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A new method of Poisson regression estimator in the presence of a Multicollinearity problem: Simulation and application

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ABSTRACT

A new estimator for the Poisson model is introduced in this study. Poisson regression model is an important log- Linear models which is the tool of modeling the dependent variable when its values are positive and as a form of count data or rates additional to be the appropriate model for analyzing rare events. The maximum likelihood estimator (MLE) suffers from the instability problem in the presence of multicollinearity for a Poisson regression model (PRM). The purpose of this study is to make a comparison of parameters estimation methods for the Poisson regression model when that model suffer from semi multicollinearity problem through the possible methods with proposed method and also propositions for the biased parameter. A Monte Carlo simulation experiment used to generate data follows Poisson regression model and suffer from multicollinearity problem according to variation factors like sample size, the value of simple correlation coefficient and the number of independent variables. So mean squared error and relative efficiency is adopted as a criteria to the comparison of the parameters estimation methods for the model. The simulation results and the real-life application evidenced that the proposed estimator performs better than the rest of the estimators.

Keywords: Estimation, Multicollinearity problem, MSE, Poisson regression, Monte Carlo simulation

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1. Introduction

There have been many of books and research studies that address the authors effects linear multicollinearity to estimate regression models parameters problem, especially non-linear, and develop ways to reduce the effects of that problem, also the proposals of these researchers in the choice of (Biased Parameters), and the following is a review of the most important studies about this topic [1-16].

The method proposed by the researchers (Hoerl and Kennard, 1970) [16, 17] that of the most common and widely used in the treatment of (Multicollinearity) problem, as suggested researchers add a small amount of positive elements diameter in a matrix of information $(X'X)$, and the method is called (Ridge regression Estimators) and explained theoretically that the estimated resulting from that addition will be estimated biased, but it is estimated will be effective more than the least-squares estimator being less variation compared by the other, making it the most efficient in spite of bias.

In (1994) the researcher (Al-Mashhadani) [18] studied the use of the principle compounds analysis method in diagnosing and treating the problem of (Multicollinearity) compared to the method of ridge regression, and applied her research to some economic phenomena by using the Cobb-Douglas model of production functions, and she concluded that the use of the ridge regression method it may address the effects of the multicollinearity problem, but it does not completely remove it, unlike the method of the principle compounds.

In (1997) the researcher (Long) [8] composed ensures regression analysis when they are approved (Dependent

Variable) in the form of (Categorical) or (Limited), as referred to the Poisson distribution as one of the cases in which the response variable is distributed according to it,, one of the first researchers who mention on Poisson regression model in terms of the fundamentals of building this model and the process of estimating the parameters.

In (2004) the researcher (Famoye et al.) [19] discussed the General Poisson Regression Model through a study that included a sample of vehicle drivers in one of the American states whose ages were (65) years and over, as they studied through that sample. Road accidents and their relationship to a number of demographic factors, driving experiences and the health aspect of those drivers, and through this study, the researchers confirmed that the general Poisson regression model is the best among the regression models in expressing the phenomenon of road accidents.

Within the framework of the research that discussed the problem of multilinearity when the distribution of random errors is not a normal distribution, the researchers (Urgan and Tez) [20] in the same year compared theoretically between the maximum likelihood estimator (MLE) and the letter regression estimator (RRE) and the Liu Estimator) for the logistic regression model, as it was shown that the Liu estimator is the best despite being a biased estimator because it is the estimator with the least mean squared error (MSE).

In (2011) the researchers began to discuss the problem of multilinearity in the diversity of regression models. The researchers (Kebria and Shukur and Mansson) [21] compared the Liu Estimator with the estimator of the maximum likelihood function of the logistic regression model when the multilinearity problem occurs. They also found the optimal value for the bias parameter (d), and the researchers used the absolute mean of errors (MAE) in the process of comparing the two methods of estimating the model parameters, and the researchers applied the comparison to real data that included a set of economic indicators as well as the use of simulation, and the results they obtained showed the superiority of the Liu estimator on the maximum likelihood estimator for the lack of mean absolute errors for the estimated model parameters

The researchers (Mansson and Shukur) [1]in (2011) also returned to suggest an estimate They call it the ridge regression estimator for the Poisson regression model, based on the basic idea put forward by the researchers (Hoerl and Kennard) in finding the ridge regression estimator for the linear regression model. The researchers also referred to several formulas for estimating the bias parameter in that estimator, and then we compared that estimator with the maximum likelihood estimator by using the Monte - Carlo method in generating data that follows the Poisson distribution and suffers at the same time from the problem of multilinearity. For comparison, the results they obtained showed the superiority of the ridge regression estimator when substituting all the ridge parameters in it to the maximum likelihood measure. They found the product of dividing the variance of the estimated random error by the largest value of the characteristic root of the information matrix, and then they calculated the largest value of the reciprocal of that indicator, which was the best estimator. The bias parameter, which indicates the great importance of the values ??of the distinct roots and the variance of random errors, as shown by some previous research by other researchers.

In the (2012), researchers (Mansson and Kibria and Shukur and Sjalander) [2] created a Liu Estimator for the Poisson regression model, based on the basic idea of the ? researcher (Liu) estimator, which he created in (1993) to address the problem of multilinearity in the linear regression model, and they found the optimal value for the bias parameter (d) and made five proposals to get the best estimate for it, and as (Liu) the researchers agreed with him that the priority of the Liu estimator is that it is a linear function in the bias parameter (d) , and then the researchers used the Monte-Carlo method in the simulation to compare between the (Liu) estimator and the maximum likelihood estimator by using the mean squared error (MSE) and the average absolute error (MAE), the researchers concluded that the (Liu) estimator is preferred for all the bias parameters calculated on the maximum likelihood method when estimating the parameters of the Poisson regression model when the multilinearity problem occurs.

In (2013) researchers (Shukur and Kebria and Mansson) [3] suggested a ridge regression estimator for the inflated zero-Poisson regression model when that model suffers from the problem of multilinearity. The researchers used the Monte-Carlo method in the simulation to show the preference of that estimator compared to the maximum likelihood estimator, as well as to find the optimal value for the bias parameter (ridge parameter). The ridge with all the proposed ridge parameters is better than the maximum likelihood method to depend on the value of the largest eigen root with the variance of the estimated error, as the researchers noted that these methods work well as the number of independent variables increases and the sample size increases, as well as the greater the value of the correlation coefficients between the independent variables.

In (2013) the researcher (Hossam) [25] studied a comparison of methods for estimating the parameters of the Poisson regression model when there is a problem of multilinearity, and the researcher used two proposed

methods in estimating the parameters of the model and compare it with the maximum likelihood estimator (MLE) method, he also used simulation in his experiment and adopted the mean squares error as a criterion for comparison.
In (2016) researchers Turkan and Ozel [22] adopted the modified jackknifed ridge regression estimator (MJRE) to the Poisson regression model as a treatment to the problem of multicollinearity.
In (2017) Asar and Genc [23] implemented the two-parameter estimator to the Poisson regression model.
In (2019) Rashad and Algamal [24] developed a new ridge estimator for the Poisson regression model by modifying Poisson modified jackknifed ridge regression.
This paper is organized as follows: In Section (2,3,4), the statistical methodology is described. In Section 5, the design of the Monte Carlo experiment is presented and the result from the simulation study is discussed. An application is presented in Section 6. Finally, a brief summary and conclusions is given in section 7.

2. Estimation of the parameters for Poisson regression model

The Poisson regression model can be expressed by the following formula [3, 4, 5, 14]

$$Y = e^{X\beta+U}, \tag{1}$$

where Y:vector of a response variable of degree $(n \times 1)$, X: matrix of explanatory variables of degree $(n \times (p+1))$, β: vector of parameters with degree $((p+1) \times 1)$, U: vector of random errors of degree $(n \times 1)$

2.1. Maximum likelihood estimators method

In order to estimate the parameters of the Poisson regression model using this method, the basic assumptions of the distribution will be based on the fact that the Poisson distribution represents the characteristic feature on which the model is built, given that the distribution is specific to the dependent variable (Y_i), if the dependent variable (Y_i) follows the Poisson distribution with a parameter of its value μ_i, the distribution function will be in the following form [1]:

$$P(Y = y_i) = \frac{e^{-\mu_i} \mu_I^{Y_i}}{Y_i!} \quad i = 1,2,\dots,n \tag{2}$$

By maximizing the observations of the dependent variable distribution (Y_i) given in the above formula, the likelihood function is as follows

$$L(Y_1, Y_2, \dots, Y_n; \mu_i) = \frac{E^{-\sum_{i=1}^{n} \mu_i} \mu_i^{\sum_{i=1}^{n} Y_i}}{\prod_{i=1}^{n} Y_i!}$$

By taking the natural logarithm of the likelihood function for the above observations, we get

$$: \log L(Y_i/x_i, \beta) = -\sum_{i=1}^{n} \mu_i + \sum_{i=1}^{n} Y_i(\log\{\mu_i\}) - \log\{\prod_{i=1}^{n} Y_i!\} \tag{3}$$

Depending on one of the basic assumptions of the Poisson regression model $\left(\mu_i = e^{\{x_i'\beta\}}\right)$, this assumption is replaced by the function (3) above, as follows:

$$\log L(Y_i/x_i, \beta) = -\sum_{i=1}^{n} e^{x_i'\beta} + \sum_{i=1}^{n} Y_i \left(\log\left\{e^{x_i'\beta}\right\} - \log\{\prod_{i=1}^{n} Y_i!\}\right. \tag{4}$$

By deriving equation (4) for the parameter vector β, we get:

$$\frac{\partial \log L}{\partial \beta} = \sum_{i=1}^{n} \left(Y_i - e^{x_i'\beta}\right) x_i \tag{5}$$

By equating the derivative of the likelihood function of the observations with respect to the parameter (β) zero, an estimate of the parameters of the Poisson regression model shown in the formula (1) can be obtained.

$$\frac{\partial \log L}{\partial \beta} = 0$$
$$\sum_{i=1}^{n} \left(Y_i - e^{x_i'\beta}\right) x_i = 0 \tag{6}$$

Equation (6) is non-linear with respect to the vector of estimators $(\hat{\beta})$, and to solve this equation, one of the iterative methods known as the Least square Iterative Weighted is used, as the estimators of the parameters β of the Poisson regression model are: [1, 2]

$$\hat{\beta}_{MLE} = \left(X'\widehat{W}X\right)^{-1}\left(X'\widehat{W}Z\right) \tag{7}$$

Where:

$\hat{\beta}_{MLE}$: vector of the parameters of a Poisson regression model estimated according to the maximum likelihood metho.

$\widehat{W}$: a diagonal matrix in which the elements of the diameter are equal to the estimated values of the parameter Poisson (μ_i) distribution according to the second assumption:

$$\widehat{W} = \begin{pmatrix} e^{\hat{\mu}} & & & \\ & e^{\hat{\mu}} & & \\ & & \ddots & \\ & & & e^{\hat{\mu}} \end{pmatrix}$$

Z: vector, and the element (i) in the vector Z is equal to:

$$Z_i = \log(\hat{\mu}_i) + \frac{Y_i - \hat{\mu}_i}{\hat{\mu}_i} \tag{8}$$

The covariance matrix of the maximum likelihood estimators of the Poisson regression model shown in the formula (5) is produced from the inverse of the second derivative of equation (8) [1].

$$Cov\left(\hat{\beta}_{ML}\right) = \left[E\left\{\frac{\partial^2 L(X;\beta)}{\partial\beta\partial\beta}\right\}\right]^{-1} = \sigma_u^2\left(X'\widehat{W}X\right)^{-1} \tag{9}$$

Where: σ_u^2: the variance of the random error of the population. Thus, the mean squared error for the parameters of the Poisson regression model estimated according to the method of maximum likelihood estimators is as follows:

$$MSE\left(\hat{\beta}_{M\mathbb{L}}\right) = E\left(\hat{\beta}_{ML} - \beta\right)'\left(\hat{\beta}_{ML} - \beta\right) = tr\left(X'\widehat{W}X\right)^{-1} = \sum_{j=1}^{p} \frac{1}{\lambda_j} \tag{10}$$

Where:

λ_j: is the eigen value of the element (j) in the matrix $(X'\widehat{W}X)$.

When there is a strong correlation and a linear relationship between the independent variables, the performance of the weighted matrix will be $(X'\widehat{W}X)$ weak, especially in the case of calculating an estimate of the model parameters, which leads to the instability and stability of the estimations, and at the same time, those linear relationships between the explanatory variables will lead to an inflated variance of the parameters estimated according to this method, Which calls for not adopting this method and the statistical inference based on it, despite the fact that the obtained estimations are unbiased [1, 3].

3. Estimating the parameters for Poisson regression model with Multicollinearity problem

Like other regression models, the explanatory variables involved in building a Poisson regression model may be exposed to a high correlation and linear correlation between two or more variables, which negatively affects the process of estimating the model parameters. linearity as well as a statement of the pros and cons of these methods on the estimation process) [1].

3.1. Ridge regression estimators method

The ridge regression estimators method is one of the alternatives for estimating the parameters of regression models when there is linear multiplicity or linear duplication between the explanatory variables. The model as a linear logarithmic Poisson regression model [8][10]. This method starts with noting that the Maximum Likelihood method for estimating the parameters of the model works by minimizing the Weighted Error Sum (WSSE), and if another estimator is tested and so β_{RR}, then the weighted sum of squared errors can be written as below [1].

$$u'u = (Y - X\beta_{ML})'(Y - X\beta_{ML}) + (\beta_{RR} - \beta_{ML})'\left(X'\widehat{W}X\right)(\beta_{RR} - \beta_{ML})$$
$$= \phi_{\min} + \phi(\beta_{RR}) \tag{11}$$

Where: $\phi_{\min}$: Represents the increase in the average of the weighted squares of errors if the estimated parameters are replaced by the Maximum Likelihood method (β_{ML}) with the parameters to be found (β_{RR}).
And through the inverse relationship between the characteristic values and the variance of the parameters estimated according to the method of Maximum Likelihood, which is indicated in the formula (10), then the parameter vector to be calculated will be reduced (β_{RR}) to the sum of the squares of the weighted errors according to the following constraint

$$MinimizeF = \acute{\beta}_{RR}\beta_{RR} + \left(\frac{1}{K}\right)(\beta_{RR} - \beta_{ML})'\left(X'\widehat{W}X\right)(\beta_{RR} - \beta_{ML} - \phi_0) \tag{12}$$

Since: Lacrange multiplier:: ($\frac{1}{K}$)

$$\phi_0 = \phi(\beta_{RR}) \tag{13}$$

$$u'u = (Y - X\beta_{MLL})'(Y - X\beta_{ML}) + \beta'_{RR}\beta_{RR}$$
$$+ \left(\frac{1}{K}\right)(\beta_{RR} - \beta_{MLL})'\left(X'\widehat{W}X\right)(\beta_{RR} - \beta_{ML} - \phi_0) \tag{14}$$

By deriving the formula (14) for the parameter vector (β_{RR}) and by equalizing the value of the derivative in the formula to zero, the ridge regression estimators for the Poisson regression model can be found, as follows:

$$\hat{\beta}_{RR} = (X'\widehat{W}X + KI)^{-1}(X'\widehat{W}X)\hat{\beta}_{ML} \tag{15}$$

$$\hat{\beta}_{RR} = Z\beta_{ML} \tag{16}$$

The ridge regression estimators are biased when they are ($k > 0$) and the amount of bias is [1].

$$Bias\left(\hat{\beta}_{RR}\right) = E\left(\hat{\beta}_{RR}\right) - \beta$$
$$= Z\beta - \beta = (Z - I)\beta \tag{17}$$

The variance matrix of character regression estimators is as follows [1].

$$Var - Cov\left(\hat{\beta}_{RR}\right) = ZVar - Cov\left(\hat{\beta}_{ML}\right)Z'$$
$$= Z\sigma_u^2\left(X'\widehat{W}X\right)^{-1}Z' \tag{18}$$

The mean squares error (MSE) for the parameters of the Poisson regression model estimated according to the ridge regression method is:

$$
\begin{aligned}
MSE(\hat{\beta}_{RR}) &= E(\hat{\beta}_{RR}-\beta)'(\hat{\beta}_{RR}-\beta) \\
&= E[\hat{\beta}_{ML}-\beta)'Z'Z(\hat{\beta}_{ML}-\beta)] + (Z\beta-\beta)'(Z\beta-\beta) \\
&= tr\cdot[\hat{\beta}_{ML}-\beta)'(\hat{\beta}_{ML}-\beta)Z'Z] + (Z\beta-\beta)'(Z\beta-\beta) \\
&= \sum_{j=1}^{J}\frac{\lambda_j}{(\lambda_j+K)^2} + \beta'\left\{(X'\widehat{W}X+KI)^{-1}(X'\widehat{W}X)-I\right\}\ \left\{(X'\widehat{W}X+KI)^{-1}(X'\widehat{W}X)-I\right\}\beta \\
&= \sum_{j=1}^{p}\frac{\lambda_j}{(\lambda_j+K)^2} + K^2\sum_{j=1}^{p}\frac{\alpha^2 j}{(\lambda_j+K)^2}
\end{aligned}
$$

since:

$$\alpha_j = \gamma\hat{\beta}_{ML} \tag{19}$$

α_j :The parameter (j) represents one of the parameters of the Poisson regression model when taking the natural logarithm of the model shown in the formula (1) γ: represents the (Eigen vector) of the matrix $(X'\widehat{W}X)$.
The ridge regression estimators of the Poisson regression model are desirable for two reasons [1]. the first is that it is a simple method and does not need any changes in the presence of software algorithms, and the second is that the mean of the error squares for the estimated parameters is less than that of the parameters estimated using the method of maximum likelihood, because the first term in the derivative of the formula (2) which represents the variance of the estimated parameters is a decreasing convergent function in(k), and that the second term in the same formula which represents the bias square is an increasing convergent function in (k) and as shown below,

$$\frac{\partial MSE(\widehat{\beta}_{RR})}{\partial K} = -2\sum_{j=1}^{p}\frac{\lambda_j}{(\lambda_j+K)^3} + 2K\sum_{j=1}^{p}\frac{\lambda_j\alpha^2{}_j}{(\lambda_j+K)^3} \tag{20}$$

Thus, we note that the first term in the above formula (20) is less than zero, which reflects the decreasing variance of the character regression estimators with the increase in the second term, which reflects the amount of bias, and (Horel and Kennard) explained that it is necessary that the value of the bias parameter (ridge parameter) and defined (k) within the field below[4]:

$$K \in \left\{0, \frac{\partial\sigma_u^2}{\alpha_{\max}}\right\} \tag{21}$$

This is in order that the derivation of the mean squared errors of the ridge regression estimators is less than zero [4].

$$\frac{\partial MSE(\widehat{\beta}_{RR})}{\partial K} < 0 \tag{22}$$

3.1.1. Ridge parameter estimators

After finding the estimators for the ridge regression and showing that these estimators are biased but more efficient than their counterparts through the method of Maximum likelihood, and as a result of the principle of the ridge regression method, which includes adding a small positive quantity (k) , which represents the cause of bias in the value of the estimator, the researchers differed in the development of many and varied formulas to propose and choose an estimator for the ridge parameter (the bias parameter) to be substituted for later in the ridge regression estimators and to indicate which of these formulas is better via common comparison criteria. The following is a review of the researchers' most prominent formulas for estimating the character parameter (the bias parameter).

- (Horel and Kennard) formula [19].

It is the first and oldest proposal for estimating the ridge parameter (the bias parameter), which was presented by the researchers in (1970), which depends on the mean of the error squares of the observations and the squared values $({\alpha^2}_j)$ within the Eigen vector as follows:

$$K_1 = \frac{\hat{\sigma}_u^2}{\hat{\alpha}^2 \max} \tag{23}$$

where: $\hat{\sigma}_u^2$: The estimated mean squared error, which is calculated in the Poisson regression model according to the formula below [20].

$$S_e^2 = \frac{\sum_{i=1}^{n} (Y_i - \hat{\mu}_i)^2}{n-p-1} \tag{24}$$

$\hat{\alpha}^2$max: represents the largest element in the formula (19)

• (Horel and Kennard) modified formula [6, 13]

In an improvement to the proposal proposed and presented by the same researchers, the improved form of the proposal took into account the existence of the linear regression theory. In the Yoisson regression model, the improved formula for the optimal value of the ridge parameter (k) is as follows:

$$K_2 = \frac{1}{{\hat{\alpha}^2}_{\max}} \tag{25}$$

• Formula (Kibria)[6]

It was presented by the researcher (Kibria) based on the improvement of the estimator (Horel and Kennard) shown in formula (23) by calculating its geometric mean and its formula as follows:

$$K_3 = \frac{S_e^2}{\left[\prod_{j=1}^{p} \hat{\alpha}_j^2\right]^{\frac{1}{p}}} \tag{26}$$

• The second formula(Kibria) [6, 7] (Kibria) came back to present another proposal to find the optimal value for the ridge parameter based on calculating the median with the idea of transforming the square root of the estimator (Horel and Kennard) shown in formula (23) and its formula as follows

$$K_4 = Median\{m_j^2\} \tag{27}$$

$$m_j = \sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}} \tag{28}$$

• Formula (AL Khamisi et al.)[1]

This estimator connects the eigen values and the variance of random errors, as well as taking into account the effect of the eigens vectors by calculating the largest value, and the optimal value of the bias parameter when analyzing the Poisson regression model are:

$$K_5 = Max\{S_j\} \tag{29}$$

Since:

$$S_j = \frac{\lambda_j \hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_j \hat{\alpha}_j^2} \tag{30}$$

λ_j: represents the distinct values (Eigen values) of the matrix $(X'\widehat{W}X)$.

• Formula (Munez and Kibria) [6, 1, 7]

The researchers (Munez and Kibria) and based on the idea of ?the inverse square root of the estimator (Horel and Kennard) shown in the formula (23) presented by (AL Khamisi and Shukur) made three proposals, the first of which is the optimal value that represents the largest value of the reciprocal of the square root transformation of the following formula :

$$K_6 = Max\left\{\frac{1}{m_j}\right\} \tag{31}$$

• Second Formula (Munez and Kibria) [1]

The second proposal was based on calculating the geometric mean with the reciprocal of the square root transformation of the estimator (Horel and Kennard) shown in formula (23) and its formula is as follows:

$$K_7 = \left\{\prod_{j=1}^{p} \frac{1}{m_j}\right\}^{\frac{1}{p}} \tag{32}$$

• The third (Munez and Kibria) formula [6, 1].

This proposal was based on combining the idea of the inverse square root of an estimator (Horel and Kennard) shown in formula (23) and finding the median and its formula as follows:

$$K_8 = Median\left\{\frac{1}{m_i}\right\} \tag{33}$$

It should be noted that the process of estimating the parameters of the Poisson regression model according to the ridge regression method for each parameter is done by substituting the eight formulas for calculating the bias parameter in the estimation formula (15).

3.2. Liu estimators method [3][14]

This method deals with the issue of inflating the variance of the estimated model parameters, as it was (Liu) who laid the foundations of this method in (1993) when he created it for the linear regression model in the presence of the problem of linear multiplicity (31).

This method is established by the same method followed by (Liu) with the different nature of the type of the Poisson regression model as it is a non-linear model, so by returning to the model formula in equation (4 - 2) and taking the natural logarithm of it, we get:

$$\log(Y) = \log\left\{e^{X\beta+U}\right\}$$
$$Y^* = X\beta + U \tag{34}$$

Since:

$$Y^* = \log(Y) \tag{35}$$

By adopting the same constraint set by (Liu)

$$\in'\in = \left(d\hat{\beta} - \beta^*\right)'\left(d\hat{\beta} - \beta^*\right) \tag{36}$$

where: $\in'\in$: Represents the amount of increase in the mean squares of the weighted error in the event that the vector of parameters estimated by the method of maximum likelihood ($\hat{\beta}$) is replaced by the vector of parameters estimated according to the method of Liu (β^*). ($\hat{\beta}$): The vector of the capabilities of the maximum likelihood when neutralizing the model to a linear model (β^*) :vector of the model parameters estimated according to the (Liu) method d: add parameter (bias parameter)

$$\begin{aligned} \in' W \in \quad &= (Y^* - X\beta^*)'\widehat{W}(Y^* - X\beta^*) + \left(d\hat{\beta} - \beta^*\right)'\left(d\hat{\beta} - \beta^*\right) \\ &= Y^*\widehat{W}Y^* - 2\beta^*X'\widehat{W}Y^* + \beta^{*\prime}X'\widehat{W}X\beta^* + d'\hat{\beta}\hat{\beta}d - 2d\hat{\beta}\beta^* + \beta^*\beta^* \end{aligned} \tag{37}$$

By differentiating equation (37) for the parameter vector (β^*) , we get:

$$\frac{\partial \in' W \in}{\partial \beta *} = -2X'\widehat{W}Y^* + 2X'\widehat{W}X\beta^* - 2d\hat{\beta} + 2\beta^* \tag{38}$$

By setting the derivation product to zero, the estimations (Liu) are obtained for the parameters of the Poisson regression model [2, 3].

$$\hat{\beta}_{Liu} = \left(X'\widehat{W}X + I\right)^{-1}\left(X'\widehat{W}Y^* + d\hat{\beta}\right) \tag{39}$$

Since the:

$$\hat{\beta} = \hat{\beta}_{ML} \tag{40}$$

Which were previously defined in equation (36) as the estimators of the maximum likelihood when neutralizing the Poisson regression model to a linear model.

$$\because \hat{\beta} = \hat{\beta}_{ML} = (X'WX)^{-1} \tag{41}$$

$$\therefore X'\widehat{W}Y^* = (X'WX)\hat{\beta}_{ML} \tag{42}$$

By substituting a value $(X'\widehat{W}X)$ with its equivalent in the formula (40), we get another form of the estimators (liu) for the parameters of the Poisson regression model [2, 3].

$$\begin{aligned} \hat{\beta}_{Liu} &= \left(\left(X'\widehat{W}X + I\right)^{-1}\left(\left(X'\widehat{W}X\right)\beta_{ML} + d\hat{\beta}_{ML}\right)\right. \\ \hat{\beta}_{Liu} &= \left(X'\widehat{W}X + I\right)^{-1}\left(X'\widehat{W}X + dI\right)\hat{\beta}_{ML} \end{aligned} \tag{43}$$

Liu's estimators are biased when the value $(d > 0)$and amount of bias is [10], [26].

$$\begin{aligned} &Bias\left(\hat{\beta}_{Liu}\right) = E\left(\hat{\beta}_{Liu}\right) - \beta \\ &= Z\beta - \beta \\ &= (Z - I)\beta \end{aligned} \tag{44}$$

since

$$Z = \left(X'\widehat{W}X + I\right)^{-1}\left(X'\widehat{W}X + dI\right) \tag{45}$$

The variance matrix of (Liu) capabilities is as follows: [2, 3].

$$\begin{aligned} &Var - Cov\left(\hat{\beta}_{luu}\right) = ZVar - Cov\left(\hat{\beta}_{ML}\right)Z' \\ &= Z\varsigma_u^2\left(X'\widehat{W}X\right)^{-1}Z' \end{aligned} \tag{46}$$

As for the mean squares error (MSE) for the parameters of the Poisson regression model estimated according to the method of (Liu) estimators, it is

$$\begin{aligned} MSE\left(\hat{\beta}_{Liu}\right) \quad &= E\left(\hat{\beta}_{Liu} - \beta\right)'\left(\hat{\beta}_{Liu} - \beta\right) \\ &= \sum_{j=1}^{p} \frac{\left(\lambda_j + d\right)^2}{\lambda_j\left(\lambda_j + 1\right)^2} + (d-1)^2 \sum_{j=1}^{p} \frac{\alpha_j^2}{\left(\lambda_j + 1\right)^2} \end{aligned} \tag{47}$$

The (Liu) estimators are a linear function in the bias parameter referred to (d), as this can be easily seen in the formulas (39) and (45), and the (Liu) estimators, although they are biased, the mean of the calculated error squares It has less than the average error squares for the same parameters if estimated according to the method of maximum likelihood, as this is shown as follows [2, 3]: First, the first derivative of the mean squared error is found in the formula (47) with respect to the bias parameter (d).

$$\frac{\partial MSE(\hat{\beta}_{Liu})}{\partial d} = 2\sum_{j=1}^{p} \frac{\lambda_j + d}{\lambda_j(\lambda_j+1)^2} + 2(d-1)\sum_{j=1}^{p} \frac{\alpha_j^2}{(\lambda_j+1)^2} \quad (48)$$

And by substituting a value (d=1) into the above equation:

$$\frac{\partial MSE(\hat{\beta}_{Liu})}{\partial d} = 2\sum_{j=1}^{p} \frac{1}{\lambda_j(\lambda_j+1)^2} \quad (49)$$

The value of the derivative in the formula (49) is greater than zero if the egine values λ_j are greater than zero, so there is a value of the bias parameter (d) that falls within the domain (0,1) and makes the mean of the error squares The parameters estimated according to the (Liu) estimators method are less than the mean squares of error for the same parameters estimated according to the maximum likelihood method.

3.2.1. Liu biased parameter estimators

We mentioned earlier that Liu's estimators are biased and that the cause of the bias is the presence of an added value (d) , and we showed that this value ranges between (0,1), and in order to find the proposed estimators for this bias parameter (d), the optimal value must be found for it, by equating the value of the derivative of the average error squares shown in the formula (48) with respect to zero.

$$\frac{\partial MSE(\hat{\beta}_{Liu})}{\partial d} = 0$$

$$2\frac{\lambda_j + \hat{d}}{\lambda_j(\lambda_j+1)^2} + 2(\hat{d}-1)\frac{\alpha_j^2}{(\lambda_j+1)^2} = 0 \quad (50)$$

Performing some algebraic operations, we get

$$\hat{d} = \frac{\frac{\alpha_j^2 - 1}{(\lambda_j+1)^2}}{\frac{\frac{1}{\lambda_j} + \alpha_j^2}{(\lambda_j+1)^2}} = \frac{\alpha_j^2 - 1}{\frac{1}{\lambda_j} + \alpha_j^2} \quad (51)$$

It is the optimal value of the parameter (d) [2], as it is noted that this value will be negative if the value α_j^2 is less than one, and positive if it exceeds the value α_j^2 of one. As (Liu) pointed out that the value of the bias parameter (d) is within the domain (0,1), so there is no specific rule for its estimation, as it is possible to find a single value for this parameter as follows:

- The first formula [1, 3]

$$D_1 = Max\left[0, \frac{\hat{\alpha}_{\max}^2 - 1}{\frac{1}{\lambda_{\max}} + \hat{\alpha}_{\max}^2}\right] \quad (52)$$

This estimator was based on the basic idea adopted by (Horel and Kennard) in adding a positive value to the matrix diameter elements ($X`X$) for the linear regression model, as well as finding the largest element in the formula (19). Where: $\lambda_{\max}$: represents the largest distinct value in the matrix ( $X`\hat{W}X$).

• The second formula [2, 3] This proposal depends on finding the value of the median as one of the measures of central tendency in relation to the bias parameter estimator because it deals with abnormal values in the data that may cause bias

$$D_2 = Max\left[0, mediar\left[\frac{\hat{\alpha}_j^2-1}{\frac{1}{\lambda_j}+\hat{\alpha}_j^2}\right]\right] \quad (53)$$

• The third version [2, 3].

The idea of this proposal comes from the same application of the multiple linear regression model by dealing with the arithmetic mean as the most prominent measure of central tendency, as well as taking into account the number of explanatory variables in the model.

$$D_3 = Max\left[0, \frac{1}{p}\left[\sum_{j=1}^{p} \frac{\hat{\alpha}_j^2-1}{\frac{1}{\lambda_j}+\hat{\alpha}_j^2}\right]\right] \quad (54)$$

• The fourth formula [1,2]

$$D_4 = Max\left[0, \max\left[\frac{\hat{\alpha}_j^2-1}{\frac{1}{\lambda_j}+\hat{\alpha}_j^2}\right]\right] \quad (55)$$

• The fifth version [2],[3]

$$D_5 = Max\left[0, \min\left[\frac{\hat{\alpha}_j^2-1}{\frac{1}{\lambda_j}+\hat{\alpha}_j^2}\right]\right] \quad (56)$$

Proposals (4) and (5) depend on calculating the largest and smallest value of the bias parameter estimator, respectively. It should be noted that finding Leo estimations for the parameters of the Poisson regression model is done by substituting the five formulas into the estimation formula (43).

4. Proposed method

In order to achieve the objectives of the research, the method was proposed to estimate the parameters of the Poisson regression model when that model suffers from the problem of multicollinearity, as method was based on the neutral model shown in formula (1) and then go in two directions in the estimation process. By multiplying the estimator proposed by (Batah) and others in (2008) by the estimator $\left(\frac{I-K^3A^{-3}}{I-K^2A^{-2}}\right)$, a method that many researchers follow in arriving at better estimators [12].

The Poisson Regression (PR) model is only applicable when the dependent variable deals with count data. Suppose, is the dependent variable and follows adistribution with parameter (μ) and is denoted as $P(\mu)$ with the following probability mass function

$$P_r(Y/\mu) = \frac{e^{-\mu}\mu^Y}{Y!} \quad Y_i = 0,1,2,\dots \quad (57)$$

where μ represents the parameter of the distribution, which has a value greater than zero $\mu > 0$. The Poisson regression model can be expressed by the formula (1) and taking its logarithm we get:

$$log(Y) = Loge^{X\beta+U} \quad (58)$$
$$Y^* = X\beta + U$$

And when the above model suffers from a problem (multicollinearity), The transformation method is used. The Poisson regression model can be rewritten as follows:

$$Y^* = XTT`\beta + U$$
$$Y^* = Z\alpha + U \quad (59)$$

where T: an orthogonal matrix of degree $p \times p$ containing (Eigen Vectors) for the matrix ($`X\widehat{W}X$). Z: matrix of explanatory (independent) variables after transformation. α: vector for form parameters after conversion. and that:

$$Z = XT \quad (60)$$
$$\alpha = \acute{T}\beta \quad (61)$$

Also:

$$\acute{Z}\widehat{W}Z = \acute{T}\acute{X}\widehat{W}XT = \mathbf{\Lambda} \quad (62)$$

$\mathbf{\Lambda}$: diagonal Matrix the elements of the diagonal represent the Eigen Values $(\lambda_1, \lambda_2, \ldots, \lambda_p)$ and are for the matrix $\acute{T}\widehat{W}X$. Thus, it will be the Maximum Likelihood Estimator to estimate the parameters of the Poisson regression model after the transformation, which is shown in the formula (59) as follows:

$$\hat{\alpha}_{MLE} = \mathbf{\Lambda}^{-1}\acute{Z}Y^* \quad (63)$$

where: $\hat{\alpha}_{MLE}$: vector Maximum Likelihood Estimators for the parameters of the Poisson regression model after transformation. As for the ridge regression estimator for the same model, it is as in the following formula:

$$\hat{\alpha}_{RR} = (\mathbf{\Lambda} + KI)^{-1}\acute{Z}Y^* \quad (64)$$

where: $\hat{\alpha}_{RR}$: The parameter vector of the ridge regression method of a Poisson regression model after transformation. I :Identity Matrix. K: A diagonal matrix whose diameter elements represent the value of the bias parameter (ridge parameter). By substituting the value $(\acute{Z}Y^*)$ shown in equation (63) with its equal, the estimator of the ridge parameter will be as follows:

$$\hat{\alpha}_{RR} = (\mathbf{\Lambda} + KI)^{-1}\Lambda\hat{\alpha}_{MLE} \quad (65)$$

By adding and subtracting the product of the two matrices (KI) to the expression $(\Lambda\hat{\alpha}_{MLE})$ on the right side of equation (65) , the ridge regression estimator for the Poisson regression model becomes:

$$\hat{\alpha}_{RR} = (\Lambda + KI)^{-1}(\Lambda + KI - KI)\hat{\alpha}_{MLE}$$
$$= (\Lambda + KI)^{-1}((\Lambda + KI) - KI)\hat{\alpha}_{MLE}$$
$$= (\Lambda + KI)^{-1}(\Lambda + KI)\hat{\alpha}_{MLE} - ((\Lambda + KI)^{-1}KI)\hat{\alpha}_{MLE}$$
$$= \hat{\alpha}_{MIE} - (KI(\Lambda + KI)^{-1})\hat{\alpha}_{MLE}$$
$$= [I - KI(\Lambda + KI)^{-1}]\hat{\alpha}_{MLE}$$
$$\therefore \hat{\alpha}_{RR} = [I - KI(A^{-1})]\hat{\alpha}_{MLE} \quad (66)$$

where: A^{-1}: a diagonal matrix in which the elements of the diameter represent the sum of the Eigen Value of a given explanatory variable with the ridge parameter $(\lambda_1 + k, \lambda_2 + k, \ldots, \lambda_p + k)$.
As a generalization of the formula (66) , the estimation of the common ridge regression of the Poisson model after the transformation is as follows:

$$\hat{\alpha}_{GRR} = (I - KA^{-1})\hat{\alpha}_{MLE}. \quad (67)$$

where:
$\hat{\alpha}_{GRR}$: estimated regression of the general ridge Several methods have been proposed stemming from the ridge regression estimator method, the most prominent of which is the Jackknifed Ridge Regression Estimator, which is in the following formula:

$$\begin{aligned}\hat{\alpha}_{JRR} &= (I + KA^{-1})\hat{\alpha}_{GRR} \\ &= (I + KA^{-1})(I - KA^{-1})\hat{\alpha}_{MLE} \\ &= (I - K^2A^{-2})\hat{\alpha}_{MLE}\end{aligned} \tag{68}$$

Within the same context of the ridge regression estimator family, (Batah) and other researchers presented a modification and improvement to the estimator shown in equation (68), and they named it the Modified Jackknifed Ridge Regression Estimator, which is as in the following equation

$$\begin{aligned}\hat{\alpha}_{JRR} &= \hat{\alpha}_{GRR}\hat{\alpha}_{JRR} \\ &= (I + KA^{-1}(I - K^2A^{-2})\hat{\alpha}_{MLE}\end{aligned} \tag{69}$$

And after all this preface, the proposed estimator will be an improvement of the modified ridge regression estimator for Jaknife in the simple linear regression model that was found for the Poisson regression model through the transformation as in the previous steps, as the improvement will be for the estimator shown in the formula (69) ,
By multiplying the estimator by the amount $\left(\frac{I-K^3A^{-3}}{I-K^2A^{-2}}\right)$, and the idea lies in this procedure in order to obtain a diagonal matrix with diagonal elements with very small values, which reduces the added value as it is a bias parameter, and thus this characteristic will be reflected on the estimation process for the parameters through the amount of little bias compared to the previous methods, as it has been the practice of that many researchers by multiplying a certain estimator. In general, the formula for the first proposed estimator will be as below:

$$\begin{aligned}\hat{\alpha}_{\text{Sug}} &= (I - KA^{-1})(I - K^2A^{-2})\left(\frac{I-K^3A^{-3}}{I-K^2A^{-2}}\right)\hat{\alpha}_{MLE} \\ &= (I - KA^{-1})(I - K^3A^{-3})\hat{\alpha}_{MLE} \\ &= (I - HA^{-1})(I - H^3A^{-3})\hat{\alpha}_{MLE} \\ &= W\hat{\alpha}_{MLE}\end{aligned} \tag{70}$$

H: a diagonal matrix is similar to the matrix (K), but has been substituted to represent the values of the bias parameter proposed later and that

$$W = (I - HA^{-1})(I - H^3A^{-3}) \tag{71}$$

Note that we will use the usual method of estimation, through which the matrix (H) will be a diagonal matrix with equal elements $\cdot \left(h_1 = h_2 = \cdots = h_p = h\right)$.
Thus, the vector of the parameters of the estimated Poisson regression model and proposed by the researcher according to the proposed method is as follows:

$$\hat{\beta}_{\text{Sug}} = T\hat{\alpha}_{\text{Sug}} \tag{72}$$

Now the bias for the proposed estimator is found as follows:

$$\begin{aligned}Bias\left(\hat{\alpha}_{\text{Sug}}\right) &= E\left[\hat{\alpha}_{\text{Sug}}\right] - \alpha \\ &= E[(I - HA^{-1})(I - H^3A^{-3})\hat{\alpha}_{MLE}] - \alpha \\ &= (I - HA^{-1})(I - H^3A^{-3})E[\hat{\alpha}_{MLE}] - \alpha \\ &= (I - HA^{-1})(I - H^3A^{-3})\alpha - \alpha\end{aligned} \tag{73}$$

Multiplying the parentheses, we get:

$$Bias(\hat{\alpha}_{\text{Sug}}) = [(I - HA^{-1}) - H^3A^{-3}(I - HA^{-1}) - I]\alpha \quad (74)$$

By taking out (A^{-1}) and (H) from the right side of equation (74), the bias value of the first proposed estimator is as shown below.

$$Bias(\hat{\alpha}_{\text{Sug}}) = H[(HA^{-1})^{-1}(I - HA^{-1}) - (HA^{-1})^{-1}H^3A^{-3}(I - HA^{-1}) - (HA^{-1})^{-1}I]A^{-1}\alpha$$
$$= H[-(HA^{-1})^{-1}\{I - (I - HA^{-1})\} - H^2A^{-2}(I - HA^{-1})]A^{-1}\alpha$$
$$= -H[(HA^{-1})^{-1}\{I - (I - HA^{-1})\} + H^2A^{-2}(I - HA^{-1})]A^{-1}\alpha$$
$$= -H\theta A^{-1}\alpha \quad (75)$$

Since:

$$\theta = [(HA^{-1})^{-1}\{I - (I - HA^{-1})\} + H^2A^{-2}(I - HA^{-1})] \quad (76)$$

The estimator resulting from the proposed estimation method shown in formula (70) is biased when it $H > 0$ and the amount of bias is:

$$Bias(\hat{\alpha}_{\text{Sug}}) = E(\hat{\alpha}_{\text{Sug}}) - \alpha$$
$$= W\alpha - \alpha$$
$$= (W - I)\alpha \quad (77)$$

The variance and covariance matrix of the proposed estimator according to the proposed method.

$$Var - Cov(\hat{\alpha}_{Sug}) = WVar - Cov(\hat{\alpha}_{MLE})W'$$
$$= W\sigma_u^2\Lambda^{-1}W' \quad (78)$$

The mean square error (MSE) for the parameters of the Poisson regression model estimated according to the proposed estimation method is:

$$MSE(\hat{\alpha}_{\text{Sug}}) = E(\hat{\alpha}_{\text{Sug}} - \alpha)'(\hat{\alpha}_{\text{Sug}} - \alpha)$$
$$= E[(\hat{\alpha}_{MLE} - \alpha)'W'W(\hat{\alpha}_{MLE} - \alpha)] + (W\alpha - \alpha)'(W\alpha - \alpha)$$
$$= tr[(\hat{\alpha}_{MLE} - \alpha)'(\hat{\alpha}_{MLE} - \alpha)W'W] + (W\alpha - \alpha)'(W\alpha - \alpha)$$

$$MSE(\hat{\alpha}_{\text{Sug}}) = \sigma_u^2 \sum_{j=1}^{p} \frac{\left(\lambda_j^4 + 3\lambda_j^3 h + 3\lambda_j^2 h^2\right)}{\lambda_j(\lambda_j + h)^8} + \sum_{j=1}^{p} \left(\frac{(\lambda_j + h)^3 h + \lambda_j h^3}{(\lambda_j + h)^4}\right)^2 \hat{\alpha}_j^2 \quad (79)$$

4.1. Proposed estimators for the bias parameter

After the proposed method has been shown in estimating the parameters of the Poisson regression model, it was noted that method is biased in estimating those parameters, and the bias is due to the presence of the diagonal matrix (H) when that matrix contains values $(h > 0)$, so several estimators will be proposed for the bias parameter, as the proposal will be based on a combination of from the suggestions of previous researchers and new ideas, the following are the suggestions of the bias parameter. We focused in our selection of the proposed estimators for the bias parameter (h) by relying on a combination of estimations adopted by researchers in previous studies [6, 1, 7, 9], which showed preference in estimating the parameters of different regression models because they contain the most important factors affecting the nature of the existence and effect of the multilinearity problem such as the Eigen values(λ_j) o?f the matrix (X`$\widehat{W}X$), the value of the estimated random error variance $\hat{\sigma}^2$ and the value of $\hat{\alpha}_j^2$.

- The first proposal:

The idea of this proposal is based on finding a correlation between the variance of the random error with the largest eigen value of the matrix (X`$\widehat{W}X$), as well as using the max as one of the measures of central tendency. The formula for this proposed estimator is as follows:

$$H_1 = \left\{Max\{q_j\}\right\} \tag{80}$$
$$q_j = \frac{\lambda_{\max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2+\lambda_{\max}\hat{\alpha}^2} \tag{81}$$

$\lambda_{\max}$:Represents the largest eigen value of the matrix ($X`\widehat{W}X$).

• The second proposal:

The idea in constructing this proposal is to find an estimator that represents the largest value for the product of two previous important estimators in estimating the bias parameters, which are (q_j) and (s_j) with the slight difference between them, which is that one of them contains the largest distinct value, while the other includes all those distinct values, and the formula for this The proposal is as follows:

$$H_2 = Max\left\{\{q_j s_j\}\right\} \tag{82}$$

where: (s_j) : As previously defined in formula (30)

• The third proposal:

Through the previous proposal, a simple mechanism will be added in calculating the third proposal by adopting the mediator in its calculation instead of the largest value for the product of the previous two estimators (q_j) and (s_j), and the formula of this proposed estimator is:

$$H_3 = \{Median\{q_i s_i\}\} \tag{83}$$

• The fourth proposal:

The idea of this proposed estimator is based on finding the mean value for the product of two variables, one of which represents the square root transformation principle of the estimator (Horel and Kennard) shown in the formula (23), and the other represents the correlation of the mean of the squares of error with the largest characteristic value, and the formula of this proposed estimator is as following

$$H_4 = \{Mean\{q_i m_i\}\} \tag{84}$$

5. The Monte Carlo simulation

This section consists of a brief description of how the data is generated together with a discussion of our findings.

5.1. The design of the experiment

The dependent variable of the Poisson regression model is generated using pseudo-random numbers from the $Po(\mu_i)$ distribution, where

$$\mu_i = \exp(\beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip}), i = 1,2, \ldots n, j = 1,2, \ldots p \tag{85}$$
$$\sum_{j=1}^{p} \beta_j^2 = 1 \tag{86}$$

Following [11], the parameter values in equation (85) are chosen so that $\sum_{j=1}^{p} \beta_j^2 = 1$ and $\beta_1 = \cdots = \beta_p$. To be able to generate data with different degrees of correlation we use the following formula to obtain the regressors:

$$x_{ij} = (1-\rho^2)^{(1/2)} z_{ij} + \rho z_{ip}, i = 1,2, \ldots n, j = 1,2, \ldots p.$$

where z_{ij} are pseudo-random numbers generated using the standard normal distribution and ρ^2 represents the degree of correlation [6]. In the design of the experiment three different values of ρ^2 corresponding to (0.90, 0.95 and 0.99) are considered which are shown in Table (1). As for the other factor that will be taken into consideration as a variable and influencing factor, it is the sample size, as four sample sizes will be taken, which are (20, 50, 100, 150, 200) as in Table (1), in order to study the comparison according to samples of different types (small, medium , big). The last factor that will be taken into consideration is the number of independent (illustrative) variables in the model as in Table (1), as we will study the effect of having two independent variables and four independent variables in the process of comparison between the different estimation methods.

Table 1. Various factors and their values in a simulation experiment

factors	values
n	(20, 50, 100, 150, 200)
ρ^2	0.90,0.95,0.99
p	2,4,6

In order to make a comparison between the different estimation methods, we will rely on the(Mean Squared Error) as a criterion for comparing the average estimation parameters β_j, according to the formula below

$$MSE = \frac{\sum_{r=1}^{R} SE_i}{R} = \frac{\sum_{r=1}^{R} (\hat{\beta}-\beta)'_r(\hat{\beta}-\beta)_r}{R} \quad r = 1,2,\dots,2000$$

where: $\hat{\beta}$: The value of the parameter estimated according to the different estimation methods. β : Parameter value in constraint (86) R : The number of times the experiment is repeated, which will be taken to be equal to (2000)

5.2. Simulation results

Tables 4-9 show the results of the simulation experiment, which were obtained by Monte Carlo Simulation by Using R. To find the mean square error (MSE) for all methods using the equation (7) for the method of Maximum Likelihood Estimator (MLE), and through direct substitution of formulas (23), (25), (26), (27), (29), (31), (32), (33) in equation (15) for the ridge regression method, and direct substitution for the formulas (52), (53),(54), (55), (56) in equation (43) for the Liu estimators method, and formulas (80), (82), (83), (84) for the proposed method by substituting into formula (72) The results reflect the mean square error (MSE) values for all previous estimation methods and the proposed method.
After the simulation experiment was conducted and implemented, the results were extracted and interpreted according to the change of the influencing research factors, each separately, degree of correlation, number of explanatory variables and sample size. The results will be interpreted by taking all the influencing factors into consideration, through the simulation result in Tables (4), (5), (6), (7), (8) and (9). We observed that increasing the sample size led to a decrease in the mean square error MSE values of all the estimators, which is one of the unique properties for any statistical estimator. The proposed estimator, consistently possessed the minimum MSE. The proposed method, according to the estimated bias parameter H_1, appears as the best method to estimate the parameters of the Poisson regression model in all different conditions.
The standard presentation for the simulation results is the tables (as in above). However, in many studies the tables are not readable, so should be present the results by the graphs.
Figures 1-3 show the performance of bias parameters for ridge and Liu estimators for sample sizes,different ρ and different independent variables P compared with maximum likelihood estimator method based on the MSE and the relative efficiency $[MSE\hat{\beta}_{RR}/MSE\hat{\beta}_{ML}]$ for ridge estimator and $[MSE\hat{\beta}_{Liu}/MSE\hat{\beta}_{ML}]$ for Liu estimator. From these figures, we have the following notes:

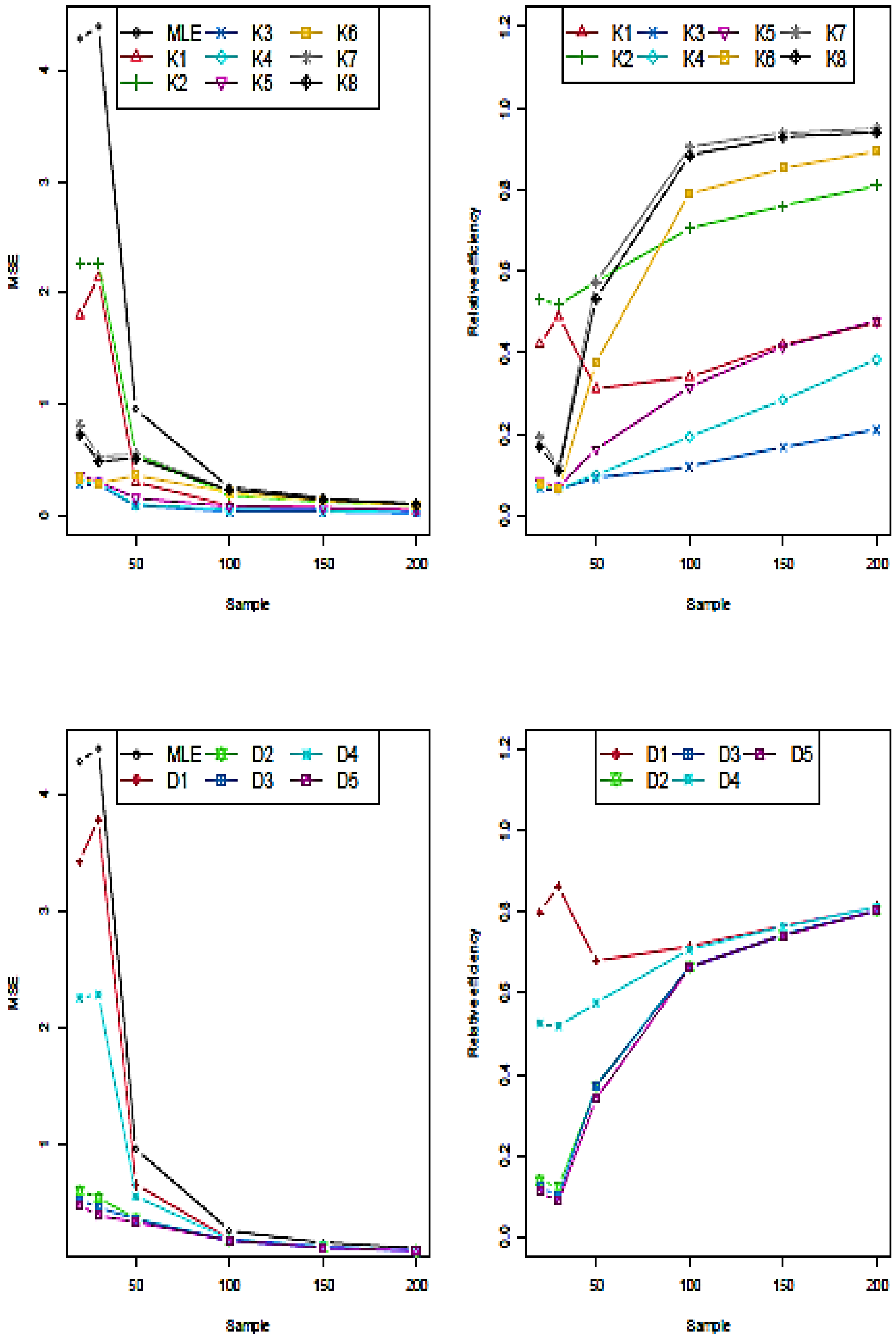

Figure 1. MSE and relative efficiency ratios of different estimators vs sample sizes n

5.2.1. Interpretation of the results according to the change in the sample size n

The acurate observation of figure 1 shows the decline in the values of mean squares of error as the sample size increases, as this is evident in all estimation methods, especially at the bias parameterK_3, D_5 and this reflects one of the good characteristics when the value of the estimator approaches the real value of the parameter by increasing the sample size. We also note the superiority of all estimation methods, over the method of maximum likelihood estimator MLE. Moreover, it has a minimum value of the relative efficiency, it starts increasing all over the range of n values.

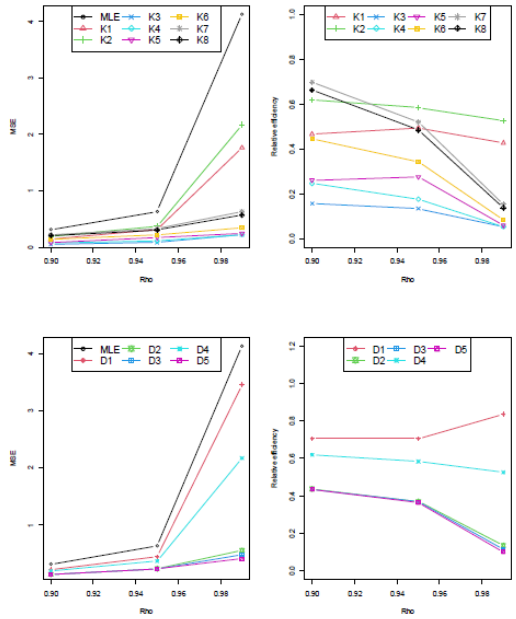

Figure 2. MSE and relative efficiency ratios of different estimators vs different correlation matrix ρ

5.2.2. Interpretation of the results according to the change in the value of the correlation matrix ρ

The simulation results were recorded when all the different factors were proven and the value of the correlation coefficient differed with the result obtained in the previous paragraph. Through Figure 2 , we can easily notice the superiority of the majority of estimation methods over the method of MLE. As the value of the correlation coefficient increases, the differences between the MSE for all methods begin to increase in the mean of the correlation coefficient 0.95. Furthermore, the efficiency of estimation methods is decreasing in moderate correlation coefficient.

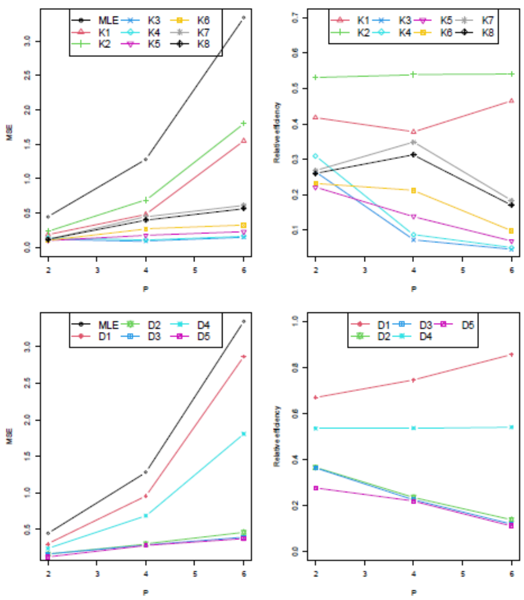

Figure 3. MSE and relative efficiency ratios of different estimators vs different independent variables P

5.2.3. Interpretation of the results according to the change in the number of independent variables $\boldsymbol{P}$

The values of the mean squares error (MSE) for the parameters of the Poisson regression model estimated according to all methods were recorded when calculating them for the model that includes four and six independent variables as in tables a noticeable increase over their calculated counterparts for the Poisson regression model that includes two independent variables. Also, the relative efficiency is less than 1 , it starts decreasing all over the number of independent variables P values.

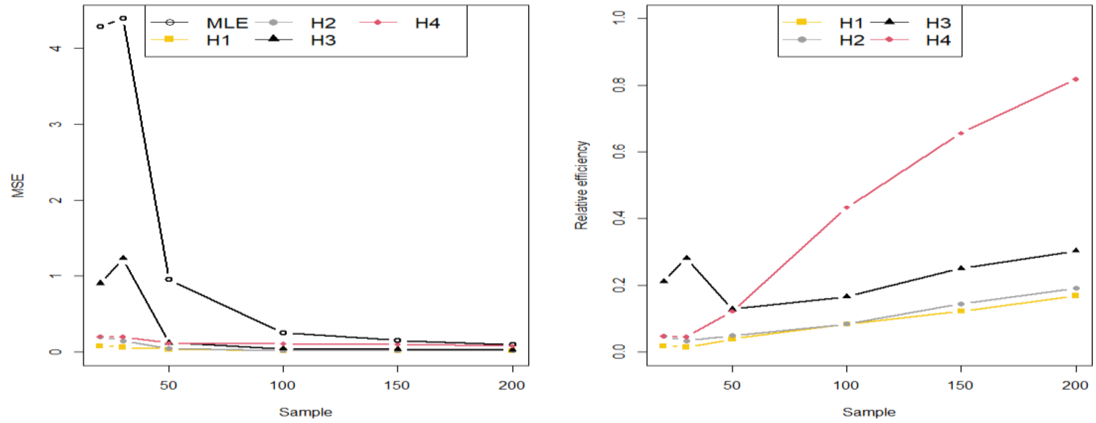

Figure 4. MSE and relative efficiency ratios of proposed method estimator vs different sample sizes n

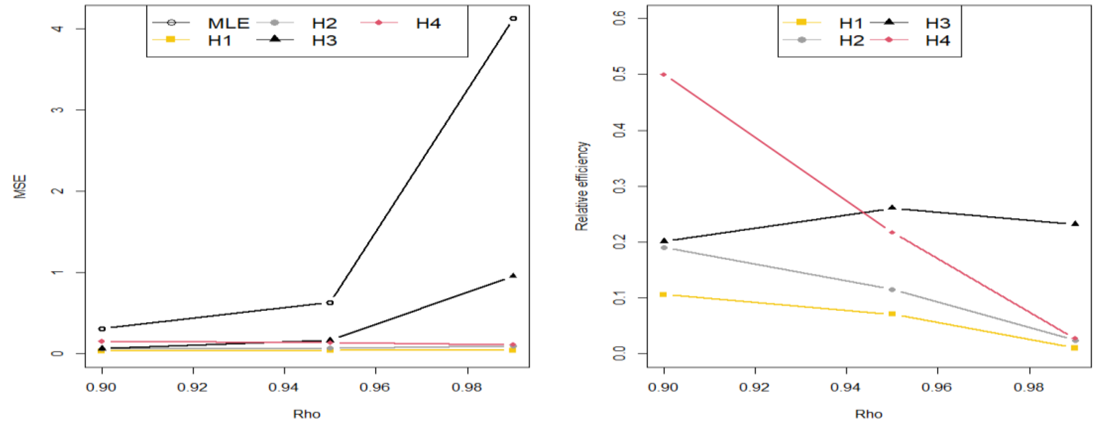

Figure 5. MSE and relative efficiency ratios of proposed method estimator vs different correlation matrix ρ

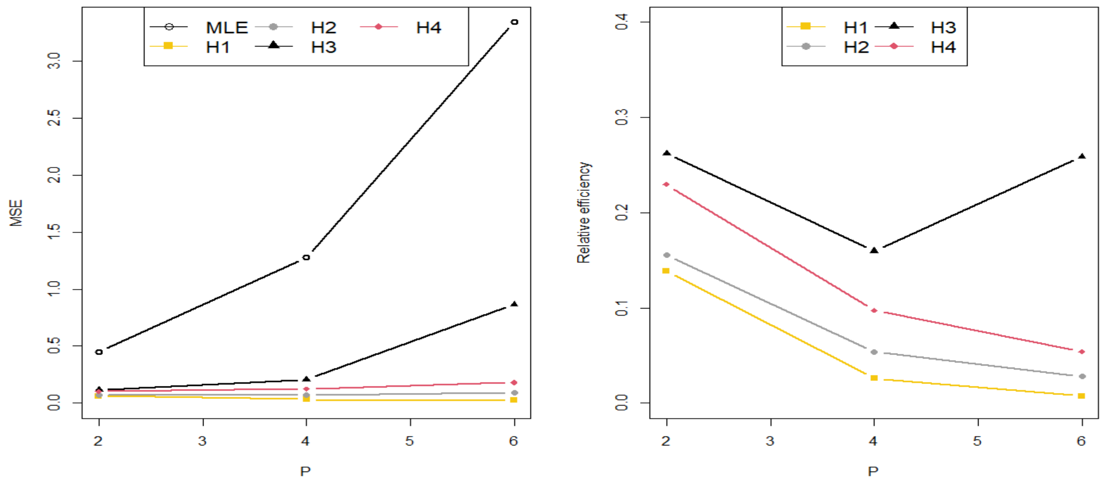

Figure 6. MSE and relative efficiency ratios of of proposed method estimator vs different independent variables P

In these Figs 4, 5, 6 display the performance of the proposed method for different n, different ρ and different independent variables P compared with maximum likelihood estimator method based on the MSE and the relative efficiency $[MSE\hat{\beta}_{sug}/MSE\hat{\beta}_{ML}]$. We also note the superiority of the proposed method over the method of the MLE with the probability of the first estimator for the bias parameter H_1 according to the suggested method, whose MSE begins to decrease significantly as the sample size increases, ρ and P.

Moreover, the relative efficiency ratio is close to zero in moderate samples. We also observe that as the value of the ρ and P increases, the relative efficiency ratio is close to zero with a value of 0.1.

6. Application

To illustrate the performance of the estimators, we consider the study data, which is in the form of count data in the shape of totals within half monthly terms (collected for every two weeks) for the term from (2006) until the end of the year (2012), which pertain to congenital defects of the heart and circulatory system in newborns In the capital, Baghdad, from the Central Children's Teaching Hospital in the Al-Iskan neighborhood, west of Baghdad, the distribution of the response variable (heart and circulatory abnormalities) will be studied and the presence of the problem of multicollinearity between the explanatory variables in the application will be revealed. There are seven explanatory variables: X_1: represents the total weights of affected children within each time period. X_2: represents the totals of the parents' ages of affected children within each time period. X_3: represents the sums of mothers' ages of affected children within each time period. X_4: Represents the number of infected male children within each time period. X_5: Represents the number of infected female children within each time period. X_6: Represents the number of infected children born from consanguineous marriages within each time period. X_7: Represents the number of infected children whose mothers were exposed to radiation or life influences such as taking certain medications and drugs during pregnancy within each time period. The response variable is$Y$: the sum of children with congenital heart and circulatory defects within each time period.
The Poisson regression model was also built as one of the appropriate models to describe the data of that study. The formula of the model was as follows:

$$Y_i = e^{\left(\begin{array}{l}\beta_1 X_{i1}+\beta_2 X_{i2}+\beta_3 X_{i3}+\beta_4 X_{i4} \\ +\beta_5 X_{i5}+\beta_6 X_{i6}+\beta_7 X_{i7}+u_i\end{array}\right)} \quad ()$$

6.1. Multilinearity problem and application data

In order to detect whether there is a duplication or multilinearity between two or more explanatory variables in apply, it is noticed through the correlation matrix that there are correlation coefficients with large values and direct direction for all explanatory variables, as each of them is associated with all other explanatory variables with strong direct linear relationships In addition, find the values of the variance inflation factor VIF shown in Table (2).
The results of Table (2) also reflect significant values of the VIF scale for all model variables, as the largest of them were those of the second, third and fourth explanatory variables, and the VIF for the remaining explanatory variables exceeded the (10) barrier, which indicates the existence of the problem of multilinearity.

Table 2. Correlation matrix and VIF between explanatory variables in the application

ρ^2	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	X_{i6}	X_{i7}
X_{i1}	1.000	0.990	0.990	0.963	0.939	0.953	0.844
X_{i2}		1.000	0.999	0.984	0.943	0.971	0.873
X_{i3}			1.000	0.987	0.932	0.976	0.878
X_{i4}				1.000	0.897	0.988	0.923
X_{i5}					1.000	0.904	0.849
X_{i6}						1.000	0.935
X_{i7}							1.000
VIF	853.2458	115493.9560	65038.1281	18048.0997	4893.6051	1138.7312	3081.5336

Eigenvalues and eigenvectors of the correlation matrix indicate the degree of multicollinearity. An eigenvalue that approaches to zero indicates a very strong linear dependency between regressors, while , the elements of the associated eigenvector display the weights of the corresponding regressor variables in the multicollinearity.
Furthermore, multicollinearity can be measured in terms of the ratio of the largest and the smallest eigenvalue. This quantity is called the condition number of the correlation matrix:

$$CN = \sqrt{\frac{\max(\text{eigenvalue})}{\min(\text{eigenvalue})}} = 46748.74$$

Large values of condition number CN are an indication of serious multicollinearity, also reflect the values of CN especially those related to the sixth and seventh variables. The extent of their direct impact on the emergence of the problem of multicollinearity in the studied model. CN of the correlation matrix X is between $(30-100)$ indicates a moderate to strong correlation and a CN greater that 100 suggest severe multicollinearity [15]. In addition to that,the number of eigenvalues near zero indicates the number of collinearities detected among the regressor variables.

6.2. Application results

After the appropriateness of the distribution of the dependent variable was observed according to the Poisson distribution as in the test of good fit, and the problem of multilinearity between the independent variables in use was revealed through VIF, the Poisson regression model became the appropriate model to express the cases of congenital defects in the heart and circulatory system in children. The proposed method with bias parameter (H_1) was used to estimate the parameters of the Poisson regression model when there is a near-perfect multilinearity problem in the application, due to its preference over all other estimation methods, as the Monte Carlo simulation program was used to calculate the estimation of the parameters as well as the standard deviation of the estimated parameters. and compared with the MLE method, as shown in Table 3.

Table 3. Parameter estimates based on the proposed and MLE method of Poisson regression model for application data

Explanatory Variable	$\hat{\beta}_{ML}$	$\hat{S}\beta_{ML}$	$\hat{\beta}_{H_1}$	$\hat{S}\beta_{H_1}$
(Intercept)	0.3662397	0.67153867	0.0000017	0.0003247
X_{i1}	0.0196046	0.07896312	0.0001601	0.0028194
X_{i2}	0.01973763	0.07653075	0.00180	0.0767312
X_{i3}	-0.00442303	0.07963316	0.0013144	0.0505632
X_{i4}	-0.72999177	1.24776644	0.0000396	0.2496800
X_{i5}	-0.64842063	1.81908397	0.0000133	0.0461439
X_{i6}	0.330966348	0.99071805	0.000011	0.0001320
X_{i7}	0.455675664	1.16882605	0.00001276	1.1543453
MSE	NA	7.68306196	NA	1.4054349

Thus, the estimated regression equation for the number of children with congenital heart and circulatory defects is as follows:

$$\hat{Y}_i = e^{\left\{\begin{matrix}0.0000017+0.0001601X_{i1}+0.00180X_{i2}+0.0013144_{i3} \\ +0.0000396X_{i4}+0.0000133X_{i5}+0.000011X_{i6}+0.00001276X_{i7}\end{matrix}\right\}}$$

The interpretation of these results indicates that the number of children with congenital defects in the heart and circulatory system depends on the extent of the increase in all parameters of the model, since all parameters are influential, but in varying proportions, in increasing the number of children with these birth defects.

Table 4. Simulated MSE when p =2

n	p	MLE	K1	K2	K3	K4	K5	K6	K7	K8
20	0.9	0.104188	0.086681	0.08969	0.097311	0.161416	0.088423	0.091679	0.093381	0.093183
20	0.95	0.131632	0.081306	0.102622	0.061351	0.093251	0.08118	0.108417	0.113427	0.112518
20	0.99	2.016914	0.81284	0.934663	0.453138	0.356054	0.27971	0.144957	0.204992	0.189483
30	0.9	0.100205	0.068432	0.084033	0.06837	0.145784	0.071384	0.088757	0.09176	0.091185
30	0.95	0.67371	0.343376	0.364123	0.287939	0.315873	0.28694	0.222807	0.257659	0.25207
30	0.99	1.940592	0.693801	0.936583	0.349549	0.26357	0.188198	0.158021	0.233261	0.21427
50	0.9	0.093507	0.066244	0.078477	0.059939	0.109796	0.069091	0.082904	0.085497	0.085017

n	p	MLE	K1	K2	K3	K4	K5	K6	K7	K8
50	0.95	0.243121	0.105665	0.163956	0.086144	0.133863	0.112654	0.182243	0.191431	0.189994
50	0.99	1.965247	0.682758	0.933773	0.32776	0.235215	0.157007	0.145002	0.220344	0.201165
100	0.9	0.021264	0.018315	0.020398	0.018644	0.068339	0.018578	0.020823	0.02099	0.020957
100	0.95	0.050069	0.036374	0.045642	0.028186	0.072568	0.037246	0.047707	0.048564	0.048379
100	0.99	0.271994	0.105828	0.171948	0.080962	0.108086	0.105476	0.188556	0.200095	0.198359
150	0.9	0.020371	0.018866	0.019676	0.019537	0.046228	0.018981	0.019927	0.020077	0.020051
150	0.95	0.035566	0.028184	0.033147	0.023196	0.076326	0.028849	0.034243	0.034791	0.034664
150	0.99	0.199672	0.09614	0.142522	0.072388	0.093952	0.101906	0.156985	0.164617	0.163364
200	0.9	0.018221	0.016435	0.017608	0.015292	0.054985	0.016557	0.017868	0.018032	0.017992
200	0.95	0.029033	0.025274	0.02758	0.021706	0.076162	0.025838	0.02816	0.134768	0.133785

Table 5. Simulated MSE when p =2

n	p	D1	D2	D3	D4	D5	H1	H2	H3	H4	Best
20	0.9	0.091034	0.090606	0.090606	0.091021	0.090567	0.083833	0.084752	0.087671	0.135617	H1
20	0.95	0.10318	0.102254	0.102254	0.103079	0.101958	0.039458	0.045943	0.056714	0.105037	H1
20	0.99	1.316376	0.497038	0.497038	0.943697	0.303073	0.169099	0.264872	0.525757	0.346063	K6
30	0.9	0.084692	0.084168	0.084168	0.084678	0.0841	0.044197	0.050034	0.058391	0.106261	H1
30	0.95	0.399252	0.304848	0.304848	0.370425	0.272414	0.237422	0.291233	0.405953	0.298239	K6
30	0.99	1.27715	0.521544	0.521544	0.941724	0.289067	0.091735	0.093466	0.250323	0.185637	H1
50	0.9	0.079168	0.078929	0.078929	0.079165	0.07891	0.049091	0.054673	0.063528	0.072197	H1
50	0.95	0.166475	0.162233	0.162233	0.165896	0.16084	0.069458	0.057799	0.09716	0.077695	H2
50	0.99	1.284326	0.507524	0.507524	0.937268	0.262965	0.066568	0.056451	0.185666	0.14475	H2
100	0.9	0.020426	0.020423	0.020423	0.020426	0.020423	0.015226	0.014938	0.015659	0.050675	H2
100	0.95	0.045736	0.045714	0.045714	0.045736	0.045714	0.020086	0.022454	0.027266	0.050099	H1
100	0.99	0.174554	0.167773	0.167773	0.173166	0.164217	0.059875	0.043138	0.088704	0.056091	H2
150	0.9	0.019705	0.019704	0.019704	0.019705	0.019704	0.017599	0.017793	0.018139	0.035439	H1
150	0.95	0.033193	0.033179	0.033179	0.033193	0.033179	0.01595	0.019668	0.023485	0.041126	H1
150	0.99	0.143663	0.141561	0.141561	0.143372	0.140553	0.058994	0.052256	0.091567	0.048548	H4
200	0.9	0.017618	0.017616	0.017616	0.017618	0.017616	0.01125	0.013844	0.01452	0.032823	H1
200	0.95	0.027613	0.027602	0.027602	0.027613	0.027602	0.019487	0.022278	0.02423	0.028776	H1
200	0.99	0.118092	0.117355	0.117355	0.118028	0.117135	0.05045	0.047857	0.079807	0.038138	H1

Table 6. Simulated MSE when p =4

n	p	MLE	K1	K2	K3	K4	K5	K6	K7	K8
20	0.9	1.967746	0.994522	1.029387	0.232643	0.305109	0.412965	0.382274	0.80349	0.648807
20	0.95	1.883315	0.995808	1.06254	0.397429	0.460243	0.691287	0.5868	0.956536	0.852852
20	0.99	8.276267	3.091269	4.09262	0.37572	0.354419	0.390684	0.365937	0.857596	0.796338
30	0.9	0.181323	0.106549	0.151112	0.047125	0.073483	0.107488	0.163766	0.176311	0.174351
30	0.95	0.542852	0.170823	0.350097	0.037139	0.047984	0.139823	0.384935	0.479243	0.452611
30	0.99	5.183919	1.779902	2.592928	0.188663	0.241152	0.282245	0.339307	1.04654	0.81814
50	0.9	0.127841	0.089714	0.112674	0.039879	0.043899	0.090022	0.11915	0.125182	0.1245
50	0.95	0.734554	0.354067	0.475734	0.104577	0.125852	0.309644	0.465245	0.597947	0.560323
50	0.99	2.239675	0.417165	1.176007	0.039792	0.043774	0.114662	0.5248	1.224439	1.068623
100	0.9	0.068535	0.055969	0.065055	0.033314	0.049075	0.056661	0.066828	0.068119	0.068057
100	0.95	0.193742	0.089469	0.15455	0.021746	0.02884	0.090341	0.173038	0.188051	0.185168
100	0.99	0.749685	0.188202	0.470045	0.032096	0.043113	0.147495	0.527601	0.648415	0.613803
150	0.9	0.022602	0.020723	0.022218	0.016761	0.031914	0.020787	0.022432	0.022574	0.022572
150	0.95	0.068661	0.049561	0.064069	0.020247	0.03686	0.050319	0.066642	0.068229	0.068129

150	0.99	0.376695	0.114284	0.266373	0.016587	0.02158	0.107645	0.309126	0.353406	0.342842
200	0.9	0.022525	0.020573	0.022151	0.017562	0.036281	0.020695	0.022368	0.022498	0.022497
200	0.95	0.032739	0.028548	0.031828	0.013436	0.022943	0.028628	0.032329	0.032672	0.032664
200	0.99	0.384462	0.123655	0.280932	0.019419	0.022488	0.119443	0.326378	0.362255	0.354326

Table 7. Simulated MSE when p =4

n	p	D1	D2	D3	D4	D5	H1	H2	H3	H4	Best
20	0.9	1.299652	0.521471	0.504378	1.030031	0.502703	0.072194	0.094894	0.507993	0.185425	H1
20	0.95	1.190709	0.725939	0.714521	1.072598	0.707311	0.182472	0.521442	1.06211	0.26261	H1
20	0.99	6.823778	0.759454	0.673982	4.02928	0.575627	0.077093	0.186295	0.983028	0.16878	H1
30	0.9	0.151905	0.151394	0.151394	0.151885	0.151394	0.026012	0.025381	0.035234	0.269279	H2
30	0.95	0.359162	0.318053	0.318053	0.351451	0.318053	0.018393	0.019356	0.052594	0.167385	H1
30	0.99	3.869278	0.653369	0.519042	2.573597	0.512773	0.026024	0.066335	0.317822	0.081585	H1
50	0.9	0.113301	0.113268	0.113268	0.113299	0.113268	0.036869	0.036641	0.038812	0.155092	H2
50	0.95	0.488961	0.432416	0.432002	0.478931	0.432002	0.037967	0.113989	0.265184	0.084202	H1
50	0.99	1.517106	0.434711	0.434134	1.172363	0.434134	0.006699	0.012819	0.056039	0.072529	H1
100	0.9	0.065169	0.065166	0.065166	0.065169	0.065166	0.027886	0.029472	0.035885	0.128092	H1
100	0.95	0.155126	0.154543	0.154543	0.155069	0.154543	0.01632	0.017171	0.032634	0.110905	H1
100	0.99	0.482121	0.418576	0.418205	0.470572	0.418205	0.005545	0.017506	0.095803	0.04551	H1
150	0.9	0.022225	0.022225	0.022225	0.022225	0.022225	0.012723	0.013688	0.01291	0.125138	H1
150	0.95	0.064151	0.064146	0.064146	0.064151	0.064146	0.016939	0.018715	0.023305	0.102199	H1
150	0.99	0.268702	0.255518	0.255518	0.266932	0.255518	0.005256	0.018716	0.056273	0.0484	H1
200	0.9	0.022159	0.022159	0.022159	0.022159	0.022159	0.012303	0.012283	0.01418	0.1030	H1
200	0.95	0.03184	0.031839	0.031839	0.03184	0.031839	0.012344	0.012657	0.014593	0.09849	H1
200	0.99	0.282229	0.275345	0.275345	0.281326	0.275345	0.004471	0.017903	0.079601	0.034562	H1

7. Conclusions

The ridge regression and Liu estimator at a different time were corresponded to the Poisson Regression Model to solve multicollinearity. However, in this study, we developed a new estimator, establish its statistical properties, carried out theoretical comparisons with the estimators mentioned above.

The increase in the sample size and the number of independent variables does not constitute any obstacles towards the efficiency of the proposed method in estimating the parameters of the Poisson regression model, while these factors affect the efficiency of some of the previous estimation methods. the proposed estimation method across all bias parameters and especially H_1 represents the optimal solution when the value of the correlation coefficient between the independent variables is increased.

Furthermore, the efficiency of proposed method estimator less than 1 (or the relative efficiency ratio is close to zero) under the effect of n, ρ and p indicates that $\hat{\beta}_{ML}$ is not at efficient as $\hat{\beta}_{sug}$ in estimating the parameter value with smaller mean square error.

In addition to that, we conducted a simulation experiment and analyzed a real-life application to display the proposed estimator effectiveness. The simulated and application results display that the proposed method estimators based on bias parameters outperform the ridge regression and Liu estimators , while MLE has the worst performance.

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