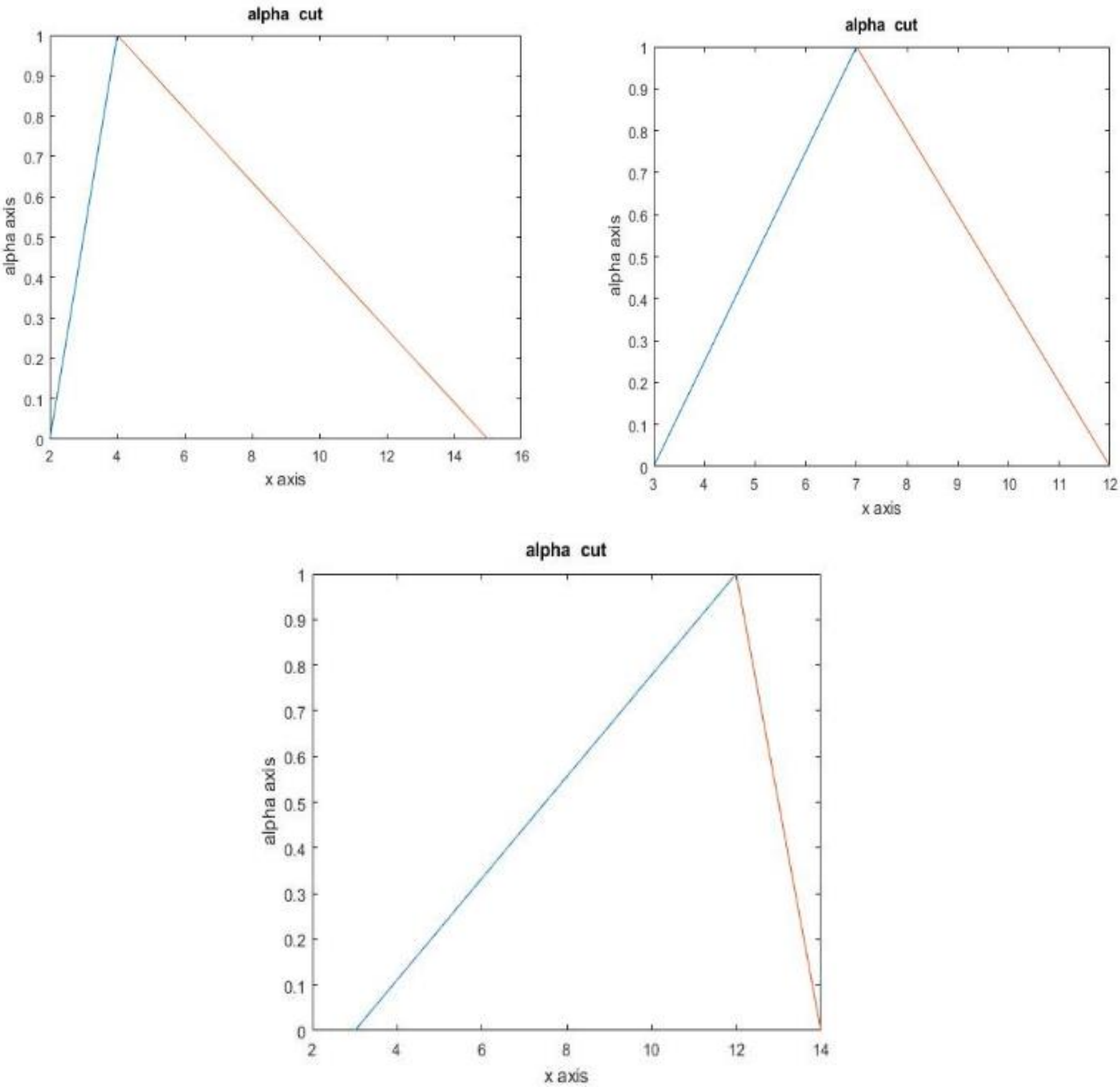

Figure 2. α-cuts of fuzzy numbers

The Quantum Genetic Turing Machine (QGTM) has the uncertain path of the Turing Machine (TM), and we are using the Quantum Fuzzy Turing Machine (QFTM) technique to solve the differential equations to determine the suitable chromosome solution in the fitness function. We also used the QGTM technique to compute α-cuts of the triangle fuzzy number, which converges to the optimal solution when it converges to one.

3. Conclusion

When it comes to using the fuzzy technique, there are two schools of thought: one uses the quantum genetic Turing machine (QGTM) to solve the fuzzy differential equations by figuring out the value of (-cut), and the other uses the quantum fuzzy Turing machine (QFTM).

Declaration of competing interest

The authors affirm that none of the materials discussed in the current work are the subject of any known non-financial or financial conflicting interests.

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