ISSN 2303-4521
Periodicals of Engineering and Natural Sciences *Original Research*
Vol. 9, No. 4, December 2021, pp.1110-1117

Comparison of the traditional exponential weighted moving average chart (EWMA) and modified EWMA chart (MEWMA)

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ABSTRACT

Any production process that needs statistical control to exit the production process as required, whether from the consumer or producer point of view, and in this research a comparison was made between the two control charts of the modified and traditional exponential weighted moving average chart.
The modified chart takes into consideration the autocorrelation that arises between the observations, unlike the traditional chart, which is affected by the autocorrelation between the values of the observations, which leads to giving inaccurate results on the progress of the production process.
To clarify the work of the modified chart and compare it with the traditional chart, data representing the temperature of a chemical industrial process was relied on, because such data are autocorrelation, meaning that small or sudden transformations may occur.
The two charts were also applied to data taken from the Quality Control Department in the General Facility for Vegetable Oils in Baghdad Governorate affiliated to the Ministry of Industry. The modified chart showed its efficiency and accuracy in detecting deviations in the production process better than the traditional chart.

Keywords: Auto correlated, (EWMA) Chart, Control chart, The Traditional Chart

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1. Introduction

The control charts are one of the statistical methods used in controlling the quality of production processes or providing any service to the customer. Perhaps one of the oldest of these charts is the Shewart's control chart, which detects deviations in the process to be achieved and monitored.
But this chart fails to detect sudden and rapid deviations because it has a memoryless trait, meaning that any observation in the chart has nothing to do with the data of previous observations, which makes this chart less sensitive in detecting small continuous changes in the process level, Such as detecting a change in the process mean within one and a half standard deviations or less, Although the tests of non-random patterns and trends that are used to interpret the chart can be applied given all observations, the use of these tests contributes to an increase in the number of false alarms.
More accurate charts appeared, such as the Moving Average chart, (EWMA) Chart, and the Accumulated Sum chart, which are the alternative charts to the Shewart chart. These charts aim to detect changes in process outputs, and it detects deviations that occur in the process better than its predecessors, The exponentially weighted moving average panel is better than the moving average panel, as it gives the observations of the short periods in time a greater weight than the distant observations, and it detects the deviations that occur in the process better than the previous ones. But if the observations of the production process suffer from the problem of autocorrelation, then it is no longer needed and the modified chart is the best in diagnosing deviations.

2. Previous studies

The first widely used control chart in statistical control processes were introduced by Shewhart, the Shewhart control chart is useful in diagnosing single large transitions in processes, when the assumptions are independent and have a normal distribution. Diversity of EWMA control chart procedures, developed by researchers such as (Montgomery & Mastrangelo), Lu & Reynolds, and recently (Patel &Divecha) Modified EWMA chart, which is very effective when diagnosing the associated transformations [1].

The control chart, Shewhart and the exponential weighted average chart are simple and effective graphical procedures for controlling the quality of the manufactured products. The EWMA control chart is designed for rapid diagnosis, either for small shifts or large shifts, but not for both. Shewhart's chart is effective in diagnosing large shifts, and it is often suggested to combine these two schemes when it is intended to diagnose both large and small shifts.

The so-called control charts, Shewhart, 1924, EWMA (Roberts, 1959), and CUSUM (Page, 1954), are frequently utilized A set of independent natural data with covariance resulting from a certain process. The Shewart chart diagnoses all single large transitions, the cumulative chart diagnoses transitions by cumulative changes in trend, and the (EWMA) Chart diagnoses transformations by cumulative changes under the influence of exponential smoothing All three charts, in general, fail to detect transformations in one example, whether a large or minor alteration in the process. Furthermore, because the observations in the processes are commonly connected, these charts cannot be employed directly in the chemical and pharmaceutical industries. The typical consequence of autocorrelation in control charts built under the concept of autonomy is to lower the average run length (ARL) under control, leading to a rapid false alarm for the autonomous process. Moreover, the auto correlated processes may have highly mutated changes; both Both situations are troublesome for these industries [2].Thus when autocorrelation appears in the data, it has a serious effect in classical control charts. This point has been made clear by several authors, including Berthouex, Hunter, and Pallensen 1978, (Harris & Ross 1991), (Montgomery &Mastrangelo 1991), and (Runger 1996), who presented a realistic model that generates autocorrelation and cross-correlation provides a useful approach to characterizing procedure data[3].

In some production processes, especially those that have a connection with human life, such as the production of drugs and medicines, these operations require a lot of accuracy, caution in such operations that result in a autocorrelation between the values of successive observations, which gives incorrect results about the progress of the production process, Therefore, care must be taken in adopting these results. In order to obtain a flawless production process, we use control charts, including the traditional charts for the exponential weighted moving average. To detect deviations in the production process that occurs suddenly as a result of the autocorrelation between observations.

The study aims to compare between the traditional chart and the modified chart and to show which is better in detecting deviations that occur in the production process, especially if the process suffers from the problem of autocorrelation between the values of observations.

3. Control charts

Control charts are used to monitor production processes. The typical assumption is that the process measurements over time are an independent random variable. However, in many circumstances, the premise of observational independence is not always accurate. The key assumptions in the production and usage of control charts are normalcy and independence. However the premise of independence can be difficult to justify for many industrial data sets. Because of the dynamic character of many chemical process industries, observations obtained at different times in time are frequently connected. Some processes produce data that is tied to each other by their very nature. The use of autocorrelation in process views has the potential to significantly alter the standard control chart. Developed on the basis of separate assumptions. When there is a lot of autocorrelation in the process data, it's not a good idea to use the classic control chart methods without making any changes, because autocorrelation has a tendency to affect the limits of control[4].

4. Traditional exponentially weighted moving average charts

The (EWMA) control chart, proposed by (Roberts 1959) [6], is widely used to diagnose and monitor small transitions in a process. It is a good alternative to the Shewhart chart and is more effective when monitoring small changes and autocorrelation data operations. The (EWMA) Chart is the optimal chart for the process with an average in period (t) related to the average in period (t-1).

Individual observations are documented using the (EWMA) Chart. The process observations of the variable (x) are assumed to have a normal distribution with a mean (μ) and variance (σ^2).

The design of this chart gives the most modern observation more weight, and the rest of the other observations have geometrically decreasing weights after the first observation.

This chart contains the following statistic [1,5]:

$$Z_i = \lambda x_i + (1-\lambda) Z_{i-1} \quad (1)$$

Since λ it represents the boot tab, and its value is $0 < \lambda < 1$

Whereas, Z_0 represents the initial value (aim value), and $Z_0 = \mu_0$

Control limits (higher and lower), and the central control limit in the (EWMA) Chart:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \quad (2)$$

$$CL = \mu_0 \quad (3)$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \quad (4)$$

Where L is (Control Width Limit), and reflects the process standard deviation.

5. Modified exponentially weighted moving average chart

This chart is effective for diagnosing transitions for all sizes according to the technical specifications. The modified (EWMA) chart takes care of the previous observation, as it is in the traditional EWMA chart. In addition, it takes care of the previous changes in addition to the last change in the process.

Little shift signals in the process, as well as abrupt transitions in the autocorrelation process, are captured by the modified (EWMA) Chart [1, 5].

The modified (EWMA) Chart is the best predictor of the process average, as it accurately predicts the state of the process average; this makes the modified (EWMA) chart free from the problem of disruption.

The modified (EWMA) Chart combine the qualities of the Shewhart chart and the (EWMA) Chart in a simple style, allowing it to diagnose basic as well as major transitions as quickly as possible, as required by various industrial processes with strong first-order autocorrelation. The modified (EWMA) statistic gives earlier observations a weight in a slightly different way than regular (EWMA) Chart, and every current change is treated as a complete weight, which prevents the (EWMA) statistic from being disrupted. The primary concept is to adjust the weight based on previous observations, changes, current observations, and changes. The modified (EWMA) Chart measures the current volatility included in the (EWMA) control statistic as in $(x_i - x_{i-1})$ the difference between two values of a consecutive process of sudden changes diagnosis.

The modified (EWMA) Chart was proposed by Patel &Divecha in 2011.

The modified (EWMA) Chart statistics are defined as follows:

$$Z_i = \lambda x_i + (1-\lambda) Z_{i-1} + (x_i - x_{i-1}) \quad (5)$$

The geometric total of earlier observations, previous changes, current observations, and the current change in the process is represented by Zi. This allows the modified (EWMA) Chart to perform similarly to the (EWMA) Chart for modest shifts while also diagnosing significant changes as the Shewhart chart does. Whereas, xi are sequence observations of the first order moving average (MA(1)) of the process.

Where i=0, 1,…, $0 < \lambda < 1$, and that $Z_0 = \mu_0 = x_0$.

The mean of this chart is (μ), and the variance is $\sigma^2\left[\left(\frac{\lambda}{2-\lambda} + \frac{2\lambda(1-\lambda)}{2-\lambda}\right)\right]$ for the autocorrelation process dependent on $\rho = 1$ the first-order autocorrelation coefficient.

The control limits (higher and lower) and the central limit of the modified (EWMA) Chart is:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda} + \frac{2\lambda(1-\lambda)}{2-\lambda}} \quad (6)$$

$$CL = \mu_0 \quad (7)$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda} + \frac{2\lambda(1-\lambda)}{2-\lambda}} \quad (8)$$

Where (μ_0) the average objective of the process, and (σ^2) the variance of the process.

The modified (EWMA) Chart constants (λ) and (L) denote the exponential weight and control width border, respectively.

6. Applied side

6.1. Modified EWMA

Table 1 shows temperature readings for a chemical industrial process [7] that are taken every two minutes, operate in under- and out-of-control scenarios, and may experience tiny fluctuations and dramatic shifts.

Table 1. Temperatures for a chemical industrial process

949	948	958	952
941	937	955	931
966	955	947	928
966	927	941	937
934	940	938	950
937	962	945	970
946	963	963	934
952	943	967	961
935	950	969	935
941	938	981	928
Total	37920		
Average	948		

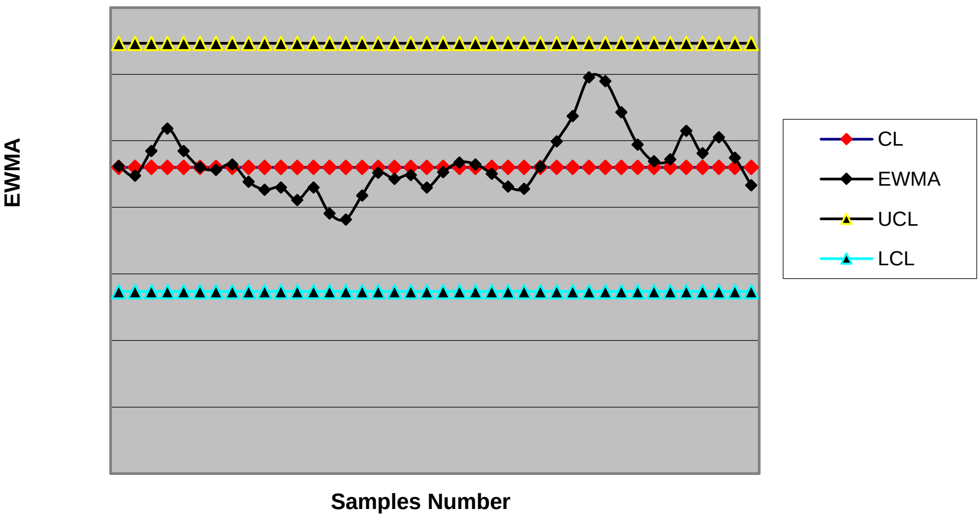

Figure 1. A conventional EWMA temperature plate for a chemical industrial process

According to the Traditional (EWMA) Chart, The process aim is (948), the standard deviation of the process is (13.54), it has been put $\lambda = 0.1$ .The value of the traditional EWMA statistics was calculated according to equation (1) and the results were placed in Table 3.

The control chart limits were also calculated according to equations (2), (3) and (4).It was (UCL=957, CL=948, LCL=938.68).Figure 1 shows a conventional EWMA temperature plate for a chemical industrial process
After drawing the observations, we note that the process is statistically controlled as longer as the values are drawn inside the control boundaries, i.e., traditional (EWMA) Chart is unable to diagnose any process transformation.

6.2. Modified (EWMA) chart

Process aim is (948), process standard deviation is (13.54), it has been put $\lambda = 0.1$ the value of the modified (EWMA) statistic was calculated according to equation (5) and the results were placed in Table 2.
Also, the control limits of the modified (EWMA) Chart was calculated according to equations (6), (7) and (8).It was (UCL=950.87, CL=948, LCL=945.13)

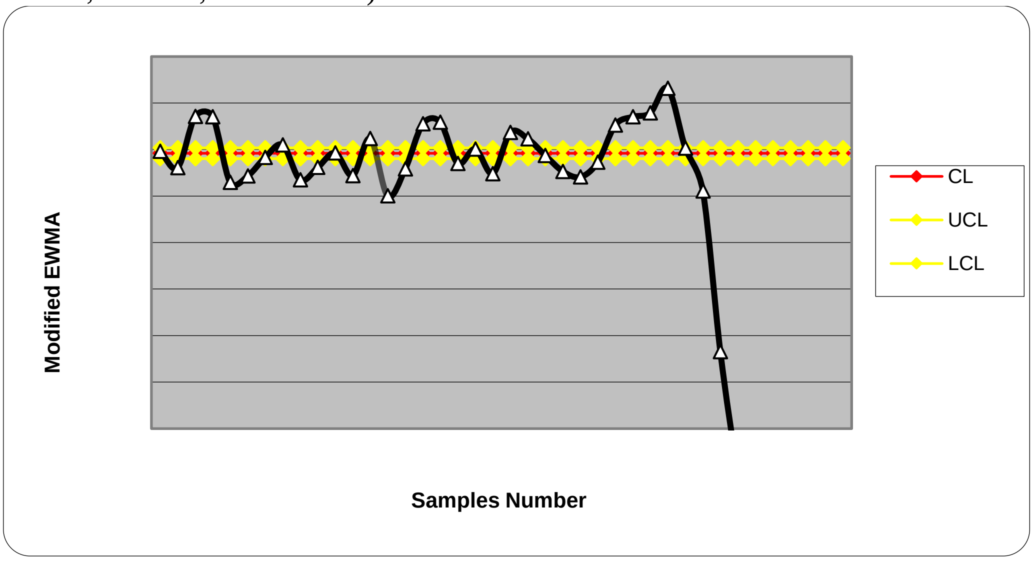

Figure 2. Modified (EWMA) Chart for temperature a chemical industrial process

Figure 2 shows the Modified (EWMA) Chart for temperature a chemical industrial process. The modified (EWMA) Chart was able to diagnose all kinds of small, large and sudden transitions, as in Figure (2). It is noted from Table 3, that (*) was placed on the large and sudden transformations, where the following observations were diagnosed:
2,3,4,5,6,9,10,12,13,14,15,16,17,18,20,21,22,24,25,27,28,29,30,32,33, 34,35,36,37,38,39,40.

6.3. Normal independent processes

The information in Table 2, which indicates the weight of a bottle of cooking vegetable oil, was examined. The data were obtained from the Quality Control Department at the General Establishment for Vegetable Oils in the Baghdad Governorate of the Ministry of Industry. The readings represent the weight of one bottle of oil, which is supposed to its standard weight is one liter, which is equivalent to 1000 grams.

Table 2. Weight of a bottle of vegetable oil (gm)

988	996	987	992	985
994	1000	1002	996	997
999	996	995	993	992
996	993	999	988	994
988	997	995	996	997
997	994	1002	1000	989
987	999	985	994	996
998	999	996	995	992
995	999	994	992	993
998	997	996	995	994
Total	49731			
Average	994.62			

6.4. Traditional (EWMA) Chart

The process aim is (994.62), the standard deviation of the process is (4.12).It has been put $\lambda = 0.1$.
The value of the traditional EWMA statistics was calculated according to equation (1) and the results were placed in Table (3). Also, the control limits for the traditional (EWMA) Chart were calculated according to equations (2), (3) and (4). It was (UCL=997.46, CL=994.62, LCL=991.79).

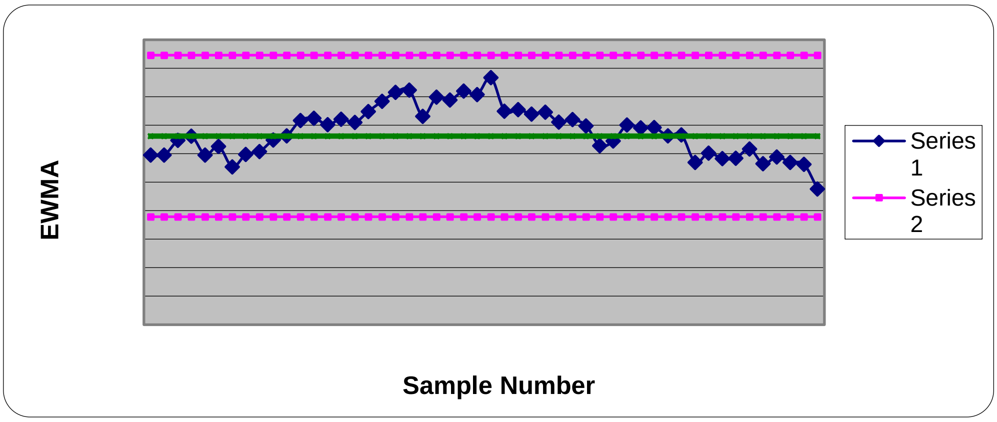

Figure 3. The traditional (EWMA) Chart for weighing a bottle of vegetable oil

After drawing the observations, we note that the process is statistically controlled as long as the values are drawn inside the control boundaries, i.e., traditional (EWMA) Chart is unable to diagnose any process transformation.

6.5. Modified (EWMA) chart

The process aim is (994.62), the standard deviation of the process is (4.12). It has been put $\lambda = 0.1$. The value of the modified EWMA statistic was calculated according to equation (5) and the results were placed in Table 3. Also, the control limits of the modified (EWMA)Chart was calculated according to equations (6), (7) and (8).It was (UCL=995.49, CL=994.62, LCL=993.75).

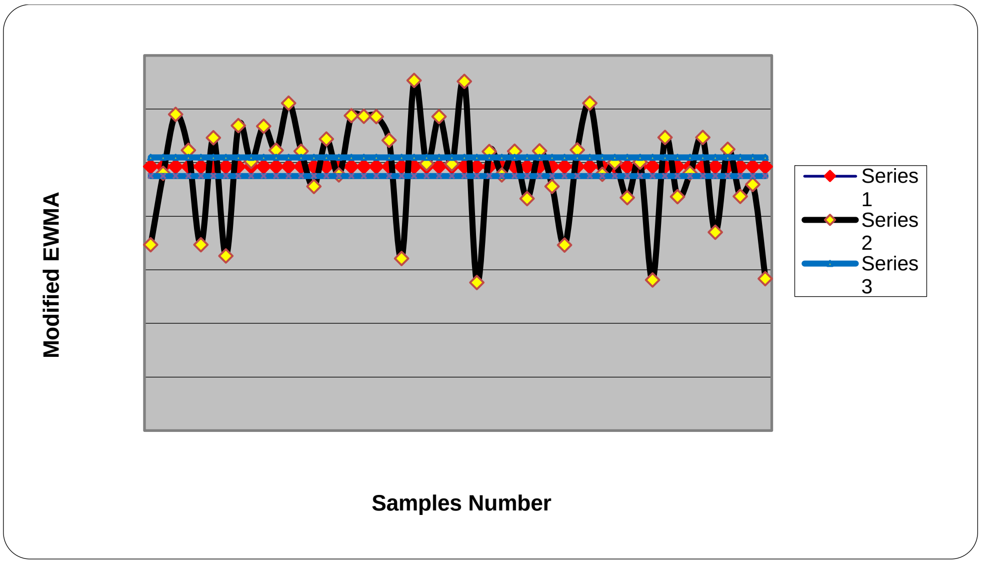

Figure 4. The modified (EWMA) Chart for the weight of a bottle of vegetable oil

The modified (EWMA) chart was able to diagnose all kinds of small, large and sudden transitions, as in Figure 4. Table 3 shows the calculation of the traditional and modified EWMA statistics for the data in Tables 1 and 2. It is noted that the temperature data for an industrial process showed that all observations were out of control except for the observations1,7,8,11,31 While the traditional panel did not reveal any deviation in the process, as for Table 2 the following observations were diagnosed that are outside the control limits: 1,3,5,6,7,8,12,21,22,26,27,34,36,41,46,50 While the traditional chart did not show any defects in the process and did not reveal the deviation in the process, this confirms the accuracy of the performance of the modified board from the traditional chart. (*) is marked for large and abrupt transitions.

Table 3. Calculation of the traditional and modified EWMA statistics for the data of Tables 1 and 2

Observation sequence	EWMA Statistics (traditional and modified) table data (2)		EWMA Statistics (traditional and modified) table data (1)	
	EWMA	Modified EWMA	EWMA	Modified EWMA
1	993.96	987.34*	948.1	949.1
2	993.96	994.01	947.39	940.29*
3	994.47	999.51*	949.25	967.86*
4	994.62	996.16	950.93	967.67*
5	993.96	987.34*	949.24	932.30*
6	994.26	997.31*	948.02	935.77*
7	993.54	986.28*	947.82	945.79
8	993.98	998.45*	948.24	952.41
9	994.08	995.11	946.92	933.67*
10	994.48	998.40	946.33	940.40*
11	994.63	996.16	946.50	948.16
12	995.17	1000.54*	945.55	936.04*
13	995.25	996.07	946.50	955.94*
14	995.02	992.76	944.55	925.05*
15	995.22	997.18	944.10	939.55*
16	995.10	993.86	945.89	963.80*
17	995.49	999.37	947.60	964.72*
18	995.84	999.33	947.14	942.55*
19	996.16	999.30	947.43	950.30
20	996.24	997.07	946.49	937.07*
21	995.32	986.06*	947.64	959.16*
22	995.99	1002.65*	948.38	955.74*
23	995.89	994.89	948.24	946.87
24	996.20	999.30	947.52	938.28*
25	996.08	994.87	946.57	935.25*
26	996.67	1002.58*	946.41	943.23
27	995.50	983.82*	948.07	963.21*
28	995.55	996.04	949.96	967.59*
29	995.40	993.84	951.86	969.73*
30	995.46	996.06	954.77	982.86*
31	995.11	991.65	954.49	950.77
32	995.20	996.09	952.14	927.79*
33	994.98	992.78	949.73	841.29*
34	994.28	987.30*	948.46	775.53*
35	994.45	996.17	948.61	720.47*
36	995.01	1000.55*	950.75	678.12*
37	994.91	993.90	949.08	583.65*
38	994.92	995.01	950.27	561.90*
39	994.63	991.71	948.74	489.06*
40	994.67	995.04	946.67	442.43*
41	993.70	984.04*		
42	994.03	997.34		
43	993.83	991.81		
44	993.85	994.03		

45	994.17	997.33		
46	993.65	988.50*		
47	993.89	996.25		
48	993.70	991.83		
49	993.63	992.95		
50	992.77	984.16*		

(*) means major or sudden changes.

7. Conclusion

1- The modified (EWMA) Chart has the ability to better diagnose the extent of transitions, when the classic chart is not perfectly designed.
2- The modified (EWMA) Chart characteristics are easy to apply in detecting shifts for all sizes, and very effective in diagnosing small transitions for monitoring operations.
3- The modified (EWMA) Chart can be used to predict the observations in the coming period, which helps the analyst to take precautionary measures before the process goes out of control.

Recommendations

1- It is recommended to use (MEWMA) chart when the data taken suffers from the problem of autocorrelation because it addresses this problem and gives accurate results in detecting deviations resulting from the process.
2- Because the observations in these processes are commonly related, the (MEWMA) Chart is considered the best to utilize in the chemical and pharmaceutical sectors. This autocorrelation has a substantial impact on control charts, since it produces incorrect results, but using the modified graph solves the problem of observation autocorrelation.

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