Periodicals of Engineering and Natural Sciences ISSN 2303-4521
Vol.6, No.1, June 2018, pp. 274~284
Available online at: http://pen.ius.edu.ba

Designing Filter for Certain Subclasses of Analytic Univalent Functions

P. Murugabharathi[1], B. Srutha Keerthi [2]
[1] Mathematics Division,School of Advanced Sciences, VIT Chennai,Chennai – 600 127, India
[1] Mathematics Division,School of Advanced Sciences, VIT Chennai,Chennai – 600 127, India

Article Info

Article history:

Received Feb 12th, 2018
Revised Apr 20th, 2018
Accepted Apr 26th, 2018

Keyword:

Bandpass filter
Highpass filter
Time response
Starlike function
Sakaguchi type function

ABSTRACT

In this paper we consider certain subclasses of analytic univalent functions and plot a frequency response of appropriate circuit for both amplitude and phase with changing source frequency.

Mathematics Subject Classification, 2010: 30C45.

Corresponding Author:

P.MURUGABHARATHI,
Mathematics Division
School of Advanced Sciences, VIT Chennai,
Chennai – 600 127, India
bharathi.muhi@gmail.com

1. Introduction

Let A be the class of functions f(z) of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disk $\Delta = \{z \in C: |z| < 1\}$.We also denoteby S the class of all functions in the normalized analytic function class A which are univalent in Δ. Goodman[4] introduced the subclasses of S. The class S*of all starlike functions with respect to the origin.

A function f ∈A is said to be starlike with respect to the origin if it maps Δ onto a Starlike domain with respect to the origin. Robertson [5] gave necessary and sufficient condition for class S* is $Re\frac{zf'(z)}{f(z)} > 0, z\epsilon\Delta$. For $0 \le \lambda \le 1, |t| \le 1$ and $t \ne 1$, a function $f \in A$ given by (1.1) is said to be in the clas$P(\alpha, \lambda, t)$ifthefollowing conditions are satisfied the analytic characterization

$$Re\frac{(1-t)zf'(z)}{(1-\lambda)[f(z)-f(tz)]+\lambda z[f'(z)-tf'(tz)]} > \alpha, \quad z\epsilon\Delta \tag{1.2}$$

Recently these kind of classes were studied by Srutha keerthi and et.al[6].

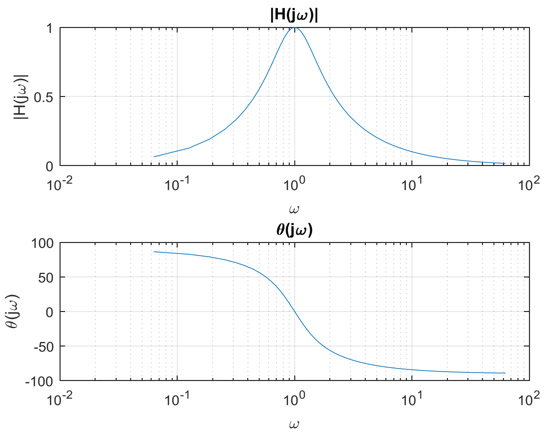
Analysis of a circuit with varying frequency of sinusoidal sources is called the frequency response of a circuit. Frequency selection in the circuits is called filters because of their ability to filter out certain input signals on the basis of frequency. Using transfer function of circuit, we plot a frequency response of the circuit for both amplitude and phase with changing source frequency. One graph of |H(jω)| versus frequency jω. It is called the Magnitude plot.

One graph of θ(jω) versus frequency ω. It is called the Phase Angle plot.
A Low-Pass filterpasses signals at frequencies lower than the cutoff frequency from the input to the output. A High-Pass filterpasses signals at frequencies higher than the cutoff frequency from the input to the output. A Band-Pass filter passes signals within the band defined by two cutoff frequencies from the input to the output.

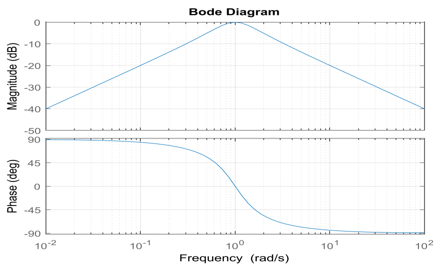
1.1 Frequency response of a rlc circuit (bandpass filter)

Let us consider the function $\frac{z}{1+z+z^2}$ belongs to the class of all analytic univalent function with the Montel normalization (ie. $\frac{z}{1+z+z^2} \in S$), which is a transfer function for the RLC circuit in series. Using transfer function of circuit, we plot a frequency response of the circuit for both amplitudeand phasewith changing source frequency. For the above transfer function RLC values are R=1;L=1;C=1;

Matlab code	Output

R=1;
L=1;
C=1;
f=0:0.01:10;
w=2*pi*f;
subplot (2,1,1)
h=abs(((j*w)*R/L)./((j*w).^2
+(j*w)*R/L+1/L*C));
semilogx(w,h)
grid on
title('|H(j\omega)|')
xlabel ('\omega')
ylabel ('|H(j\omega)|')
theta=angle(((j*w)*R/L)./((j*w).^2
+(j*w)*R/L+1/L*C));
subplot (2,1,2)
degree=theta*180/pi;
semilogx(w,degree)
grid on
title('\theta(j\omega)')
xlabel('\omega')
ylabel('\theta(j\omega)')
	

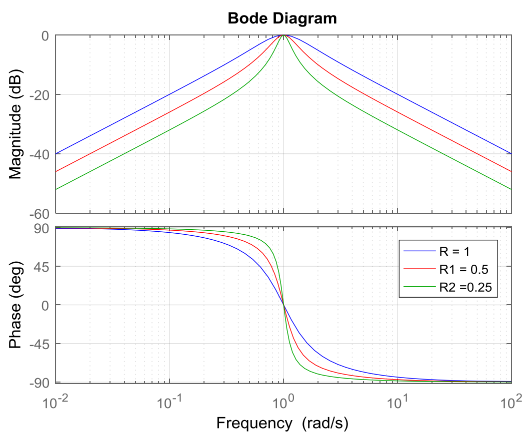
The Bode plot is a convenient tool for investigating the bandpass characteristics of the RLC network. Use tf to specify the circuit's transfer function for the values

<table>
<tr><th>Matlab code using bode</th><th>Output</th></tr>
<tr><td>R = 1; L = 1; C = 1;
G = tf([R/L 0],[1 R/L 1/(L*C)])
bode(G),
grid on</td><td>G = s
----
s^2 + s + 1
Continuous-time transfer function.

</td></tr>
</table>

The transfer function from input to output voltage is:

$$G(s)\frac{\frac{sR}{L}}{s^2+\frac{sR}{L}+\frac{1}{LC}}$$

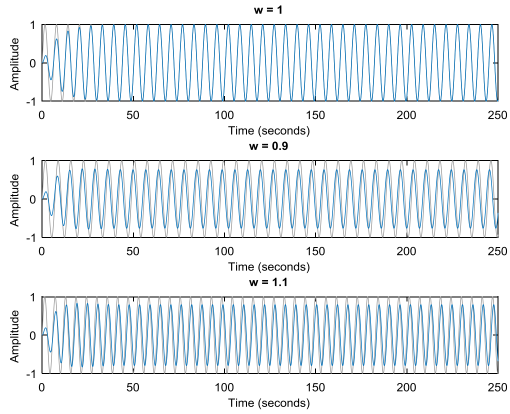
To build a bandpass filter tuned to the frequency 1 rad/s, set L=C=1 and use R to tune the filter band.To get a narrower passing band, try decreasing values of R as follows

<table>
<tr><td>R = 1; L = 1; C = 1;
G = tf([R/L 0],[1 R/L 1/(L*C)])
bode(G), grid
R1 = 0.5; G1 = tf([R1/L 0],[1 R1/L 1/(L*C)])
R2 = 0.25; G2 = tf([R2/L 0],[1 R2/L 1/(L*C)])
bode(G,'b',G1,'r',G2,'g'), grid
legend('R = 1','R1 = 0.5','R2 =0.25')</td><td>
</td></tr>
</table>

The resistor value R=0.25 gives a filter narrowly tuned around the target frequency of 1 rad/s.

1.1. Time response of the circuit

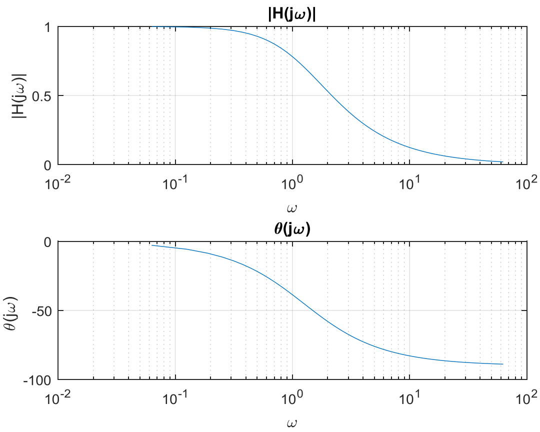
We can confirm the attenuation properties of the circuit G2 (R=0.25) by simulating how this filter transforms sine waves with frequency 0.9, 1, and 1.1 rad/s:

t = 0:0.05:250;
opt = timeoptions;
opt.Title.FontWeight = 'Bold';
subplot(311), lsim(G2,sin(t),t,opt),
title('w = 1')
subplot(312), lsim(G2,sin(0.9*t),t,opt),
title('w = 0.9')
subplot(313), lsim(G2,sin(1.1*t),t,opt),
title('w = 1.1')
	

The waves at 0.9 and 1.1 rad/s are considerably attenuated. The wave at 1 rad/s comes out unchanged once the transients have died off.

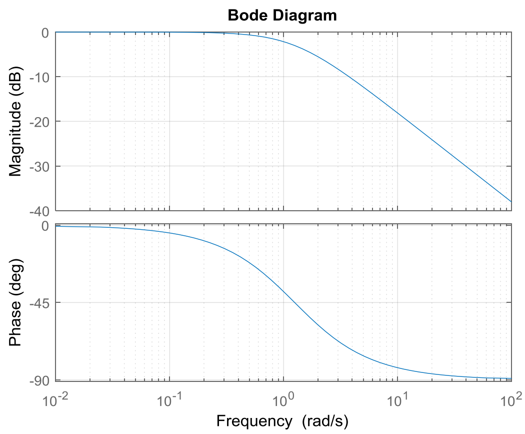
2. Frequency response of rl circuit (lowpass filter)

For the case $\lambda = 0.2$, $f = \frac{z}{(1+z)}$, $t = 0$ and $\alpha = 0$ in $P(\alpha,\lambda,t)$ we get the function $H(z) = \frac{5}{4z+5}$ which is a transfer function for the RL circuit in series. Using transfer function of circuit, we plot a frequency response of the circuit. For the above transfer function RL values are R=1; L=0.8;

Matlab code	Output
>> R=1; >> L=0.8; >> f=0:0.01:10; >> w=2*pi*f; >> subplot (2,1,1) >> h=abs((R/L)./(j*w+(R/L))); >>semilogx(w,h) >> grid on >> title('|H(j\omega)|') >>xlabel ('\omega') >>ylabel ('|H(j\omega)|') >> theta=angle((R/L)./(j*w+(R/L))); >> subplot (2,1,2) >> degree=theta*180/pi; >>semilogx(w,degree) >> grid on >> title('\theta(j\omega)') >>xlabel('\omega') >>ylabel('\theta(j\omega)')	

The Bode plot is a convenient tool for investigating the lowpass characteristics of the RL network. Use tf to specify the circuit's transfer function for the values.

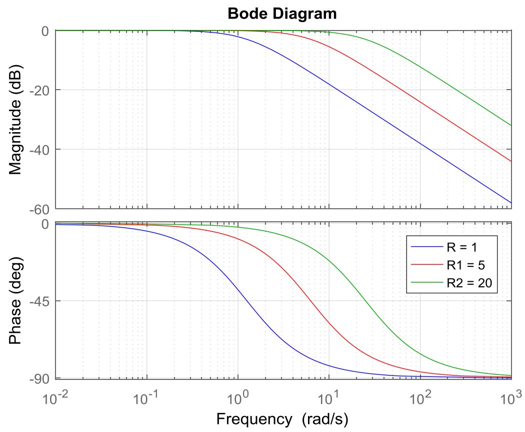
Matlab code using bode	Output

syms s
n=[0 5/4];
d=[1 5/4];
G=tf(n,d)
bode (G)
grid on
	g =
	1.25
	-------- 	
	s + 1.25
	Continuous-time transfer function.
	

The transfer function from input to output voltage is:

$$G(s)\frac{\frac{R}{L}}{s+\frac{R}{L}}$$

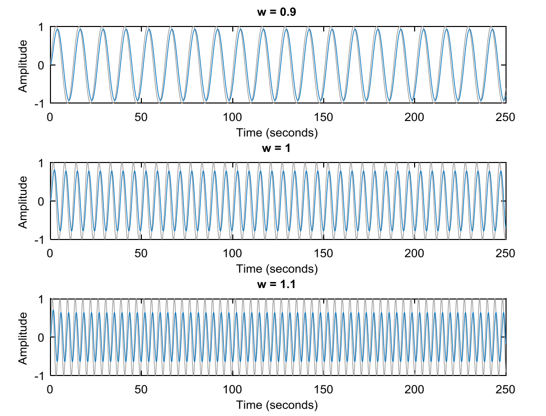
To build a bandpass filter tuned to the frequency 1 rad/s, set L=4/5 and use R to tune the filter band. To get a narrower passing band, try decreasing values of R as follows

R = 1; L = 4/5; G = tf([R/L],[1 R/L]) bode(G), grid R1 = 5; G1 = tf([R1/L],[1 R1/L]); R2 = 20; G2 = tf([R2/L],[1 R2/L]); bode(G,'b',G1,'r',G2,'g'), grid legend('R = 1','R1 = 5','R2 = 20')	

The resistor value R=1 gives a filter narrowly tuned around the target frequency of 1 rad/s.

2.1 time response of the circuit

We can confirm the attenuation properties of the circuit G2 (R=1) by simulating how this filter transforms sine waves with frequency 0.5, 1, and 1.5 rad/s:

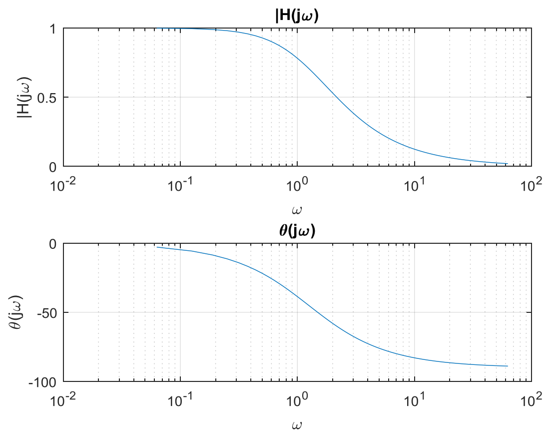
t = 0:0.05:250;
opt = timeoptions;
opt.Title.FontWeight = 'Bold';
subplot(311), lsim(G,sin(0.5*t),t,opt), title('w = 0.5')
subplot(312), lsim(G,sin(1*t),t,opt), title('w = 1')
subplot(313), lsim(G,sin(1.5*t),t,opt), title('w = 1.5')
	

The waves at 1 and 1.1 rad/s are considerably attenuated. The wave at 0.9 rad/s comes out unchanged once the transients have died off.

3. Frequency response of rc circuit (lowpass filter)

H(z) is also a transfer function for the RC circuit in series. Using transfer function of circuit, we plot a frequency response of the circuit for both amplitude and phase with changing source frequency. For the above transfer function RC values R=1; C=0.8;

Matlab code	Output

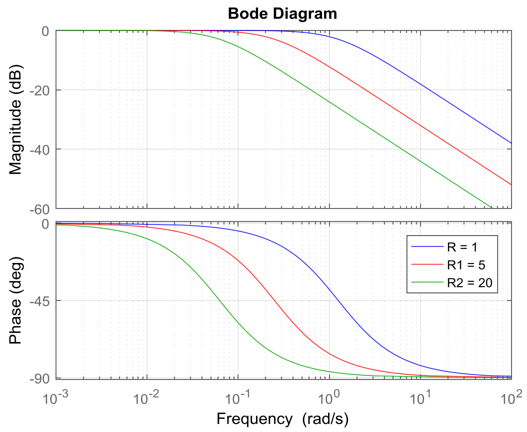
R=1;
C=0.8;
f=0:0.01:10;
w=2*pi*f;
h=abs((1/(R*C))./(j*w+1/(R*C)));
subplot (2,1,1)
semilogx(w,h)
grid on
title('|H(j\omega)')
xlabel ('\omega')
ylabel ('|H(j\omega)')
theta=angle((1/(R*C))./(j*w+1/(R*C)));
subplot (2,1,2)
degree=theta*180/pi;
semilogx (w, degree)
grid on
title('\theta(j\omega)')
xlabel('\omega')
ylabel('\theta(j\omega)')
	

The transfer function from input to output voltage is:

$$G(s)\frac{\frac{1}{RC}}{s+\frac{1}{RC}}$$

To build a lowpass filter tuned to the frequency 1 rad/s, set C=0.8 and use R to tune the filter band. To get a narrower passing band, try increasing values of R as follows

```
R=1; C=0.8;
G = tf([1/(R*C)],[1 1/(R*C)])
bode(G), grid
R1 = 5;   G1 = tf([1/(R1*C)],[1 1/(R1*C)]);
R2 = 20;  G2 = tf([1/(R2*C)],[1 1/(R2*C)]);
bode(G,'b',G1,'r',G2,'g'), grid
legend('R = 1','R1 = 5','R2 = 20')
```

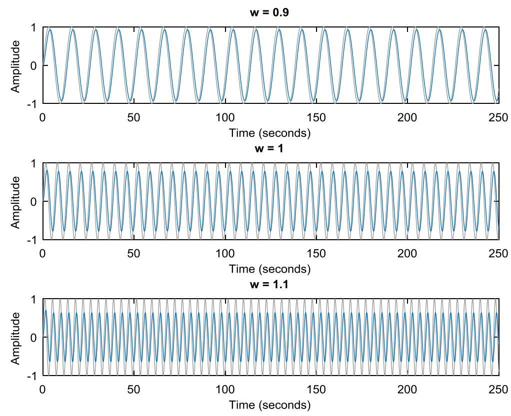


The resistor value R=20 gives a filter narrowly tuned around the target frequency of 1 rad/s.

3.1 analyzing the time response of the circuit

We can confirm the attenuation properties of the circuit G (R=1) by simulating how this filter transforms sine waves with frequency 0.5, 1, and 1.5 rad/s:

```
t = 0:0.05:250;
opt = timeoptions;
opt.Title.FontWeight = 'Bold';
subplot(311), lsim(G,sin(0.5*t),t,opt), title('w = 0.5')
subplot(312), lsim(G,sin(1*t),t,opt), title('w = 1')
subplot(313), lsim(G,sin(1.5*t),t,opt), title('w = 1.5')
```

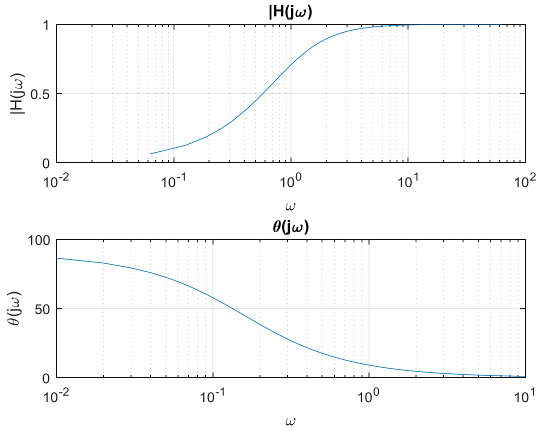


The waves at 1 and 1.1 rad/s are considerably attenuated. The wave at 0.9 rad/s comes out unchanged once the transients have died off.

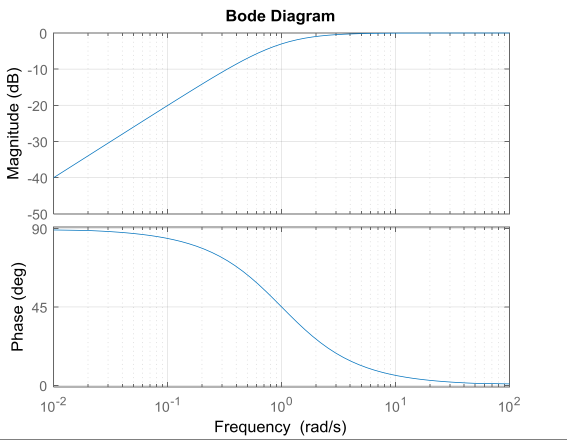
4. Frequency response of rl circuit (highpass filter)

Let us consider the function $\frac{z}{1+z}$ belongs to the class of all analytic univalent function with the Montel normalization (ie. $\frac{z}{1+z} \in S$), which is a transfer function for the RL circuit in series.Using transfer function of circuit,we plot a frequency response of the circuit. For the above transfer function RL values R=1; L=1;

Matlab code	Output

R=1;
L=1;
f=0:0.01:10;
w=2*pi*f;
h=abs((j*w)./(j*w+R/L));
subplot (2,1,1)
semilogx(w,h)
grid on
title('|H(j\omega)')
xlabel ('\omega')
ylabel ('|H(j\omega)')
theta=angle((j*w)./(j*w+R/L));
subplot (2,1,2)
plot (w, theta)
degree=theta*180/pi;
semilogx (f, degree)
grid on
title('\theta(j\omega)')
xlabel('\omega')
ylabel('\theta(j\omega)')
	

The Bode plot is a convenient tool for investigating the highpass characteristics of the RL network. Use tf to specify the circuit's transfer function for the values

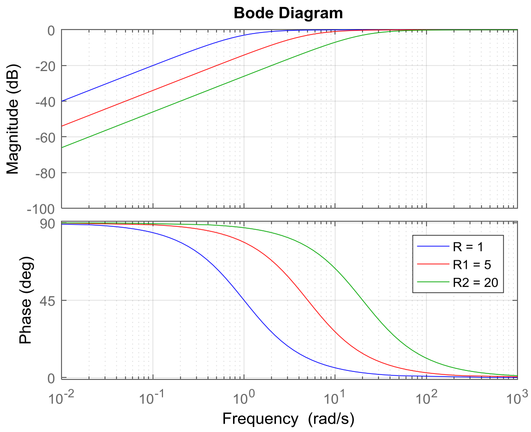
symss
n=[1 0];
d=[1 1];
G=tf(n,d)
bode (G)
grid on
	

The transfer function from input to output voltage is:

$$G(s)\frac{s}{s+\frac{R}{L}}$$

To build a bandpass filter tuned to the frequency 1 rad/s, set L=1 and use R to tune the filter band. To get a narrower passing band, try decreasing values of R as follows

```
R = 1; L = 1;
G = tf([1 0],[1 R/L])
bode(G), grid
R1 = 5;   G1 = tf([1 0],[1 R1/L]);
R2 = 20;  G2 = tf([1 0],[1 R2/L]);
bode(G,'b',G1,'r',G2,'g'), grid
legend('R = 1','R1 = 5','R2 = 20')
```

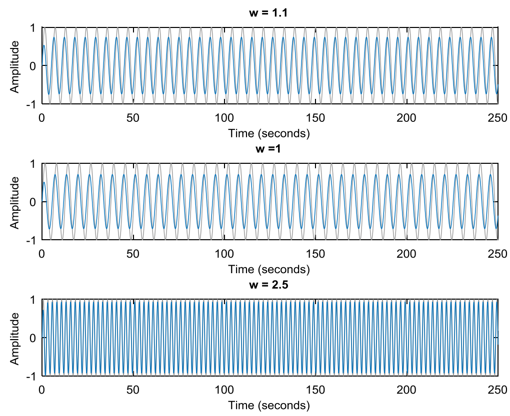


The resistor value R=20 gives a filter narrowly tuned around the target frequency of 1 rad/s.

4.1 analyzing the time response of the circuit

We can confirm the attenuation properties of the circuit G (R=1) by simulating how this filter transforms sine waves with frequency 1, 1.1, and 2.5 rad/s:

```
t = 0:0.05:250;
opt = timeoptions;
opt.Title.FontWeight = 'Bold';
subplot(311), lsim(G,sin(1.1*t),t,opt), title('w =
1.1')
subplot(312), lsim(G,sin(1*t),t,opt), title('w =1')
subplot(313), lsim(G,sin(2.5*t),t,opt), title('w =
2.5')
```

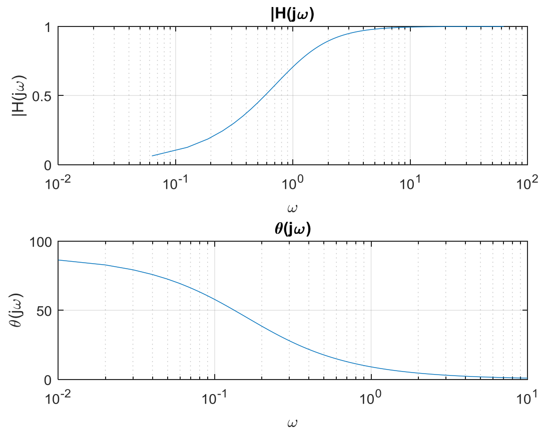


The waves at 1 and 1.1 rad/s are considerably attenuated. The wave at 2.5 rad/s comes out unchanged once the transients have died off.

5. Frequency response of rc circuit (highpass filter)

Let us consider the function $\frac{z}{1+z}$ belongs to the class of all analytic univalent function with the Montel normalization (ie. $\frac{z}{1+z} \in S$), which is a transfer function for the RC circuit in series.Using transfer function of circuit, we plot a frequency response of the circuit. For the above transfer function RC valuesareR=1; C=1;

```
R=1;
C=1;
f=0:0.01:10;
w=2*pi*f;
h=abs((j*w)./(j*w+R*C));
subplot (2,1,1)
semilogx(w,h)
grid on
title('|H(j\omega)')
xlabel ('\omega')
ylabel ('|H(j\omega)')
theta=angle((j*w)./(j*w+R*C));
subplot (2,1,2)
plot (w, theta)
degree=theta*180/pi;
semilogx (f, degree)
grid on
title('\theta(j\omega)')
xlabel('\omega')
ylabel('\theta(j\omega)')
```

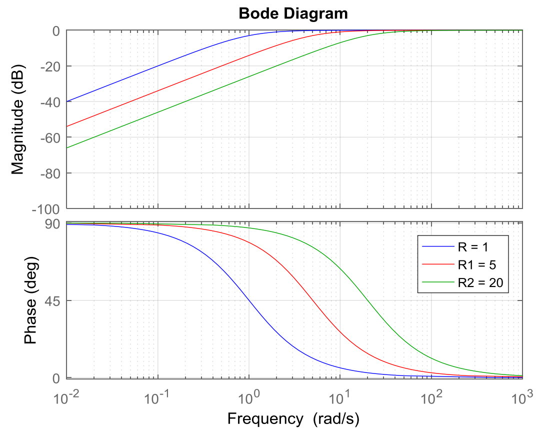


The transfer function from input to output voltage is:

$$G(s)\frac{s}{s+RC}$$

To build a bandpass filter tuned to the frequency 1 rad/s, set C=1 and use R to tune the filter band. To get a narrower passing band, try decreasing values of R as follows

```
R = 1; C = 1;
G = tf([1 0],[1 R*C])
bode(G), grid
R1 = 5;   G1 = tf([1 0],[1 R1*C]);
R2 = 20;  G2 = tf([1 0],[1 R2*C]);
bode(G,'b',G1,'r',G2,'g'), grid
legend('R = 1','R1 = 5','R2 = 20')
```

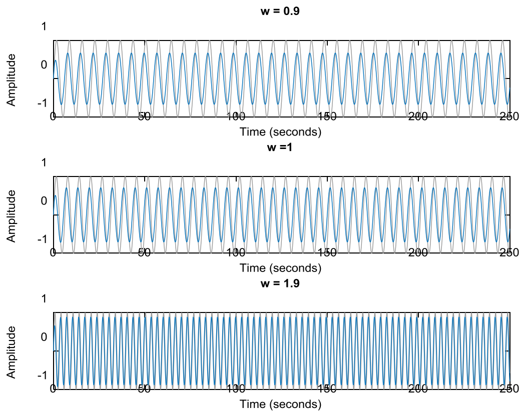


The resistor value R=1 gives a filter narrowly tuned around the target frequency of 1 rad/s.

5.1 analyzing the time response of the circuit

We can confirm the attenuation properties of the circuit G (R=1) by simulating how this filter transforms sine waves with frequency 0.9, 1, and 1.9 rad/s:

```
t = 0:0.05:250;
opt = timeoptions;
opt.Title.FontWeight = 'Bold';
subplot(311), lsim(G,sin(0.9*t),t,opt), title('w = 0.9')
subplot(312), lsim(G,sin(1*t),t,opt), title('w =1')
subplot(313), lsim(G,sin(1.9*t),t,opt), title('w = 1.9')
```



The waves at 0.9 and 1 rad/s are considerably attenuated. The wave at 1.9 rad/s comes out unchanged once the transients have died off.

References

1. http://www.mathworks.com/. Dec. 1998.
2. Matlab, Signal Processing Toolbox User’s Guide. The MathWorks, Inc., 1996.
3. H.J. Orchard and G.C. Temes. “Filter Design Using Transformed Variables”. IEEE Trans. Circuit Theory, Vol. 15, pp. 385-408, Dec. 1968.
4. P.L. Duren, Univalent Functions, Grundlehren der MathematischenWissenschaften Springer, New York, USA, 259 1983.
5. A.W. Goodman, On uniformly starlike functions, J. Math. Anal. Appl., 155 (1991), 364–370.
6. S. Prema, B. S. Keerthi, Coefficient bounds for certain subclasses of analytic functions, J. Math.Anal., 4 (1) (2013), 22–27.
7. Robertson, M.S., On the theory of univalent functions, \textit{Annals. Math.}, (1936), 374--408.