ISSN 2303-4521
Original Research
Periodicals of Engineering and Natural Sciences
Vol. 10, No. 2, April 2022, pp.219-230

Comparison of the two hybrid models, Wavelet-ARIMA and Wavelet-ES, to predict the prices of the US dollar index

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ABSTRACT

The US dollar index is one of the most important measures to compare the value of the US dollar against a basket of foreign currencies. The strategic importance of this index lies in avoiding risks and fluctuations in the basket of major global currencies. It is known that the process of accurate prediction must take place after understanding the nature of the data of the phenomenon under study, and accordingly we can employ the most appropriate models to obtain the best predictive values. In this paper, we made a comparison between two models from the hybrid wavelet transform models, namely Wavelet-ARIMA and Wavelet-ES, by applying to data representing the weekly rates of the last price of the US dollar index from 2011 to 2022, in order to get the best predictive values for this indicator. The results of the comparison criteria AIC, RMSE and MAPE indicated the preference of the hybrid Wavelet-ARIMA model, which was used to predict the weekly rates of the index (USDX). These results indicated that there would be no significant changes or fluctuations during the next sixteen weeks, the weekly average of the index price will be ($96), the lowest predictive value of the index will be ($95.24), which will be recorded in the fourteenth week, and that the fifteenth week will record the highest predictive value of the index, as it will amount to ($96.31).

Keywords: ARIMA, wavelet transform, exponential smoothing, Wavelet-ARIMA, Wavelet-ES

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1. Introduction

Time series models are widely used to predict the behavior of phenomena in many fields such as economics, engineering, natural sciences and humanities, many studies have been conducted using linear models such as ARIMA models or using non-linear models such as GARCH, ANN, Wavelet models, as well as studies that have been conducted Using hybrid models such as Wave-ARIMA, Wave-ES and Wave-ANN. The application of ARIMA models depends on the analysis of time for stable linear time series, but it is not necessary that the studied time series be linear and stable, so the application of nonlinear time series models is used, the wavelet transform, for example, is the best way to analyze any signal (data) in the time series, as the wavelet transform analyzes the time series in terms of time and frequency, which helps to perform accurate, detailed and flexible smoothing and filtering.

Hybrid models have been used in many studies, Kumar used the ARIMA and Wave-ARIMA hybrid models in order to predict the prices of gas and oil for two companies, by comparing these models, they conclude that the Wave-ARIMA model gives better results in prediction than the direct use of the ARIMA model on the data, [7]. Nury compared between Wave-ANN and Wave-SARIMA models to predict the maximum and minimum temperature in an area located in eastern Bangladesh. He concluded that Wave-SARIMA is more effective than Wave-ANN, [12]. Valvi compared between three models to predict the closing prices of IT companies' stocks, the first model depends on smoothing the time series through the wavelet transformation and then finding the appropriate ARIMA model for the smoothed partial time series, the second model is the Wave-ARIMA hybrid model, and the third is the ARIMA model, it was concluded that the second model is more accurate than the rest of the models, [16]. Muhammad worked on selecting the best model among the two models, Wave-ARIMA and ARIMA, to predict the drought phenomenon in Malaysia, he concluded that the Wave-ARIMA model was preferred over the ARIMA model, [10]. Singh and others worked to predict the numbers of deaths from the Corona virus for the countries of Italy, America, Spain, France and Britain, by comparing the ARIMA models with the Wave-ARIMA models, and the results came to prove the preference of hybrid models, [18]. Jankova conducted an empirical study aimed at obtaining the best model for forecasting seasonal time series, by comparing SARIMA with Wave-SARIMA models, and the experimental results showed the effectiveness and accuracy of prediction for hybrid models, and that Wave-SARIMA model can be used to predict suitable seasonal time series, [6]. Finally, in a study that aims to determine the appropriate model for predicting the daily and monthly water flow in western Nishnaputna and Trinity River with different water conditions, Nourani made a comparison between non-linear models, SARIMA, ANN, ES, with hybrid models which are Wave-SARIMA, Wave-ANN, Wave -ES, as well as a proposed hybrid model which is Wave-WES, the results of the comparison indicated that the hybrid models are better in reducing the impact of noise and seasonal changes in the time series than the traditional SARIMA models, and that the proposed hybrid model Wave-WES is the most suitable model for predicting water flow ,[11].

In this research, the US dollar index is predicted, which is a measure of the value of the US dollar relative to the value of the group of currencies of the most important trading partners of the United States. This indicator is calculated based on the exchange rates of six major currencies: the euro, the Japanese yen, the Canadian dollar, the pound sterling, the Swedish krona and the Swiss franc. The US Dollar Index allows traders to monitor the value of the US dollar against the group of the six currencies in a single transaction in order to be wary of their dollar-related investments. It is possible to integrate contract strategies based on the US dollar index, as well as the possibility for investors to benefit from the index to be wary of currency movements and speculation on them. The most difficult task here is where and when investors can buy and sell index shares, in general, these difficulties apply to most trading and investment operations in the currency and financial markets, so forecasting the future values of financial indicators must be efficient and accurate. To our knowledge, studies related to forecasting this indicator are rare and if available, they included studies that depended on the employment of linear and non-linear models only, we mention the study (Liu, Z), which included prediction and description of the volatility in the prices of the US dollar index, using the ARIMA and GARCH models, the study concluded that there are differences in the prediction of the US dollar index between the two models, and that the ARIMA model is an appropriate and effective method when forecasting for a month, while the GARCH model is an appropriate method for predicting the fluctuations of the US dollar index in the long term. Based on the foregoing, and on the basis of the nature of the data of the series of weekly rates of dollar index prices, we will employ the two hybrid models Wave-ARIMA and Wave-ES, and then compare between them with the aim of obtaining a suitable model for forecasting weekly price rates.

2. Wavelet transformation

Wavelet transformation used to analyze the signal into a set of multiple levels solution in both time and frequency. This transformation use variate width window to get the frequency changes throughout the wavelength, in able to produce a limited length signal with zero average value called wavelet, this wavelet compressed with two functions, the first is called mother function to obtain a set of coefficients called detailed coefficients, and the second is called father function which is a measurement function to get the approximation coefficients. Wavelet transformation Wavelet transformation can be categorized into two types, the first is the continuous Wavelet Transformation (CWT), and the second is the discrete Wavelet Transformation (DWT). [17]

In the field of time series, the wavelet transform is an alternative to the original time series, as it summarizes the information received from the data as a function of frequency, and through it the time series can be analyzed into a set of multi-level solutions, both in terms of frequency or time, which makes it efficient and flexible with linear and non-linear time series, financial time series are not static and show very complex patterns over time such as seasonal changes, sudden volatilities, etc. wavelet defined as a function in time denoted by $\varphi(t)$, it is a local function similar in form to a wavelet, used to transform the signal $y(t)$ under analysis to another representation that displays the signal information more usefully. This transformation of the signal through the wavelet function is known as the wavelet transformation. [7],[13]. wavelet transformation depend on the function $\varphi \in L_2(R)$, which called wavelet Which is compressed and converted to extract the signal specifications through its relationship with time and frequency at the same time, so that at least some of the coefficients of the function φ must be different from zero, and that is by combining several sets of displacement and extension of the mother wavelet, and in order for the wavelet transformation to be able to extract all the information in the time series and link it to specific time periods and locations., the function φ must fulfill the following characteristics and conditions, [4],[18]:

a. Admissibility Condition, that is:

$$C_{\varphi} = \int_{0}^{\infty} |\hat{\varphi}(\omega)| \ \omega^{-1} \, d\omega < \infty \tag{1}$$

where $\hat{\varphi}$ is the Fourier transform of the function φ, such that, $\hat{\varphi}(\omega) \to 0$ whenever $\omega \to 0$, to ensure that $C_{\varphi} < \infty$ It is necessary that $\hat{\varphi}(0) = 0$, that is:

$$\int_{-\infty}^{\infty} \varphi(\omega) \, d\omega = 0 \tag{2}$$

b. The integral of the energy function achieves the following relationship:

$$\int_{-\infty}^{\infty} |\varphi(\omega)|^2 \, d\omega = 1 \tag{3}$$

The last two conditions imply that at least some of the wavelet transform coefficients must be different from zero and that these deviations from zero must cancel out. Here we will give an idea of the father and mother wavelets, as the father wavelets generate the scaling coefficients and represent the long-range smooth component of the signal, while the mother wavelets generate the different coefficients and represent deviations from the smooth component, the father wavelet acts as a low-pass filter, while the mother wavelet acts as a high-pass filter, The application of father and mother wavelets allows separating the low-frequency components of the time series from its high-frequency components, for the function $\vartheta \in L_2(R)$, the wavelets of the mother and father functions can be described mathematically as follows, [9],[13].[18]:

$$\Psi_{j.k}(t) = 2^{\frac{j}{2}} \Psi\left(2^{j} \, t - k\right) \quad , j = 1,2,3, \dots, J \tag{4}$$

$$\int_{-\infty}^{\infty} \Psi(t) \, dt = 0$$

$$\vartheta_{J.k}(t) = 2^{\frac{J}{2}} \vartheta(2^{J} \, t - k) \quad , k \in Z \, , J \in Z^{+} \tag{5}$$

$$\int_{-\infty}^{\infty} \vartheta(t) \, dt = 1$$

Where $\Psi_{j.k}(t)$ represent the mother wavelet function, and $\vartheta_{J.k}(t)$ represent the father wavelet function. Through these base functions, we can define a series of coefficients for the two wavelet functions father and mother, respectively, as follows:

$$d_{j,k} = \int_{-\infty}^{\infty} g(t) \, \Psi(t) \, dt \, , j = 1,2,3, \dots, J \tag{6}$$

$$C_{J,k} = \int_{-\infty}^{\infty} g(t) \, \vartheta(t) \, dt \tag{7}$$

Where $C_{J,k}$ represent the coefficient of the mother wavelet function or the Detail Coefficients, and $d_{j,k}$ represent the coefficient of the father wavelet function or Smooth Coefficients.

There are some programs available for the applications of wavelet transform, in this paper the libraries available in the program R will be used. In order to decompose the time series, one of the known discrete transform wavelets must be available, there is a large group of these wavelets, such as, Daubechies**,** Meyer, and Morlet, the selection of these wavelets depends on the characteristics of the studied data. In this research, (Daubechies) (db) wavelets were used, which is one of the orthogonal wavelets commonly used by researchers in signal processing, the formulas of db wavelet were derived in (1988) by the Belgian researcher Ingrid Daubechies, the derivation of these formulas is based on the use of recursive relationships to generate accurate discrete samples that gradually approach the original wavelet, [2],[10]. In the figure (1) below, a description of the process of decomposing the time series using the discrete wavelet transformation, while the figure (2) shows the nature of the mother and father functions of the wavelets of db4, db8, db16.

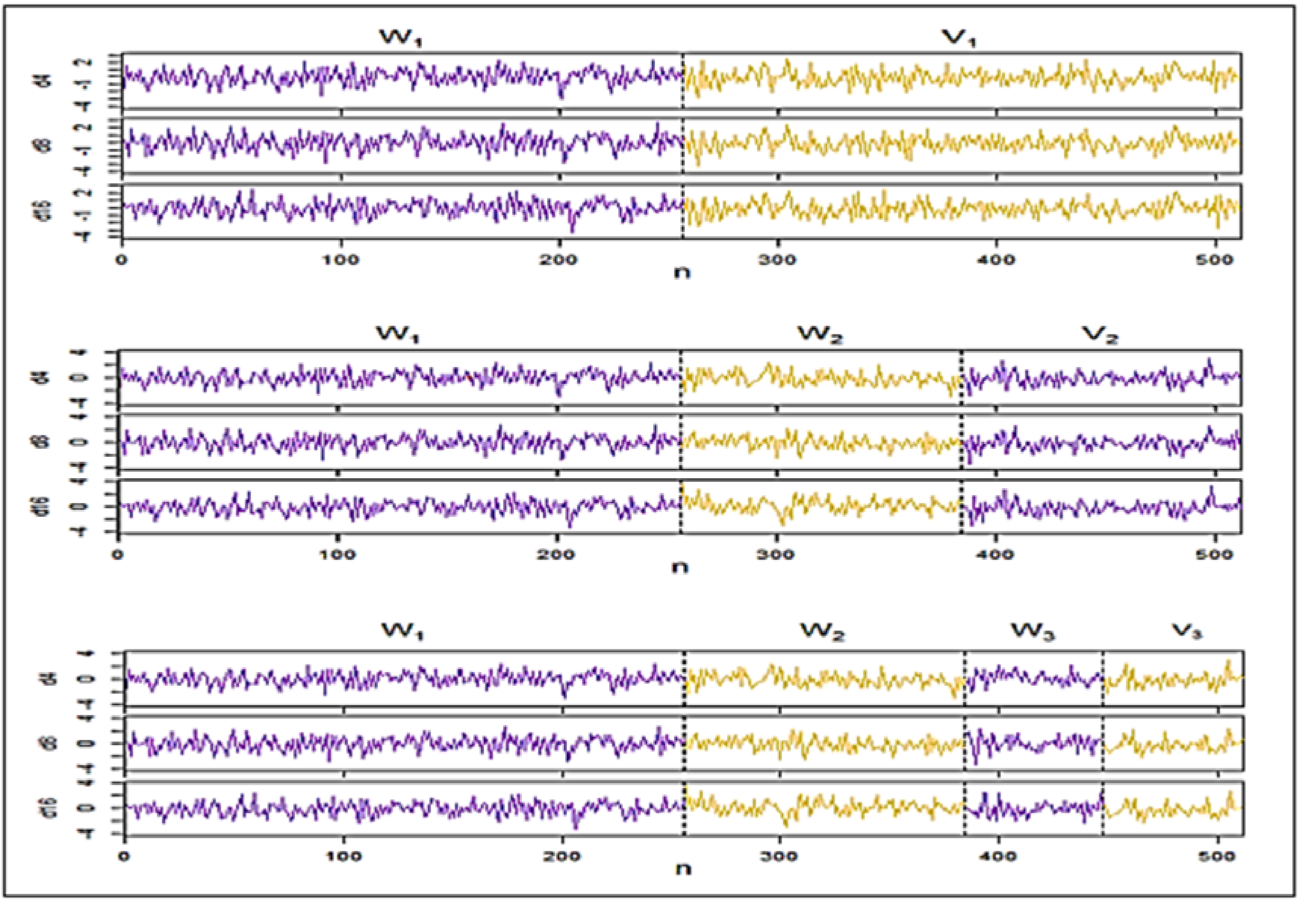

Figure 1. Time series decomposition using discrete wavelet transform

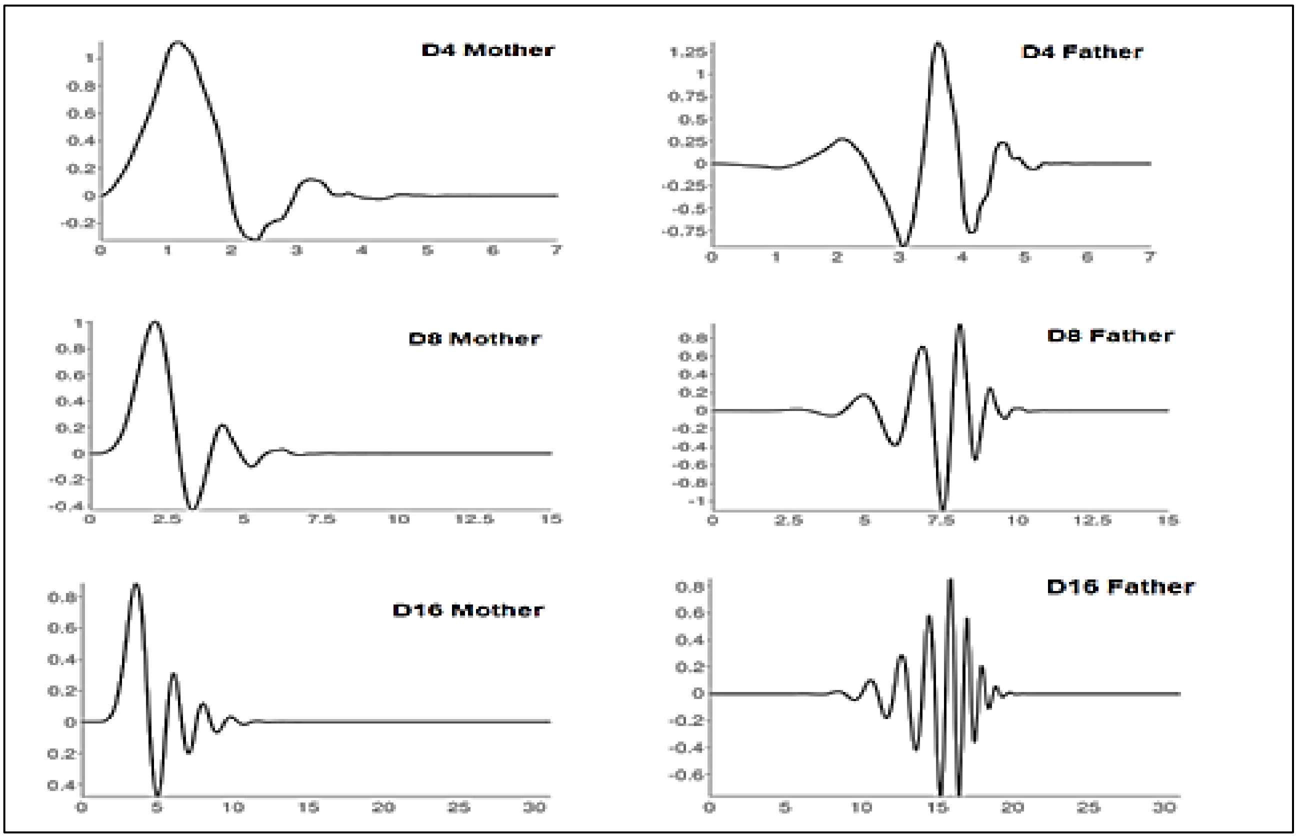

Figure 2. The nature of the mother's and father's functions of wavelets for some Daubechies functions

3. ARIMA models

Autoregressive integrated moving average models (ARIMA) are the most widely used time series models, as all models can be derived from them, whether autoregressive, moving averages, or mixed models, these models consist of three parts, the first part being an autoregressive model AR(p), the second part being an moving averages model MA(q), while the third part represent the degree of integrated which denoted by (d) and defined as the number of differences that must be made on the time series in order for it to be stable, the mathematical formula of the ARIMA(p,d,q) defined as following, [1]:

$$y_t = \varphi_1\, y_{t-1} + \cdots + \varphi_p\, y_{t-p} + e_t - \theta_1\, e_{t-1} - \cdots - \theta_q\, e_{t-q} \tag{8}$$

This formula can be rewrite using the back shift operator B as following:

$$\varphi(B) y_t = \theta(B) e_t \tag{9}$$

Where $\varphi(B)$ and $\theta(B)$ are two polynomials in the back shift operator B with degree p and q respectively which define as following, e_t are the white noise process with zero mean and constant variance:

$$\varphi(B) = 1 - \varphi B - \varphi_2 B^2 - \cdots - \varphi_p B^p$$

$$\theta(B) = 1 - \theta B - \theta_2 B^2 - \cdots - \theta_q B^q$$

4. Exponential smoothing

Exponential smoothing is one of the most popular estimation and prediction techniques as it involves a wide range of models. Exponential smoothing models are used when the data under investigation do not fulfill the assumptions of classical models such as the assumption of a normal distribution of errors. Exponential smoothing (ES) can be defined as an estimation or prediction technique based on an exponentially weighted average of past observations, used for future predictions of data that follows unstable or non-linear behavior. ES models have many formulas according to the nomenclature of Hyndman et al 2008. These models are named with three capital letters, the first letter describing the error, the second letter describing the general trend, and the third letter describing seasonality, [3],[5]. In this research, given that there is no general trend or seasonal trend in the time series resulting from the wavelet transformation, so a simple exponential smoothing will be employed with two models: the first model (A,N,N) which is the errors model, and the second model (M,N,N) which is the double error model. The estimation mechanism by exponential smoothing is summarized by giving weights to the observations so that the weight of each observation is determined by using the smoothing parameter θ. The simple exponential smoothing model can be described mathematically as follows:

$$\hat{y}_t = \theta y_{t-1} + \theta(1-\theta) y_{t-1} + \theta(1-\theta)^2\, y_{t-2} + \cdots + \theta(1-\theta)^{t-1}\, y_1 \tag{10}$$

Where $0 < \theta \leq 0$, $\hat{y}_t$ is the expected value of $\hat{y}_{t-1}$.

4. Hybrid models

The idea of hybrid models is to use the wavelet transform to decompose the time series into several levels in order to treat the non-stationary of the time series and increase the accuracy of predictions, and then apply ARIMA models or exponential smoothing on each level as these levels are sub-time series. The mechanism for applying hybrid DWT models to the g(t) signal representing time series data can be described in the following three steps, [10],[12]:

The first step: the time series $g(t)$is decomposed through the discrete wavelet transform (DWT) into several series that are different in precision, and through smoothing and detail coefficients defined in equations (6) and (7), the signal $g(t)$ canbe expressed mathematically as follows:

$$g(t) = \sum_{k \in Z} C_{J,k}\, \vartheta_{J.k}(t) + \sum_{k \in Z} d_{j,k}\, \Psi(t) + \cdots + \sum_{k \in Z} d_{1,k}\, \Psi(t) \tag{11}$$

Equation (8) can be briefly rewritten as follows:

$$g(t) = V_J + W_J + W_{J-1} + \cdots + W_1 \quad (12)$$

Where; $V_J = \sum_{k \in Z} C_{J,k}\, \vartheta_{J.k}(t)$, $W_J = \sum_{k \in Z} d_{j,k}\, \Psi(t)$, and $j = 1,2,3, \dots, J$.

The number of decomposition levels (J) is determined by the following formula:

$$J = int\,\{log(n)\} \quad (13)$$

Where n is the number of the time series observations. This means that the time series is decomposed into a number of components according to the specified levels, which represent different frequency components of the original data, and each of these components plays a special and different role in the original time series data, each component is used as an input for the model instead of using a one component in order to increase and improve the ability and accuracy of the estimating or predicting as expected from the models.

The second step: Reconstructing the signal to reduce the high-frequency wavelet noise, and this is done by integrating or summing the wavelet coefficients, that is mean removing the high frequency by applying the inverse discrete wavelet transformation (IDWT) as following:

$$G(t) = IWDT\left(V_J, W_J, W_{J-1}, \dots, W_1\right) \quad (14)$$

The third step: Building the hybrid models WHA or HWE, by combining the discrete wavelet transform (DWT) and the estimation methods ARIMA or ES, after the original time series is decomposed into several sequential subcomponents and reconstructed again, the resulting series is used as input to the ARIMA model or ES model which gives an additional improvement to the accuracy of the model, the diagram in Figure (3) below shows the steps for building the HWA and HWE hybrid models.

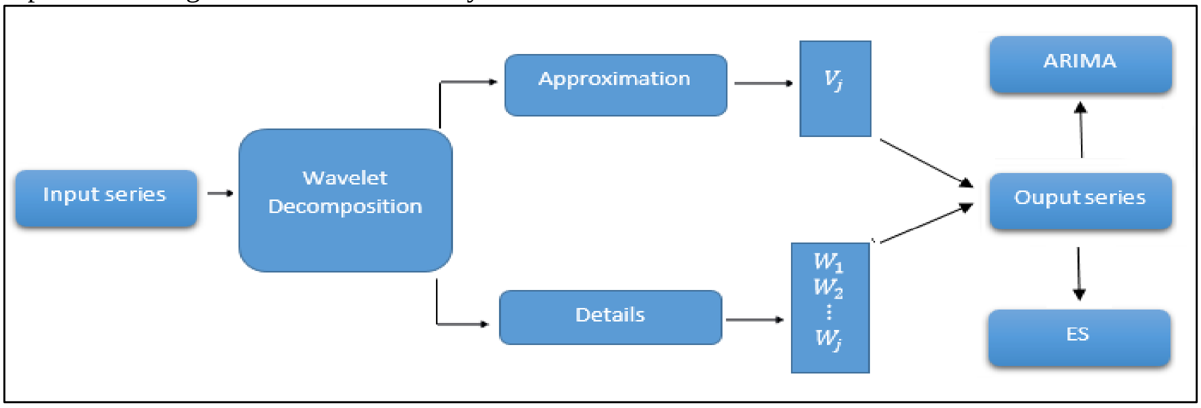

Figure 3. The structure of the hybrid models HWA and HWE.

5. Application

In this part, the time series of the weekly rates of the latest price of the US dollar index will be analyzed in order to predict the weekly rates of this index. The dollar index describes the relationship of the US dollar to the following six currencies: the euro, the British pound, the Swiss franc, the Canadian dollar, the Swedish krona, and the Japanese yen. In other words, it is the ratio of the US currency against the currencies of other countries. This index is calculated as a weighted average of the prices of the six currencies, so that the largest share of the euro is 57.6% of the currencies included in the index, while the proportion of the rest of the currencies of the index is as follows: the Japanese yen 13.6%, the pound sterling 11.9%, the Canadian dollar 9.1%, the krona Sweden 4.2%, Swiss franc 3.6%. The index is greatly influenced by macroeconomic factors, including inflation and deflation in the dollar and foreign currencies included in the comparison group, as well as recessions and economic growth in those countries. The US Dollar Index allows traders to monitor the value of the US dollar in relation to a group of currencies in a single transaction, in order to be wary of their dollar bets. It is possible to combine contract strategies based on the US dollar index, and these financial strategies are currently traded on the New York Board of Trade. In addition to the above, investors can take advantage of the index to be wary of currency movements and speculate on them. The research data included the time series of the weekly rates of the latest price of the dollar index in the global markets for the period from 1/1/2011 to 1/1/2022 with (512) observations, and these data are classified as a long-term time series. Figure 4 represents the time series, the autocorrelation and partial autocorrelation functions, while Table 1 represents some statistics and properties of that series.

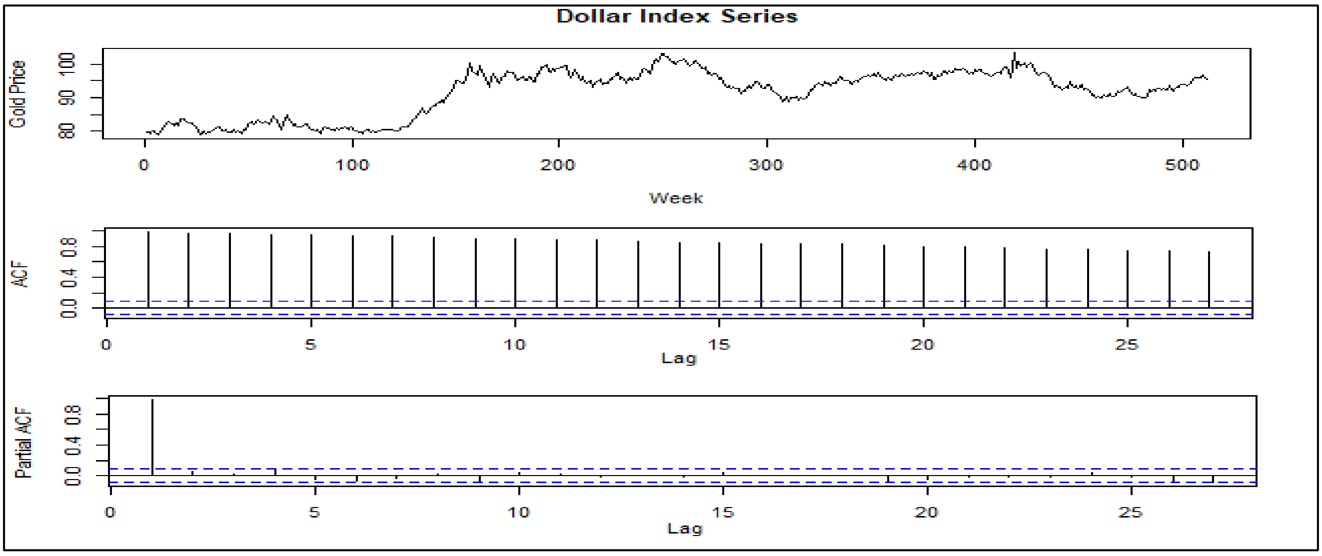

Figure 4. The time series of the weekly rates of the latest price of the dollar index, the autocorrelation and partial autocorrelation functions

Table 1. Some statistics and properties of the time series under consideration.

Min	Max	Mean	Median	Skewness	Kurtosis	Sd
78.76	103.5	91.6	93.93	-0.61	-1.02	6.84
J.B, P-value		ADF, P-value		Tsay, P-value		
53.832 (2.045e-12)		-1.8089 (0.6592)		10.55 (0.00124)		

Through Figure (1), it is clear that the time series is fluctuating and that the coefficients of the autocorrelation and partial autocorrelation functions are outside the confidence limit. We also note that the highest price of the dollar index reached ($103.5) and the lowest price amounted to ($78.76) and that the average world prices of the dollar index for this period has reached ($91.6) and the skewness and kurtosis coefficients are negative. From Table (1), it can be seen that the P-value of the (JB) statistic is small, which indicates that the series errors do not follow a normal distribution, as well as that the P-value of the (ADF) test is large, which indicates that the time series is unstable, as the P-value of the Tsay test is small, so it can be said that the time series follows a non-linear behavior.

5.1. Application of the two hybrid models

For the purpose of applying the two hybrid models Hybrid Wavelet-ARIMA (HWA) and the Hybrid Wavelet-Exponential (HWE) on the time series of dollar index prices, the discrete wavelet transformation with Daubechies filters, where three filters were used which are D4, D8 and D16. It was first necessary to determine the best levels at which to estimate the HWA and HWE models for each Wavelet filter, to achieve this, the comparison criteria were used, the mean squared error (MSE) and the mean absolute relative error (MAPE), as shown in table (2). With regard to the HWA model, it was found that the first level was the best level of estimation for filters D4 and D16, and for filter D8, the best level of estimation was the sixth level, and with regard to the HWE model, the sixth level was the best level of estimation for filters D4 and D8, while the third level was Best level when applying the D16 filter.

Table 2. The comparison of the levels of the Wavelet functions

Filter	HWA			HWE		
	Level	MSE	MAPE	Level	MSE	MAPE
D4	**1**	**0.14603**	**0.38823**	**6**	**0.40134**	**0.41984**
	2	0.15345	0.39976	7	0.40142	0.41992
	6	0.15351	0.40979	5	0.40205	0.42058

Filter	HWA			HWE		
	Level	MSE	MAPE	Level	MSE	MAPE
	7	0.15354	0.40987	4	0.40444	0.42308
	5	0.15212	0.40990	2	0.40447	0.42311
	6	**0.15212**	**0.40801**	**6**	**0.39073**	**0.40874**
	4	0.15213	0.40802	5	0.39074	0.40875
D8	5	0.15215	0.40804	4	0.39087	0.40889
	1	0.15215	0.40805	3	0.39208	0.41015
	3	0.15311	0.40932	2	0.39907	0.41747
	1	**0.15201**	**0.40786**	**3**	**0.39202**	**0.41009**
	3	0.15369	0.41010	4	0.39204	0.41009
D16	4	0.15370	0.41012	5	0.39205	0.41020
	5	0.15372	0.41015	2	0.39260	0.41069
	2	0.15410	0.41064	1	0.39304	0.41115

After determining the best level for the three filters, the details of the wavelet transform of the original time series with its detailed and approximate parts were obtained, as shown in Figures (5,6,7), It is clear from it that the wavelet transformation of the HWA model with filters D4 and D16 included one detailed part, while the wavelet transformation of the filter D8 included six detailed parts, as for the wavelet transformation of the HWE model with filter D16, it included three detailed parts and six detailed parts with filters D4 and D8.

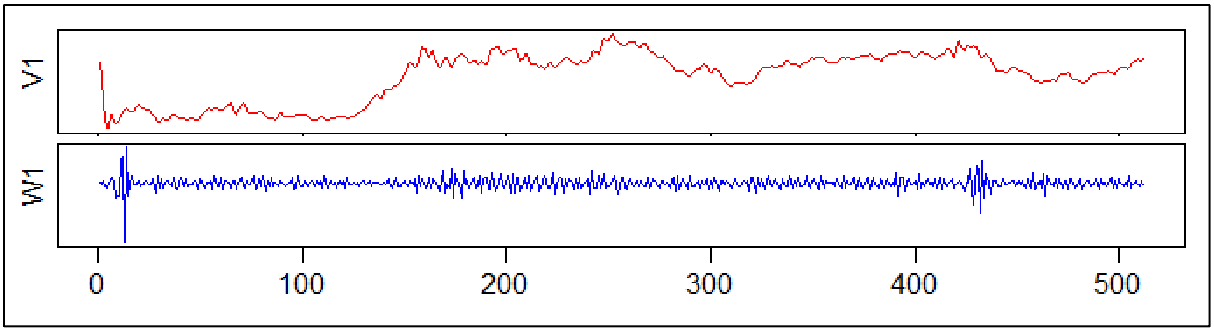

Figure 5. Wavelet transformation of the HWA model with filters D4 and D16

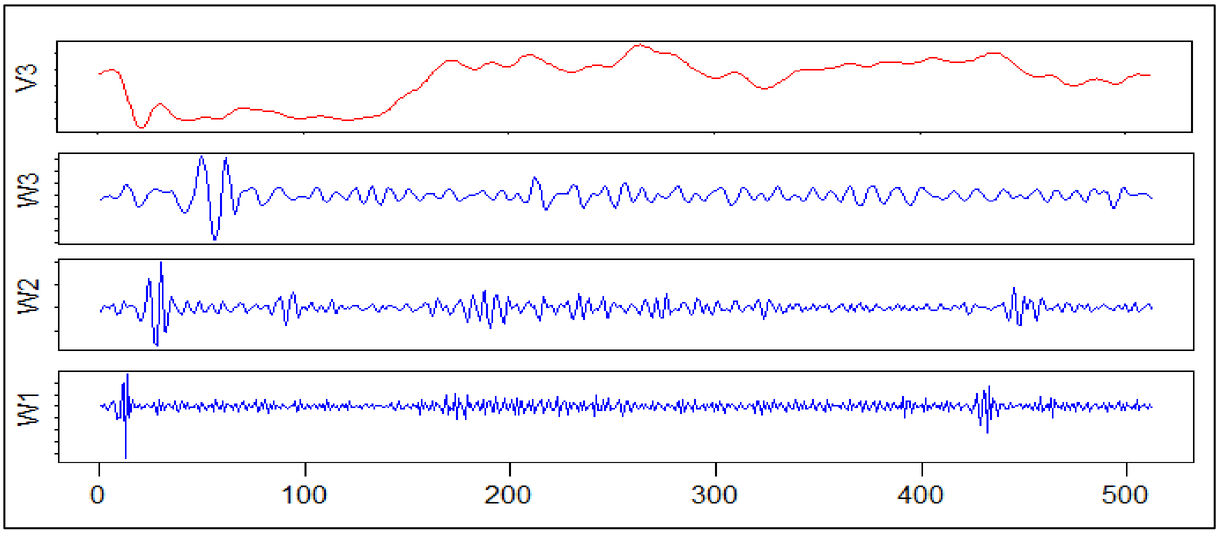

Figure 6. Wavelet transformation of the HWE model with filter D16

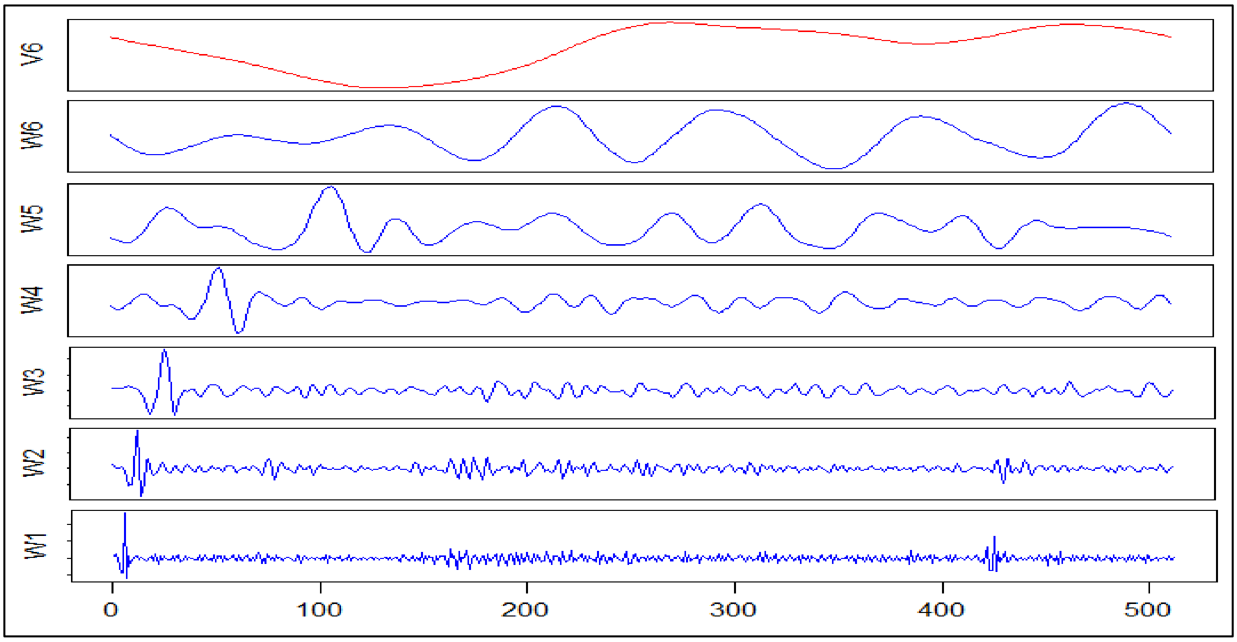

Figure 7. Wavelet transformation of the HWA model with filter D8 and the HWE model with filters D4 and D8

After performing the wavelet transformation on the dollar index price series and getting to know the details of the detailed and approximate series, the final time series can be built by collecting those parts and then matching the ARIMA models or the exponential smoothing models with the final series, and thus we get the estimates of the two hybrid models HWA and HWE, Table 3 includes the results of the comparison criteria between the two hybrid models for all the filters used, as well as the details and types of the resulting models. It is clear that the final HWA hybrid models for filters D4, D8 and D16 were, respectively, ARIMA(4,1,0), ARIMA(2 ,1,2) and ARIMA(5,1,2), while all HWE final hybrid models were ETS(A,N,N) for all filters. Through the comparison criteria AIC, RMSE and MAPE, the best HWA hybrid models were obtained using the filter D16, followed by the model with the filter D4, then the model with the filter D8, while the best HWE models was the model with the filter D8, followed by the model with the filter D4, then the model with the filter D16, and therefore the best hybrid model for the dollar index price series is the HWA hybrid model with filter D16.

Table (3): Details and types of estimated hybrid models.

Filter	HWA					
	Model	AIC	RMSE	MAPE	Q-stat	P-
D4	ARIMA(4,1,0)	1559.03	1.09336	0.79599	7.0426	0.532
D8	ARIMA(2,1,2)	1565.67	1.10766	0.84042	7.7655	0.4567
D16	ARIMA(5,1,2)	**1553.42**	1.08398	0.79225	4.6443	0.7948
Filter	HWE					
	Model	AIC	RMSE	MAPE	Q-stat	P-
D4	ES(M,N,N)	1582.09	1.09423	0.84043	8.5142	0.3849
D8	ES(A,N,N)	**1572.31**	1.10922	0.84669	8.2164	0.4126
D16	ES(A,N,N)	1624.18	1.14963	0.88788	12.321	0.1375

Based on the foregoing, the final model that will be applied to estimate the dollar index price series is the ARIMA (5,1,2) model, defined according to the following mathematical formula:

$$y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4} + \beta_5 y_{t-5} + a_t + \delta_1 a_{t-1} + \delta_2 a_{t-2} \quad (15)$$

The estimated formula is as following:

$$\hat{y}_t = -0.25\, y_{t-1} - 0.75\, y_{t-2} - 0.22\, y_{t-3} - 0.07\, y_{t-4} + 0.06\, y_{t-5} + a_t + 0.2\, a_{t-1} + 0.74\, a_{t-2} \qquad (16)$$

Through figure (8), which relates to the residual test of the final model, it was found that the values of the autocorrelation function were within confidence limits, and that these residuals are random and follow a normal distribution.

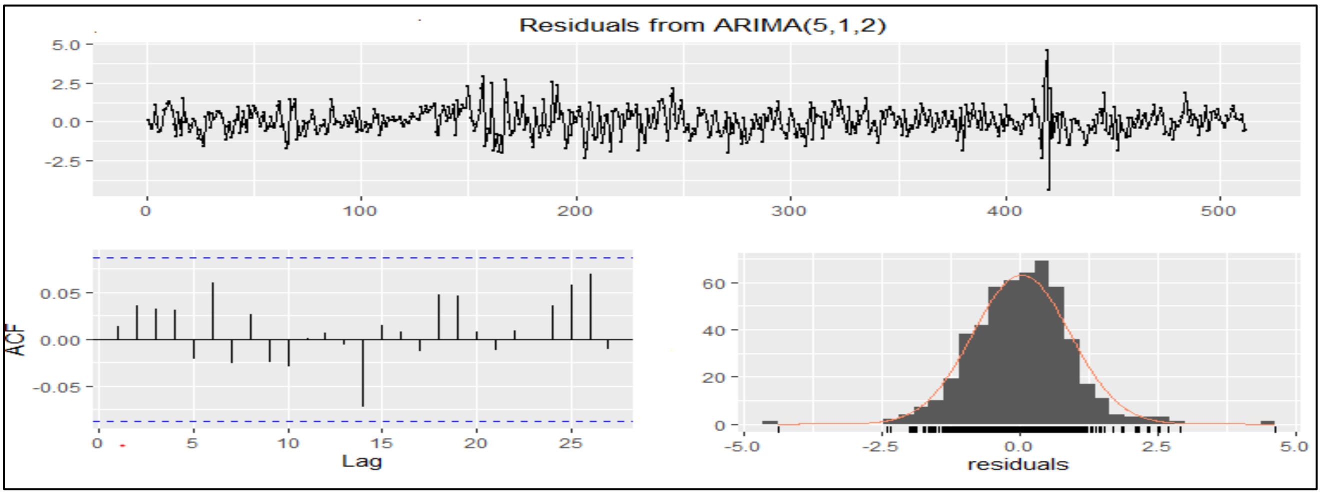

Figure 8. Residual test for the ARIMA (5,1,2) model

The weekly values of the dollar index were predicted according to the (D16)-ARIMA(5,1,2) hybrid model for a period of (16) weeks, Table 4 and Figure 9 show the predictive values and confidence limits at the level of significance (0.05). It is clear from Table 4 that the lowest predictive value will amount to (95.24$), while the highest predictive value for the index will be (96.31$), and the average predictive values for the index will be (96.01$).

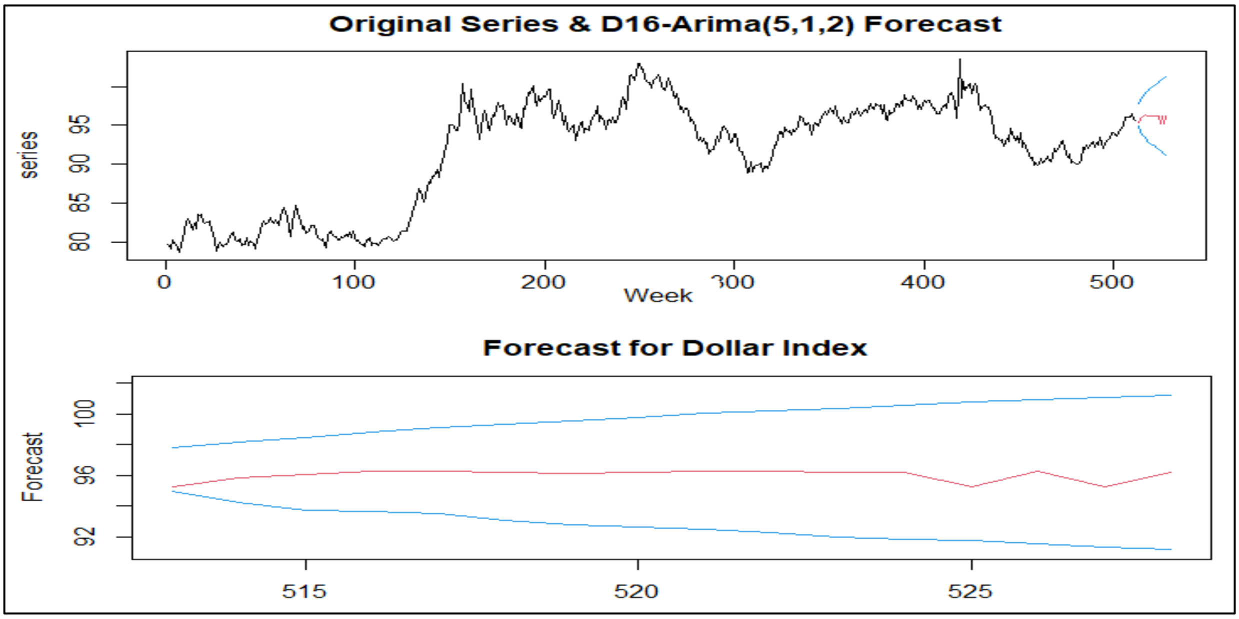

Figure (9): The original series, predictive values, and confidence limits for the US dollar index

Table 4. Predictive values for the US dollar index

Obs.	513	514	515	516	517	518	519	520	521
Forecast	96.27	96.2	96.16	96.23	96.31	96.24	96.09	95.82	95.29
Upper	92.5	92.61	92.76	93.11	93.49	93.64	93.72	94.25	94.98
Lower	100.04	99.8	99.57	99.35	99.12	98.85	98.46	98.15	97.81
Obs.	522	523	524	525	526	527	528		
Forecast	96.21	95.26	96.24	95.24	96.2	96.19	96.24		

Upper	91.2	91.36	91.56	91.73	91.84	92.01	92.27		
Lower	101.21	101.06	100.92	100.75	100.56	100.38	100.22		

6. Conclusions

Through the obtained results and graphic forms, the following conclusions can be written:

1. The Dollar Index Weekly Averages are a time series that is not linear, unstable, and does not follow a normal distribution.
2. The application of discrete wavelet transform with Daubechies filters on the dollar index series resulted in smoothing and filtering of random errors of the series by transforming the time series field into the wavelet field.
3. The appropriate levels when applying the discrete wavelet transform to the dollar index series vary with the different Daubechies filters.
4. For the HWA hybrid model, the best filter for the decomposition of the dollar index time series is the D16 filter, while the D8 filter was the best filter to apply the HWE hybrid model.
5. The types of final models generated when applying the HWA hybrid model differ according to the three wavelet filters employed in the process of decomposing and rebuilding the time series of the dollar index, while the types of final models resulting from HWE may be similar, but we find a difference in the values of the estimated parameters.
6. In general, the HWA model is better than the HAE model in decomposing and rebuilding the dollar index time series, and the D16-ARIMA(5,1,2) hybrid model is the best model among all other hybrid models.
7. The process of forecasting the weekly rates of the dollar index showed that the future prices of the index will not show significant changes or fluctuations during the next sixteen weeks, as the weekly average of the dollar index price will be ($ 96), and that the lowest value of the index will be recorded in the fourteenth week and the highest value of the index will be recorded in the fifth week.

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