Construction of the spatial development model of a city based on vertical planning concepts

Susanne Braun [1]**, Monika Peters** [2]**, Abram Burnell** [3]

[1] The Hebrew University of Jerusalem
[2] Macquarie University
[3] Hanoi University of Science and Technology

ABSTRACT

Formation of an urbanistic approach to the study of the urban environment is based on the understanding of how the systems of support and spatial planning can be implemented. The relevance of the study is determined by the fact that urban planning in a traditional way when using horizontal space, may not always affect areas that will ensure expansion and improve the quality of life in cities. The novelty of the study is determined by the fact that vertical planning does not always allow using the general forms and structures of urban planning concepts. The authors have determined that particular importance in planning should be given to the design of the visual environment for the urban area. The paper presents a model for designing a vertical environment and defines the principles of modelling high-rise design concept. It is shown that modelling should be carried out online using real-time technologies. The practical significance of the study is determined by the fact that the construction of the developed model is possible only with the use of spatial modelling technologies on high-resource systems. Obtaining three-dimensional models is assumed in the structure of stereo modelling not only of buildings, but also of accompanying objects that can act as infrastructural assets.

Corresponding Author:

Susanne Braun
Rachel & Selim Benin School of Computer Science & Engineering
The Hebrew University of Jerusalem
9190401, Derech Brodetski Str., Jerusalem, Israel
E-mail: braun.susann09@gmail.com

1.   Introduction

The construction of skyscrapers can be beneficial not only to their owners but also to the authorities and residents of the city, since very often agreements are concluded between the authorities and owners, one of the conditions of which is the additional improvement of the territories adjacent to the future object, in particular the creation of new public spaces [1-12]. The business structure is fundamental in a skyscraper, so everything must be built around it [2; 13; 14]. In this context, apartments located in a skyscraper should not be targeted at everyone, but only at the employees working in this building [3;15-22]. At the same time, all apartments must be rented out (prices will be regulated by the building owners or tenants), without the possibility of purchase and privatization [23-30]. This would, firstly, reduce the burden on the city by reducing commuting and carbon footprint. Second, it will allow the skyscraper to live a fulfilling life without stagnation, as after being fired, employees will vacate apartments, opening those spaces for their successors [31-44].

The problem of unsustainable construction mainly dates back to the 20th century. High-rise buildings consume significantly less energy than low-rise buildings, and modern skyscrapers are more environmentally friendly than all other buildings [45]. In fact, many of them are self-sustaining, which has a beneficial effect on the entire global ecology [46-51]. It also means decentralization and antitrust regulation in this area. This forms an independent city within another city; including in relation to the conditions of cataclysms and epidemics, this is a particularly useful factor. Moreover, sustainability is not the only merit of high-rise

buildings, as skyscrapers are the innovators of many modern technologies in completely different fields of science – from roof gardens, green walls and multi-story greenhouses as part of new vertical forms of urban gardening and agriculture, to carbon fiber technology allowing elevators to safely climb 1000 meters at a time and not burst in the event of a fire [52-63]. Thus, in the future, the skyscraper can be developed as a laboratory for research universities [64]. In this context, skyscrapers not only symbolize the power of new technologies but also use them – even if they symbolize completely different technologies than those used [65; 66; 67].

At the same time, the model that shows the possibility of integrating skyscrapers into the urban environment does not take into account the visual component and the development of a multidimensional research model is relevant. It would allow simulating the general plan of the city online with minimal loss of time.

The use of stereoscopic principles for three-dimensional objects visualization, in the general case, adds complexity in this sense, since, among other things, it is necessary to ensure that all tridimensional development procedures are performed [68-88]. If we use the anaglyphic method of separating stereopairs, when only two mutually complementary colors (red and turquoise) are used, then by reducing the color palette, it is possible to somewhat compensate for the additional difficulties that arise [89-97]. For specialized stereoscopic systems, it is possible to propose a method for speeding up computational procedures based on the idea of using group arithmetic processors in modelling and display [98-102].

2. Materials and methods

A group arithmetic operation should, on the one hand, be dominant in modelling and in computer graphics procedures, and on the other hand, it should allow its implementation on the basis of combinational high-speed structures that require a minimum of control. Previous studies have shown that if the arithmetic operation of scalar multiplication of two real vectors (the sum of paired products) is chosen as the main group operation, then typical problems of linear algebra (matrix-vector operations, systems of linear algebraic equations) admit the effective application of this operation. Many problems of calculating power grids, problems of physics and technology, both directly and when solving differential equations (ordinary and in partial derivatives), are reduced to solving systems of linear algebraic equations of high orders. The results of further research on the subject of accelerating computational procedures based on the indicated group arithmetic operation is presented.

The tasks of modelling and displaying three-dimensional objects require significant computational resources, and often even a powerful CPU cannot meet the modern requirements of the "man-machine" dialogue due to the large volume of three-dimensional graphic information processed during modelling and display [103]. In order to remove most of the computational load associated with manipulating graphic procedures during the display of three-dimensional information and thereby ensure a fast response of the computing environment to a human request, more and more new graphics cards and video accelerators are constantly being developed and put on the market [104]. Modern graphics cards are inherently fast specialised graphics processors designed to perform specific image processing [105]. Display resolutions are constantly increasing; the colour gamut of each screen pixel is constantly growing – therefore, GPUs are becoming more sophisticated and often cost more than a CPU [106-110]. In addition, intensive work is underway to find new algorithms and high-performance computing structures to meet the increased requirements for processing speed and realism of 3D images [111-115].

A system of differential equations is given:

$$\frac{d\vec{x}}{dt} = A\vec{x} + \vec{F}(t) \tag{1}$$

$$\vec{x}(t_0) = x_0$$

where A – square matrix of constant values; $\vec{F}(t)$ – vector of the right-hand sides.

To find a solution, the following sequence of actions was proposed: replacing system (1) with a system of the form

$$\frac{d\vec{y}}{dt} = A\vec{y} + L_1[\vec{F}(t)] \tag{2}$$

$$\vec{y}(t_0) = \vec{y}_0$$

where $L_1[\vec{F}(t)]$ – linear operator, the application of which to the vector of the right-hand sides $\vec{F}(t)$ brings system (1) either to a simpler form or to a form for which the answer is known; determination of the vector of variables $\vec{y}(t)$; finding the required vector $\vec{x}(t)$ using the so-called inverse operator $L_1^{-1}[\vec{y}(t)]$, acting by the latter on the vector $\vec{y}(t)$, that is

$$\vec{x}(t) = L_1^{-1}[\vec{y}(t)] \tag{3}$$

Previously, two pairs of linear operators were considered:
first pair,

$$\begin{cases} L_{1m}[\vec{F}(t)] = \frac{d^m}{dt^m}[F(t)] \\ L_{1m}^{-1}[\vec{y}(t)] = \underbrace{\int_{t_0}^{t} \int_{t_0}^{t} \ldots \int_{t_0}^{t}}_{m} \underbrace{d\tau d\tau \ldots d\tau}_{m} \end{cases} \tag{4}$$

with constraints

$$\begin{cases} \vec{F}(t_0) = 0 \\ \frac{d^i[\vec{F}(t)]}{dt^i}\big|_{t=t_0} = 0 \\ i = 1,2,\ldots,m-1 \end{cases} \tag{5}$$

The only constraints of the second pair (6)

$$\begin{cases} L_{2m}[\vec{F}(t)] = \underbrace{\int_{t_0}^{t} \int_{t_0}^{t} \ldots \int_{t_0}^{t} \vec{F}(t)}_{m} \underbrace{d\tau d\tau \ldots d\tau}_{m} \\ L_{2m}^{-1}[\vec{y}(t)] = \frac{d^m}{dt^m}[\vec{y}(t)] \end{cases} \tag{6}$$

with the conditions of vector integration $\vec{F}(t), \int_{t_0}^{t} \vec{F}(\tau)d\tau, \int_{t_0}^{t} \int_{t_0}^{t} \vec{F}(\tau)d\tau d\tau$.

3. Results and discussions

It is clear that it is expedient to use these pairs of operators only in those cases when the components of the vector $\vec{F}(t)$ are the same, since the operators act identically on all components of the vectors $\vec{F}(t)$ and $\vec{y}(t)$. This narrows down the class of these systems. In order to expand this class, it is proposed to use operator matrices instead of pairs of linear operators. In this case, instead of direct linear operators $\vec{y}(t)$ the vector

$\vec{y}(t)$ will be acted upon by the diagonal matrix of linear operators L, in which the direct operators L_{1m}, L_{2m} can stand on the diagonal in any combination and in any order:

$$L = \begin{bmatrix} L_{1m} & 0 & 0 & 0 & 0 \\ 0 & L_{2k} & 0 & 0 & 0 \\ 0 & 0 & L_{2l} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & L_{1f} \end{bmatrix}$$

(7)

The vector $\vec{y}(t)$ is acted upon by the diagonal matrix of inverse linear operators L^{-1}:

$$L^{-1} = \begin{bmatrix} L_{1m}^{-l} & 0 & 0 & 0 & 0 \\ 0 & L_{2k}^{-l} & 0 & 0 & 0 \\ 0 & 0 & L_{2l}^{-l} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & L_{1f}^{-l} \end{bmatrix}$$

(8)

It is convenient to prove the possibility of using the proposed matrices of inverse operators in the Laplace image domain. Suppose that the vector $\vec{F}(t)$ of system (1) under zero initial conditions is acted upon by the matrix of operators (7) under constraints (5) on all necessary operators. Then,

$$\frac{d\vec{y}}{dt} = A\vec{y} + L[\vec{F}(t)]$$

(9)

Apply the Laplace transform

$$p\vec{Y}(p) = A\vec{Y}(p) + L(p)\vec{F}(p)$$

(10)

where

$$L(p) = \begin{bmatrix} p^{\pm i} & 0 & 0 & 0 \\ 0 & p^{\pm j} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & p^{\pm k} \end{bmatrix}$$

(11)

From where

$$\vec{Y}(p) = (pE - A)^{-1}L(p)\vec{F}(p)$$

(12)

In the operator domain, the influence of the matrix of operators on the vector $L[\vec{F}(t)]$ is reduced to the multiplication of matrix (11) by the vector $\vec{F}(p) : L(p)\vec{F}(p)$.

The required vector of unknowns of system (1) in the image domain, on the one hand, equals

$$\vec{X}(p) = (pE - A)^{-1} \circ \vec{F}(p)$$

(13)

On the other hand, applying a matrix of inverse operators in the image domain to a vector gives

$$L^{-1}(p)\vec{Y}(p) = L^{-1}(p)(pE - A)^{-1}L(p)\vec{F}(p)$$

(14)

where

$$L^{-1}(p) = \begin{bmatrix} p^{\pm i} & 0 & 0 & 0 \\ 0 & p^{\pm j} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & p^{\pm k} \end{bmatrix}$$

(15)

Since the matrix $L^{-1}(p)$ is diagonal, expression (14) can be transformed to the form

$$L^{-1}(p)\vec{Y}(p) = (pE - A)^{-1}L^{-1}(p)L(p)\vec{F}(p) = (pE - A)^{-1}E\vec{F}(p) = \vec{X}(p)$$

(16)

which it was necessary to prove.
The fact that the Laplace image domain:

$$L^{-1}(p)L(p) = E$$

,

(17)

served as the fact that it was accepted to write the matrix of inverse operators $L^{-1}(p)$, corresponding to the spelling adopted for the image of the inverse matrix. Formally, in the time domain, the action of the matrix of direct operators on the vector $\vec{F}(t)$ is reduced to three steps: multiplying the components of the matrix L by the vector $\vec{F}(t)$; introducing each component of the vector $\vec{F}(t)$ under the sign of the corresponding linear direct operator; performing actions provided by direct operators. Similarly, from the formal standpoint, the actions of the matrix of inverse operators L^{-1} over the vector $\vec{y}(t)$ are performed:
1) components of the matrix L^{-1} multiplied by the vector $\vec{y}(t)$;
2) each component of the vector $\vec{y}(t)$ is introduced under the sign of the corresponding linear inverse operator; the actions provided by the inverse operators are performed.
Example. Given the system:

$$\begin{cases} \dfrac{dx_1}{dt} - (a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4) = b_1 t^5 \\ \dfrac{dx_2}{dt} - (a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4) = b_2 t^{-4} \\ \dfrac{dx_3}{dt} - (a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4) = b_3 t^3 \\ \dfrac{dx_4}{dt} - (a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4) = b_4 t^{-3} \\ (x_1, x_2, x_3, x_4)|_{t=0} = 0 \end{cases}$$

(18)

It is necessary to determine the vector $x(t)$ provided that the answer for the case

$$\begin{cases} \frac{d\vec{y}}{dt} = A\vec{y} + \vec{\varphi}(t) \\ \vec{y}(t_0) = \vec{y}_0 \end{cases}$$

$$(19)$$

Where

$$\varphi_1(t) = c_1,$$

$$(20)$$

$$\varphi_2(t) = c_2 t^{-1},$$

$$(21)$$

$$\varphi_3(t) = c_3,$$

$$(22)$$

$$\varphi_4(t) = c_4 t^{-1}$$

$$(23)$$

c_1, c_2, c_3, c_4 – known constants.

In this case, it is advisable to choose the matrix of direct linear operators of the form:

$$L(p) = \begin{bmatrix} p^5 & 0 & 0 & 0 \\ 0 & p^{-3} & 0 & 0 \\ 0 & 0 & p^3 & 0 \\ 0 & 0 & 0 & p^{-2} \end{bmatrix}$$

$$(24)$$

which corresponds to the matrix in the time domain

$$L = \begin{bmatrix} \frac{d^5}{dt^5}[0] & 0 & 0 & 0 \\ 0 & \int_{t_0}^t \int_{t_0}^t \int_{t_0}^t d\tau d\tau d\tau & 0 & 0 \\ 0 & 0 & \frac{d^3}{dt^3} & 0 \\ 0 & 0 & 0 & \int_{t_0}^t \int_{t_0}^t d\tau d\tau \end{bmatrix}$$

$$(25)$$

Step 1.

$$L[F_1(t)] = \begin{bmatrix} L_{15} & 0 & 0 & 0 \\ 0 & L_{23} & 0 & 0 \\ 0 & 0 & L_{13} & 0 \\ 0 & 0 & 0 & L_{22} \end{bmatrix} \circ \begin{bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ F_4(t) \end{bmatrix} = \begin{bmatrix} L_{15}F_1(t) \\ L_{23}F_2(t) \\ L_{13}F_3(t) \\ L_{22}F_4(t) \end{bmatrix}$$

$$(26)$$

Step 2.

$$\begin{cases} L_{14}[F_1(t)] = \dfrac{d^5}{dt^5}[F_1(t)] = \dfrac{d^5}{dt^5}[b_1 t^5] \\ L_{23}[F_2(t)] = \int_{t_0}^{t}\int_{t_0}^{t}\int_{t_0}^{t} F_2(\tau)d\tau d\tau d\tau = \int_{t_0}^{t}\int_{t_0}^{t}\int_{t_0}^{t} b_2 \tau^{-4}d\tau d\tau d\tau \\ L_{13}[F_3(t)] = \dfrac{d^3}{dt^3}[F_3(t)] = \dfrac{d^3}{dt^3}[b_3 t^3] \\ L_{22}[F_4(t)] = \int_{t_0}^{t}\int_{t_0}^{t} F_4(\tau)d\tau d\tau = \int_{t_0}^{t}\int_{t_0}^{t} b_4 \tau^{-3}d\tau d\tau \end{cases}$$

(27)

Step 3.

$$\begin{cases} \dfrac{d^5}{dt^5}[b_1 t^5] = 120 b_1, c_1 = 120 b_1 \\ \int_{t_0}^{t}\int_{t_0}^{t}\int_{t_0}^{t} b_2 \tau^{-4}d\tau d\tau d\tau = -\dfrac{1}{6}b_2 t^{-1}, c_2 = -\dfrac{b_2}{6} \\ \dfrac{d^3}{dt^3}[b_3 t^3] = 6 b_3; c_3 = 6 b_3 \\ \int_{t_0}^{t}\int_{t_0}^{t} b_4 \tau^{-3}d\tau d\tau = \dfrac{1}{2}b_4 t^{-1}, c_4 = \dfrac{b_4}{2} \end{cases}$$

(28)

Constants (20) make it possible to determine the vector $\vec{y}(t)$ from the system (19). Then, to obtain the required answer, the matrix of inverse operators is applied to the vector $\vec{y}(t)$, which in this case takes the form:

Reverse step 1.

$$L^{-1} = \begin{bmatrix} \dfrac{1}{c_1}\underbrace{\int_0^t \ldots \int_0^t}_{5} \underbrace{d\tau \ldots d\tau}_{5} & 0 & 0 & 0 \\ 0 & \dfrac{1}{c_2}\dfrac{d^3}{dt^3} & 0 & 0 \\ 0 & 0 & \dfrac{1}{c_3}\underbrace{\int_0^t \ldots \int_0^t}_{3} \underbrace{d\tau \ldots d\tau}_{3} & 0 \\ 0 & 0 & 0 & \dfrac{1}{c_4}\dfrac{d^2}{dt^2} \end{bmatrix}$$

(29)

or in the image domain:

$$L^{-1}(p) = \begin{bmatrix} \dfrac{1}{c_1}p^{-5} & 0 & 0 & 0 \\ 0 & \dfrac{1}{c_2}p^{3} & 0 & 0 \\ 0 & 0 & \dfrac{1}{c_3}p^{-3} & 0 \\ 0 & 0 & 0 & \dfrac{1}{c_4}p^{2} \end{bmatrix}$$

(30)

Reverse step 2.

$$\begin{cases} L_{15}^{-1}[y_1(t)] = \frac{1}{c_1} \underbrace{\int_0^t \dots \int_0^t}_{5} y_1(\tau) \underbrace{d\tau \dots d\tau}_{5} \\ L_{23}^{-1}[y_2(t)] = \frac{1}{c_2} \frac{d^3}{dt^3}[y_2(t)] \\ L_{22}^{-1}[y_4(t)] = \frac{1}{c_3} \underbrace{\int_0^t \dots \int_0^t}_{3} y_1(\tau) \underbrace{d\tau \dots d\tau}_{3} \\ L_{22}^{-1}[y_4(t)] = \frac{1}{c_4} \frac{d^2}{dt^2}[y_4(t)] \end{cases}$$

$$(31)$$

Reverse step 3, as indicated above, is to implement the mathematical operations provided by the inverse operators. An analysis of the formulas obtained by the T-transformation method as applied to differential equations shows that most of the computational load falls on the group arithmetic operation – the sum of paired products. On the other hand, it is known that such an operation is effectively implemented in a non-positional number in residue system (RS), since it consists only of modular operations of addition and multiplication. To increase the efficiency of solving differential equations, the idea arose to combine these two approaches.

System of linear differential equations with constant coefficients, Cauchy problem. The system (32)

$$\begin{cases} \frac{d\vec{x}}{dt} = A\vec{x} + \vec{f}(t) \\ \vec{x}_{|t=t_0} = \vec{x}_0 \end{cases}$$

$$(32)$$

where $\vec{x}$ – vector of unknowns; $\vec{f}(t)$ – vector of the right-hand sides; $\vec{x}_0$ – vector of initial values; $\vec{x}_0$ – matrix of constant coefficients.

In the region of T -transformations, system (32) takes the form

$$D\vec{x}(k) = A\vec{x}(k) + \vec{F}(k)$$

$$(33)$$

where D – symbol of the Taylor derivative, which in this case applies to each component of the vector $\vec{x}(k)$; $\vec{x}(k), \vec{F}(k)$ – corresponding images of the vectors $\vec{x}(t)$ and $\vec{f}(t)$; k – discrete argument taking on values 0,1,2, ...

After the implementation of the T -derivative, the calculation formula will take the form:

$$\vec{x}(k+1) = \frac{H}{k+1}\left[A\vec{x}(k) + \vec{F}(k)\right]$$

$$(33)$$

where H – constant with the dimension of time t .

The main computational load in formula (33) falls on the operation $A\vec{x}(k)$, that is, on a finite number of sums of paired products. However, it is not possible to fully implement formula (19) on computational structures operating in the residue system, because at each step of searching for the next discrete vector of unknowns $\vec{x}_i$, it is necessary to perform the operation of dividing by a value that is so laborious in a non-positional number system that the advantage that residue system gives when performing modular operations.

Therefore, the task arose – to transform formula (33) so that the calculation of the further vector of unknowns (or a function of the vector) occurs without using the division operation over the vector (or its function) at the previous step. It was proposed to solve this problem by introducing a new vector of variables,

$$\vec{y}(k) = \varphi(k)\vec{x}(k)$$

(34)

where $\varphi(k)$ – function of an integer argument (meaning that its calculation does not require non-modular operations).

Substitute (34) in (33):

$$\frac{\vec{y}(k+1)}{\varphi(k+1)} = \frac{H}{k+1}\left[\frac{A\vec{y}(k)}{\varphi(k)} + \vec{F}(k)\right]$$

.

(35)

After transformation we obtain:

$$\frac{\vec{y}(k+1)}{\varphi(k+1)} = \frac{H}{k+1\varphi(k)}\left[A\vec{y}(k) + \varphi(k)\vec{F}(k)\right]$$

(36)

From formula (36) it follows that to satisfy the above conditions, the function $\varphi(k)$ must have the following property:

$$\varphi(k + 1) = (k + 1)\varphi(k)$$

(37)

Indeed, if condition (37) is satisfied, then formula (36) is transformed into the form (38):

$$\vec{y}[k + 1] = H\left[A\vec{y}(k) + \varphi(k)\vec{F}(k)\right]$$

(38)

from which it follows that to calculate any subsequent vector of unknowns $\vec{y}(k + 1)$, only modular operations are needed, that is, the results of their execution remain in the class of integers if there were initial data in this class. The function $\varphi(k)$ that simultaneously satisfies both formula (37) and the conditions mentioned above (41) is the factorial of the variable:

$$\varphi(k) = !$$

(39)

The validity of this statement is confirmed, first, by the fact that for calculating $k!$ only the operation of multiplication is needed, and secondly, by the fact that

$$(k + 1)! = (k + 1)k!$$

(40)

Satisfies (43).

Finally, formulas (41) and (42) have the form:

$$\vec{y}(k) = k!\vec{x}(k)$$

(41)

$$\vec{y}[k+1] = H\left[A\vec{y}(k) + k!\vec{F}(k)\right]$$

(42)

The opinion is erroneous that the change of the vector of variables (41) does not give anything significant on the grounds that we are ultimately interested in the vector $\vec{x}(k)$, and not in the vector $\vec{y}(k)$. Note, however, that the transition to the vector of variables $\vec{y}(k)$ makes it possible to combine non-modular division operations $\frac{\vec{y}(k)}{k!}$ and modular operations for calculating the next discrete vector $\vec{y}(k+1)$. Moreover, it is important that by the time the next discrete vector $\vec{y}_i$ is calculated, all the necessary components in the residue system (RS) are already available.

The original formula (33) does not provide such possibility. Indeed, suppose that the numerator of formula (33) is calculated on the RS basis; and the operation of division by $(k+1)$, is associated with the calculation of the vector of unknowns $\vec{x}(k+1)$, similar to the case considered above, which is performed in the positional number system. All this is possible, but during the transformation of the numerator of formula (33) from the RS into a positional basis and during the further execution of the operation of division by $(k+1)$, and then during the transformation of the vector $\vec{x}(k+1)$ from the positional form into the RS – non-positional blocks are in the waiting state, since to calculate the discrete vector of the numerator of formula (33) at the next step, the vector $\vec{x}(k+1)$ is needed, moreover, in a non-positional basis.

Returning to the new formulas (41), (42), the following distribution of computational functions between positional and non-positional blocks is proposed. A specialised processor of the group operation, working in the RS, is busy calculating the discrete vector by the formula (42). Calculation of the discrete vector $\vec{x}(k+1)$, checking for accuracy, as well as obtaining points of the required vector of a given differential equation (43) is performed in blocks with a positional basis,

$$\vec{x}(i+1) = \vec{y}_i(0) + \vec{y}_i(1) + \frac{\vec{y}_i(2)}{2!} + \frac{\vec{y}_i(3)}{3!} + \dots$$

(43)

Moreover, as noted above, at the moment of calculating by the non-positional processor, for example, the vector $\vec{y}_i(4)$, the transformation from the RS to the positional basis of the previously obtained value $\vec{y}_i(3)$ occurs in parallel; the vector $\frac{\vec{y}_i(3)}{3!}$ in the positional number system; the desired result is accumulated in positional memory by summing $\frac{\vec{y}_i(3)}{3!}$ to the previously accumulated value $\vec{y}_i(0) + \vec{y}_i(1) + \frac{\vec{y}_i(2)}{2!}$. When calculating using formula (26), the components of the vector $\vec{y}(k+1), HA\vec{y}(k), Hk!\vec{F}(k)$, should be stored separately in memory, because when performing the next division operation $\frac{\vec{y}(k+1)}{k+1}$ the range of representation of numbers for separate division is less than for the division of the entire $\vec{y}(k+1)$ vector by $(k+1)!$. In other words, it is necessary to transform the value $HA\vec{y}(k)$ from RS into positional form and here to determine the first component $\frac{HA\vec{y}(k)}{(k+1)!}$.

Second component:

$$\frac{Hk!\vec{F}(k)}{(k+1)!} = \frac{H\vec{F}(k)}{k+1}$$

(44)

must be fully calculated in positional form, since there is all the data for this operation in positional memory.

Further, it remains only to add the resulting components of the vector $\vec{y}(k+1)$.

System of linear differential equations with variable coefficients, the Cauchy problem. The system (67)

$$\begin{cases} \frac{d\vec{x}}{dt} = A(t)\vec{x} + \vec{f}(t) \\ \vec{x}_{|t=t_0} = \vec{x}_0 \end{cases}$$

(45)

where the coefficients of the matrix $A(t)$ depend on time. According to the technique, system (67) in the region of T-images is represented as follows:

$$D\vec{x}(k) = A(k) * \vec{x}(k) + \vec{F}(k)$$

(46)

where $A(k) * \vec{x}(k)$ – product in the T-domain.

The calculation formula after the implementation of the T-derivative and T-product has the form:

$$\vec{x}(k+1) = \frac{H}{k+1}\left[\sum_{l=0}^{l=k} A(l)\vec{x}(k-l) + \vec{F}(k)\right]$$

(47)

As can be seen from formula (47), the main complication in setting the system of differential equations (48) on computational structures with mixed coding is the presence of the division by the coefficient $(k+1)$ at each step of calculating the discrete vector $\vec{x}(k+1)$.

Let us try to use the change of variables found earlier (25). The proof will be carried out by the method of mathematical induction. First, let us verify the efficiency of substitution (41) in (47) for the cases when $k = 0,1,2,3$:

$k = 0$

$$\vec{x}(1) = H\left[A(0)\vec{x}(0) + \vec{F}(0)\right] = \vec{y}(1)$$

(48)

division operation is not necessary; $k = 1$:

$$\vec{y}(1) = H\left[A(0)\vec{y}(0) + \vec{F}(0)\right]$$

(49)

$$\vec{x}(2) = \frac{H}{2}\left[A(0)\vec{x}(1) + A(2)\vec{x}(0) + \vec{F}(1)\right] = \frac{\vec{y}(2)}{2!}$$

(50)

division operation is not necessary; $k = 2$

$$\vec{y}(2) = H\left[A(0)\vec{y}(1) + A(1)\vec{y}(0) + \vec{F}(1)\right]$$

(51)

$$\vec{x}(3) = \frac{H}{3}\left[A(0)\vec{x}(2) + A(1)\vec{x}(1) + A(2)\vec{x}(0) + \vec{F}(2)\right] = \frac{H}{3}\left[A(0)\frac{\vec{y}(2)}{2!} + A(1)\vec{y}(1) + A(2)\vec{y}(0) + \vec{F}(2)\right] = \frac{H}{3!}\left[A(0)\vec{y}(2) + 2!\,A(1)\vec{y}(1) + 2!\,A(2)\vec{y}(0) + 2!\,\vec{F}(2)\right] = \frac{\vec{y}(3)}{3!}$$

(52)

division operation is not necessary; $k = 3$:

$$\vec{y}(3) = H\left[A(0)\vec{y}(2) + 2!\,A(1)\vec{y}(1) + 2!\,A(2)\vec{y}(0) + 2!\,\vec{F}(2)\right]$$

(53)

$$\vec{x}(4) = \frac{H}{4}\left[A(0)\vec{x}(3) + A(1)\vec{x}(2) + A(2)\vec{x}(1) + A(3)\vec{x}(0) + \vec{F}(3)\right] = \frac{H}{4}\left[A(0)\frac{\vec{y}(3)}{3!} + A(1)\frac{\vec{y}(2)}{2!} + A(2)\vec{y}(1) + A(3)\vec{y}(0) + \vec{F}(3)\right] = \frac{H}{4!}\left[A(0)\vec{y}(3) + 3!\,A(1)\vec{y}(2) + 3!\,A(2)\vec{y}(1) + 3!\,A(3)\vec{y}(0) + 3!\,\vec{F}(3)\right] = \frac{\vec{y}(3)}{4!}$$

(54)

division operation is not necessary:

$$\vec{y}(4) = H\left[A(0)\vec{y}(3) + 3!\,A(1)\vec{y}(2) + 3!\,A(2)\vec{y}(1) + 3!\,A(3)\vec{y}(0) + 3!\,\vec{F}(3)\right]$$

(55)

Suppose that the above procedure is also valid for the case when $k = n$, that is, the vector $\vec{y}(n)$ is determined excluding the division operation, and under this condition, consider the case:

$$k = n + 1$$

(56)

$$\vec{x}(n+1) = \frac{H}{n+1}\left[A(0)\vec{x}(n) + A(1)\vec{x}(n-1) + A(2)\vec{x}(n-2) + \ldots + A(n-1)\vec{x}(1) + A(n)\vec{x}(0) + \vec{F}(n)\right]$$

(57)

Substituting (41) into (53), obtain:

$$\vec{x}(n+1) = \frac{H}{n+1}\left[A(0)\frac{\vec{y}(n)}{n!} + A(1)\frac{\vec{y}(n-1)}{(n-1)!} + A(2)\frac{\vec{y}(n-2)}{(n-2)!} + \ldots + A(n-1)\vec{y}(1) + A(n)\vec{y}(0) + \vec{F}(n)\right]$$

(58)

After conversion:

$$\vec{x}(n+1) = \frac{H}{(n+1)!}\big[A(0)\vec{y}(n) + nA(1)\vec{y}(n-1) + (n-1)A(2)\vec{y}(n-2) + \cdots + n!\,A(n-1)\vec{y}(1) + n!\,A(n)\vec{y}(0) + n!\,\vec{F}(n)\big]$$

(59)

or more compact form:

$$\vec{x}(n+1) = \frac{H}{(n+1)!}\big[A(0)\vec{y}(n) + \sum_{l=1}^{n} A(l)\vec{y}(n-l)\prod_{0}^{l-1}(n-l) + n!\,\vec{F}(n)\big]$$

(60)

Substituting (41) into (60), obtain:

$$\vec{y}(n+1) = H\big[A(0)\vec{y}(n) + \sum_{l=1}^{n} A(l)\vec{y}(n-l)\prod_{0}^{l-1}(n-l) + n!\,\vec{F}(n)\big]$$

(61)

As can be seen, the vector $\vec{y}(n+1)$ can be calculated without the division operation. By induction, the above procedure is valid for any values of the integer argument k.

Thus, the proposed change of variables (41), applied to the calculation formula (53), makes it possible to parallelise the computation process in blocks operating in the positional and non-positional number system.

As in the system of linear differential equations with constant coefficients, calculations related to the vector $\vec{y}(k)$, formula (61), it is advisable to perform in a non-positional basis (on a special processor of a group operation). In addition, specialised structures operating in the residue system (RS) can be effective in representing the value of $k!$. Calculations related to the implementation of the division operation are performed in a positional basis.

Approximate methods for solving systems of nonlinear equations requires a large number of arithmetic operations of the same type. Therefore, the computational process corresponding to this problem, when the latter is set up on universal computers, takes a lot of computer time. The carried-out elaboration of problems of linear algebra and analysis prove the efficiency of the use of group arithmetic operation of the dot product of two real vectors. Below, an analysis will be carried out for the application of a group operation to speed up the computational process when solving systems of nonlinear equations.

Newton method. A system of nonlinear equations with real left-hand sides is given:

$$\begin{cases} f_1(x_1, x_2, \ldots, x_n) = 0 \\ f_2(x_1, x_2, \ldots, x_n) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ f_n(x_1, x_2, \ldots, x_n) = 0 \end{cases}$$

(62)

Or $\vec{F}(\vec{x}) = 0$, where $\vec{x}$ – vector of unknowns, $\vec{F}(\vec{x})$ – vector function.

Assuming that the function $\vec{F}(\vec{x})$ is continuously differentiated in some convex domain that contains:

$$\vec{x}^{(k)} = \left(x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)}\right),$$

(63)

it is proved that the iterative process of finding the required vector can be organised by the formula:

$$\vec{x}^{(k+1)} = \vec{x}^{(k)} - W^{-1}(\vec{x}^{(k)})\vec{F}(\vec{x}^{(k)}), k = 0,1,2,\dots$$

(64)

where $W^{-1}(\vec{x}^{(k)})$ – the inverse Jacobi matrix of the system of functions $f_1, f_2, \dots, f_n$ with respect to the variables $x_1, x_2, \dots, x_n$ for k-th approximation,

$$W^{-1}(\vec{x}^{(k)}) = \begin{bmatrix} \dfrac{df_1}{dx_1^{(k)}} & \dfrac{df_1}{dx_2^{(k)}} & \cdots & \dfrac{df_1}{dx_n^{(k)}} \\ \dfrac{df_2}{dx_1^{(k)}} & \dfrac{df_2}{dx_2^{(k)}} & \cdots & \dfrac{df_2}{dx_n^{(k)}} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{df_n}{dx_1^{(k)}} & \dfrac{df_n}{dx_2^{(k)}} & \cdots & \dfrac{df_n}{dx_n^{(k)}} \end{bmatrix}^{-1}$$

(65)

From the standpoint of the computational procedure complexity, the inconvenience of Newton method is the need to calculate the inverse matrix $W^{-1}(\vec{x}^{(k)})$ at each step. It is proved that if the matrix $W^{-1}(\vec{x})$ is continuous in proximity to the required $\vec{x}$, then approximately assuming the solution $\vec{x}^{(0)}$ and the initial approximation:

$$W^{-1}(\vec{x}^{(k)}) \approx W^{-1}(\vec{x}^{(0)}),$$

(66)

come to a modified Newton process:

$$\vec{y}^{(k+1)} = \vec{y}^{(k)} - W^{-1}(\vec{x}^{(0)})F((\vec{y})^{(k)}, k = 0,1,2,\dots,\vec{y}^{(0)} = \vec{x}^{(0)})$$

(67)

In this case, the inverse matrix $W^{-1}(\vec{x}^{(0)})$ needs to be calculated only once during the task execution time; therefore, this work and the calculations of the vector $\vec{F}(\vec{y}^{(i)})$ can be assigned to the CPU; the rest of the cumbersome computations of the same type, associated with multiplying a matrix by a column at each iteration step, will be effectively performed by a special processor of the above group arithmetic operation. Denoting

$$W^{-1}(\vec{x}^{(0)}) = G,$$

(68)

$$G = \begin{bmatrix} g_{11} & g_{11} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1} & g_{n2} & \cdots & g_{nn} \end{bmatrix}$$

(69)

and representing (67) by component, the formulas consisting only of the sums of even products are obtained:

$$
\begin{cases}
y_1^{(k+1)} = y_1^{(k)} - \left(g_{11} f_1(y^{(k)}) + g_{12} f_2(y^{(k)}) + \ldots + g_{1n} f_n(y^k) \right) \\
y_2^{(k+1)} = y_2^{(k)} - \left(g_{21} f_1(y^{(k)}) + g_{22} f_2(y^{(k)}) + \ldots + g_{2n} f_n(y^k) \right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
y_n^{(k+1)} = y_n^{(k)} - \left(g_{n1} f_1(y^{(k)}) + g_{n2} f_2(y^{(k)}) + \ldots + g_{nn} f_n(y^k) \right)
\end{cases}
\tag{70}
$$

If it is necessary to apply the unmodified Newton method (66), the operation of finding the elements of the inverse matrix in terms of complexity comes to the fore. From the standpoint of using the group arithmetic operation, the most suitable of the general methods for finding the inverse matrix are the border method and the completion method. Border method. The idea of the method is that the original n-th order matrix A_n is considered as the result of bordering the matrix $A_{n-1}(n-1)$-th order:

$$
A = A_n =
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n-1} & a_{1n} \\
a_{21} & a_{21} & \cdots & a_{2n-1} & a_{2n} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
a_{n-11} & a_{n-12} & \cdots & a_{n-1n-1} & a_{n-1n} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
a_{n1} & a_{n2} & \cdots & a_{nn-1} & a_{nn}
\end{bmatrix}
=
\begin{bmatrix}
A_{n-1} & \vec{u}_n \\
\vec{v}_n & a_{nn}
\end{bmatrix}
\tag{71}
$$

where

$$
\vec{v}_n = (a_{n1}, \ldots, a_{nn-1}),
\tag{72}
$$

$$
\vec{u}_n = (a_{1n}, \ldots, a_{n-1n})'.
\tag{73}
$$

The matrix A^{-1} is also sought in the form of a bordered matrix. It has been proven that:

$$
A^{-1} =
\begin{bmatrix}
\left[A_{n-1}^{-1} + \dfrac{A_{n-1}^{-1} \vec{u}_n \vec{v}_n A_{n-1}^{-1}}{\alpha_n} \right] & \left[-\dfrac{A_{n-1}^{-1} \vec{u}_n}{\alpha_n} \right] \\
\left[-\dfrac{\vec{v}_n A_{n-1}^{-1}}{\alpha_n} \right] & \dfrac{1}{\alpha_n}
\end{bmatrix}
\tag{74}
$$

where

$$
\alpha_n = \vec{v}_n A_{n-1}^{-1} \vec{u}_n.
\tag{75}
$$

Formula (75) is the basis for the sequential border method.
Inverse matrices are sequentially constructed for matrices:

$$
(a_{11}), \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
\tag{76}
$$

of which each subsequent one leaves the previous one with the help of a border on the basis of formula (68).

This process combines the following actions (provided that A_{n-1}^{-1} is already known, the elements of the matrix A_n^{-1} are sought:

1. Column $A_{n-1}^{-1}\vec{u}_n(\beta_{1n},\dots,\beta_{n-1n})$ is calculated.

2. String $\vec{v}_n A_{n-1}^{-1}$ with elements $\gamma_{n1},\dots,\gamma_{nn-1}$ is calculated

3. The number is calculated

$$a_n = a_{nn} + \sum_{i=1}^{n-1} a_{ni}\beta_{in} = a_{nn} + \sum_{i=1}^{n-1} a_{in}\gamma_{ni} \tag{77}$$

4. The elements d_{ik} of the inverse matrix are calculated by the formulas:

$$d_{ik} = d'_{ik} + \frac{\beta_{in}\gamma_{nk}}{\alpha_n}, (i,k \le n-1) \tag{78}$$

$$d_{in} = \frac{\beta_{in}}{\alpha_n}, \tag{79}$$

$$d_{nk} = \frac{\gamma_{nk}}{\alpha_n}, (i,k \le n-1), \tag{80}$$

$$d_{nn} = \frac{1}{\alpha_n} \tag{81}$$

where d'_{ik} – elements of the matrix A_{n-1}^{-1}.

Calculations by formulas (81) can be successfully performed on a special processor of a group operation. It is advisable to assign the implementation of dependencies by the item 4 to the CPU.

Replenishment method. The idea of the method is as follows, let B be a non-personal matrix for which the inverse is known. It is necessary to determine the inverse matrix A^{-1}, related to the matrix B by the formula

$$A = B + \vec{u}\vec{v}, \tag{82}$$

where $\vec{u}$ – some well-known column; $\vec{v}$ – some known string;

$$\vec{u}\,\vec{v} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \circ \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix} = \begin{bmatrix} u_1v_1 & u_1v_2 & \dots & u_1v_n \\ u_2v_1 & u_2v_2 & \dots & u_2v_n \\ \dots & \dots & \dots & \dots \\ u_nv_1 & u_nv_2 & \dots & u_nv_n \end{bmatrix} \tag{83}$$

It is proved that the inverse matrix

$$A^{-1} = B^1 - \frac{1}{\gamma}B^{-1}\vec{u}\vec{v}B^{-1}, \tag{84}$$

where

$$\gamma = 1 + \vec{v}\, B^{-1} \qquad (85)$$

provided $\gamma \neq 0$.

This idea is usually extended to the special case when the matrix A leaves the matrix B by changing one string, that is

$$A = B + V \qquad (86)$$

where V – matrix, all elements of which are equal to zero, except for elements of a variable string, for example, with k. Then,

$$V = \vec{u}\,\vec{v} = \vec{e}_k\,\vec{v} \qquad (87)$$

where $\vec{v}$ – nonzero strings of the matrix V; $V; \vec{e}_k$ – column, k-th element of which is equal to one, and the rest is zero. In this case,

$$\vec{a}_j = \vec{\beta}_j - \frac{(\vec{v}', \vec{\Phi}_j)}{1 + (\vec{v}', \vec{\beta}_k)}\, \vec{\beta}_k \qquad (88)$$

where $\vec{\beta}_k$ – k- th column of the matrix $B^{-1}; \vec{a}_j - j$ -th column of the matrix $A^{-1}; (\vec{v}', \vec{\beta}_j), (\vec{v}', \vec{\beta}_k)$ – scalar products of vectors.

The question arises of where to take for an arbitrary matrix A the matrix B satisfying formula (70), for which the inverse B^{-1} is known. The completion method provides that a given matrix A is considered as the last member of the sequence $A_0, A_1, A_2, \ldots, A_n = A, A_0 = E$ (identity matrix).

The transition from the previous matrix A_{k-1} o the next A_k is carried out by replacing the k-th string of the matrix A_{k-1} with the k-th string of the matrix A_k with n iterations.

The transition formulas at the k-th step are as follows:

$$\vec{a}_j^{(k)} = \vec{a}_j^{(k-1)} - \frac{\left(\vec{v}_k', \vec{a}_j^{(k-1)}\right)}{1 + \left(\vec{v}_k', \vec{a}_j^{(k-1)}\right)}\, \vec{a}_k^{(k-1)} \qquad (89)$$

where $\vec{a}_j^{(k)}$ – j-th column of the matrix

$$A_k^{-1}; \vec{v}^{(k)} = (a_{k1}, a_{k2}, \ldots, a_{kn}) \qquad (90)$$

Operations in brackets $\left(\vec{v}_k', \vec{a}_j^{(k-1)}\right), \left(\vec{v}_k', \vec{a}_j^{(k-1)}\right)$ of formula (90) are the scalar product of two vectors, which are expedient perform group operations on the processor. The operation of finding the elements of the inverse matrix can be accelerated if $\tilde{A}^{-1}$. It makes sense for the first calculations to be performed with

rounding by Newton method. In the future, as the solution is approached, it may be necessary to refine the value of the inverse Jacobian – more accurately find the elements of the inverse matrix. The use of a special processor of the group operation will be effective in this case as well. Indeed, first, we will effectively obtain the control relation $A\,\tilde{A}^{-1}$, to find which n^2 is necessary so the norm of the indicated group operations if the relation $A\,\tilde{A}^{-1}$:

$$\|E - A\,\tilde{A}^{-1}\| \leq K < 1$$

(91)

where $\|-A\,\tilde{A}^{-1}\|$ – the first or second norm of the matrix, then in this case the elements of the inverse matrix A^{-1} can be calculated with an arbitrarily high accuracy using an iterative process:

$$\begin{cases} A_1^{-1} = A_0^{-1}(E + R_0), R_1 = E - A\,A_1^{-1} \\ A_2^{-1} = A_1^{-1}(E + R_1), R_2 = E - A\,A_2^{-1} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ A_m^{-1} = A_{m-1}^{-1}(E + Rm - 1), R_m = E - A\,A_{m-1}^{-1} \end{cases}$$

(92)

It is proved that if condition (40) is met, the convergence of the process is very fast – the number of correct decimal places grows exponentially. Secondly, the form of formulas (92) convincingly testifies to the efficiency of using the processor of sums of paired products, since in these formulas the most laborious and preferable operation is the multiplication of two square matrices, which requires n^2 of the indicated group operations.

Fastest descent method. Assuming that the functions $f_1, f_2, \dots, f_n$ of formulas (65) are real and continuously differentiable in their common domain of definition, it is proved that the iterative process of the fastest descent that converges is performed as follows:

$$\vec{x}^{(k+1)} = \vec{x}^{(k)} - \mu_k W_k' \vec{F}^{(k)}, k = 0,1,2,\dots$$

(93)

where

$$\vec{F}^k = \vec{F}(\vec{x}^{(k)});$$

(94)

$$W_k = W(\vec{x}^{(k)})$$

(92)

Jacobi matrix:

$$W(\vec{x}) = \frac{d\vec{F}}{d\vec{x}}\,-;$$

(95)

$W'(\vec{x})$ – transposed matrix

$$\mu_k = \left(\frac{\vec{F}^{(k)}, W_k W_k' \vec{F}^{(k)}}{W_k W_k' \vec{F}^{(k)}, W_k W_k' \vec{F}^{(k)}} \right)$$

(96)

introduce the notation

$$W_k \vec{F}^{(k)} = \vec{\delta}^{(k)}$$

.   (97)

Represent the matrix $W_k \; W_k'$ in another way:

$$W_k = \begin{bmatrix} \vec{w}_{1k} \\ \vec{w}_{2k} \\ \vdots \\ \vec{w}_{nk} \end{bmatrix} \quad \vec{w}_{ik} = \begin{bmatrix} w_{i,1k} & w_{i,2k} & \cdots & w_{i,nk} \end{bmatrix} \quad i = 1,2,\ldots,n$$

$$W_k' = \begin{bmatrix} \vec{w}_{1k}' \\ \vec{w}_{2k}' \\ \vdots \\ \vec{w}_{nk}' \end{bmatrix} \quad \vec{w}_{ik} = \begin{bmatrix} w_{i,1k}' & w_{i,2k}' & \cdots & w_{i,nk}' \end{bmatrix} \quad i = 1,2,\ldots n$$

(98)

Represent the vector $\vec{\delta}^{(k)}$ in expanded form:

$$\vec{\delta}^{(k)} = \begin{bmatrix} \delta_1^{(k)} \\ \delta_2^{(k)} \\ \vdots \\ \delta_n^{(k)} \end{bmatrix}, \quad \begin{cases} \delta_1^{(k)} = \left(\vec{w}_{1k}', \vec{F}^{(k)} \right) \\ \delta_2^{(k)} = \left(\vec{w}_{2k}', \vec{F}^{(k)} \right) \\ \cdots\cdots\cdots\cdots\cdots \\ \delta_n^{(k)} = \left(\vec{w}_{nk}', \vec{F}^{(k)} \right) \end{cases}$$

(99)

Let us denote
$$W \vec{\delta}^{(k)} = \vec{\varepsilon}^{(k)}$$
(100)

$$\vec{\varepsilon}^{(k)} = \begin{bmatrix} \delta\varepsilon_1^{(k)} \\ \varepsilon_2^{(k)} \\ \vdots \\ \delta\varepsilon_n^{(k)} \end{bmatrix}, \quad \begin{cases} \delta\varepsilon_1^{(k)} = \left(\vec{w}_{1k}', \vec{\delta}^{(k)} \right) \\ \varepsilon_2^{(k)} = \left(\vec{w}_{2k}', \vec{\delta}^{(k)} \right) \\ \cdots\cdots\cdots\cdots\cdots \\ \varepsilon_n^{(k)} = \left(\vec{w}_{nk}', \vec{\delta}^{(k)} \right) \end{cases}$$

(101)

$$c^{(k)} = \left(\vec{F}^{(k)}, \vec{\varepsilon}^{(k)} \right)$$

(102)

$$k^{(k)} = \left(\vec{\varepsilon}^{(k)}, \vec{\varepsilon}^{(k)} \right)$$

(103)

$$\mu^{(k)} = \frac{c^{(k)}}{k^{(k)}}$$

(104)

$$\begin{cases} x_1^{(k+1)} = x_1^{(k)} - \mu^k \delta_1^{(k)} \\ x_2^{(k1+1)} = x_2^{(k)} - \mu^k \delta_2^{(k)} \\ \dots\dots\dots\dots\dots\dots \\ x_n^{(k+1)} = x_n^{(k)} - \mu^k \delta_n^{(k)} \end{cases}$$

(105)

Determination of the components of the vector $\vec{\delta}^{(k)}$ (99), vector $\vec{\varepsilon}^{(k)}$ (100), the quantities $c^{(k)}$ (101) and $k^{(k)}$ (102) is carried out by calculating the scalar product of two vectors. Therefore, it is advisable to perform all the above calculations in the group operation processor. Calculations by formulas (95), (104), (105) must be performed in the CPU.

After the computing system finishes one of the modelling stages, which involves the visualisation of three-dimensional scenes of the concepts obtained, the process of forming stereo display files begins. Computational procedures that are associated with the transformation of spatial images are an integral part of the interactive mode of graphic display systems in general and stereographic systems in particular, since they serve the purpose of better perception of complex three-dimensional pictures. The operation of transforming three-dimensional machine images, such as transformations, rotations, scaling, mirroring, etc., is usually carried out in the so-called uniform coordinates, where a point in three-dimensional space is represented by a four-dimensional string vector (X, Y, Z, H) or column vector; H – some arbitrary number. It is shown that the transformation operations of three-dimensional machine images can be performed on the basis of the generalised transformation matrix T of size (4x4):

$$T = \begin{bmatrix} a & b & c & p \\ d & e & f & q \\ h & i & j & r \\ l & m & n & s \end{bmatrix}$$

(106)

Below are some examples of particular types of the matrix T for the case when a point in three-dimensional space is represented by a four-dimensional column vector with $H = 1$:

T_1 – partial rescaling matrix;

T_2 – general rescaling matrix;

T_3 – three-dimensional shift matrix;

$$T_1 = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & f & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_3 = \begin{bmatrix} 1 & b & c & 0 \\ d & 1 & f & 0 \\ h & i & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(107)

T_4 – matrix of three-dimensional rotation around the OX axis by $\angle\theta$;

T_5 – matrix of three-dimensional rotation around the OY axis by $\angle\varphi$;

T_6 – matrix of three-dimensional rotation around the OZ axis by $\angle\phi$;

T_7 – matrix of rotation by $\angle\theta$ around an arbitrary axis of rotation given by the direction cosines $n_1 = \cos\alpha, n_2 = \cos\varphi, n_3 = \cos\phi$,

877

$$T_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_5 = \begin{bmatrix} \cos\varphi & 0 & \sin\varphi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\varphi & 0 & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_4 = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(108)

$$T_7 = \begin{bmatrix} n_1^2 + (1 - n_1^2 \cos\theta) & n_1 n_2(1-\cos\theta) - n_3 \sin\theta & n_1 n_3(1-\cos\theta) + n_2 \sin\theta & 0 \\ n_1 n_2(1-\cos\theta) + n_3 \sin\theta & n_2^2 + (1 - n_2^2 \cos\theta) & n_1 n_3(1-\cos\theta) - n_1 \sin\theta & 0 \\ n_1 n_2(1-\cos\theta) - n_2 \sin\theta & n_2 n_3(1-\cos\theta) + n_1 \sin\theta & n_3^2 + (1 - n_3^2 \cos\theta) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(109)

T_8 – matrix of 3D image mirror reflection relative to the XOY plane;

T_9 – matrix of 3D image mirror reflection relative to the YOZ plane;

T_{10} – matrix of 3D image mirror reflection relative to the XOZ plane,

T_{11} – spatial transfer matrix,

$$T_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(110)

$$T_9 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

(111)

$$T_{10} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

(112)

$$T_{11} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & m \\ 0 & 0 & 1 & n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(113)

The examples considered are quite enough to make sure that a wide range of graphic transformations can be carried out on the basis of a square matrix of the fourth order; and that the implementation of the considered transformations is associated with the group operation of scalar multiplication of two real vectors.

In computational procedures related to 3D development based on the proposed operator method showed that in them the indicated group operation is also the most time-consuming computational operation. This applies to:

– procedures that implement forward and backward stereo operators of constant variable and rotary angles of stereo modelling – procedures related to the analysis of modelling zones;

– procedures for analysing geometric distortions in various stereometric systems;

procedures for modelling high-rise buildings, interpolation, movement, video environment of the area.

4.   Conclusion

The analysis of the listed computational procedures showed that the most time-consuming computational operation in them is the group arithmetic operation of scalar multiplication of two real vectors, which is dominant in many computational processes of algebra, analysis, in matrix transformations, in 3D graphics and in spatial modelling of urban environment. For those specialized stereometric systems that are included in the operational control loop of important critical dynamic processes, a specialized processor of the sums of paired applications will significantly increase the computing performance in the procedures of the proposed urban environment model, which ultimately, if other necessary above mentioned conditions are met, will lead to an increase in the speed of adequate perception of wireframe or full-scale wireframe models of vertical design of the urban environment.

References

[1] H. Schmidt, and M. Heimann, "Probabilistic Modelling of HSC and UHPC Slender Columns in High-Rise Buildings," in: *Applications of Statistics and Probability in Civil Engineering -Proceedings of the 11th International Conference on Applications of Statistics and Probability in Civil*, Berlin: Springer, pp. 1169-1174, 2011.

[2] L. Wang, X. Lin, S. Liang, A. Yan, and L. Zou, "An Analysis on the Instability of Vortex Induced Resonance of Super High-Rise Buildings," *Journal of Vibration and Shock*, vol. 37, no. 12, pp. 53-59, 2018.

[3] S. Rastgoo Moghadam, and D. Konstantinidis, "Finite Element Study of the Effect of Support Rotation on the Horizontal Behavior of Elastomeric Bearings," *Composite Structures*, vol. 163, pp. 474-490, 2017.

[4] A. M. Belostotsky, S. B. Penkovoy, S. V. Scherbina, P. A. Akimov, and T. B. Kaytukov, "Correct Numerical Methods of Analysis of Structural Strength and Stability of High-Rise Panel Buildings part 2: Results of Modelling," Key Engineering Materials, vol. 685, pp. 221-224, 2016.

[5] H. Schmidt, and M. Six, "Probabilistic Modelling of HPC Slender Columns in High-Rise Buildings," *Beton- Und Stahlbetonbau*, vol. 102, no. 12, pp. 859-868, 2007.

[6] L. Caracoglia, "Unified Stochastic Dynamic and Damage Cost Model for the Structural Analysis of Tall Buildings in Thunderstorm-Like Winds," *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems*, vol. 4, no. 4, pp. 1-17, 2018.

[7] T. M. Naser, I. Kanda, T. Ohara, K. Sakamoto, S. Kobayashi, H. Nitta, and T. Nataami, "Analysis of Traffic-Related NOx and EC Concentrations at Various Distances from Major Roads in Japan," *Atmospheric Environment*, vol. 43, no. 15, pp. 2379-2390, 2009.

[8] B. Hajra, T. Stathopoulos, and A. Bahloul, "Assessment of Pollutant Dispersion from Rooftop Stacks: ASHRAE," *ADMS and wind tunnel simulation, Building and Environment*, vol. 45, no. 12, pp. 2768-2777, 2010.

[9] A. Z. Dhunny, Z. Allam, M. R. Lollchund, K. Dookhitram, and S. D. D. V. Rughooputh, "A Decentralized Model of Wind Turbine Optimization," *Wind Engineering*, vol. 44, no. 2, pp. 208-222, 2020.

[10] J. Nováček, and M. Zich, "Contribution of Settlement of a High-Rise Building with Recession in a Ground Plan to Punching Shear," *Solid State Phenomena*, vol. 259, pp. 198-202, 2017.

[11] S. Shafaei, A. H. Varma, M. R. Broberg, and J. Seo, "An Introduction to Numerical Modeling of Composite Plate Shear Walls / Concrete Filled (C-PSW/CF)," in: *Structural Stability Research Council Annual Stability Conference 2019, SSRC 2019,* Buffalo: University at Buffalo, pp. 976-989, 2019.

[12] Y. Bao, W. Guo, G. Wang, W. Gan, M. Zhang, and J. S. Shen, "Millimeter-Accuracy Structural Deformation Monitoring Using Stand-Alone GPS: Case Study in Beijing, China," *Journal of Surveying Engineering*, vol. 144, no. 1, 2018, Available from: https://ascelibrary.org/doi/10.1061/%28ASCE%29SU.1943-5428.0000242

[13] A. Shaz, E. T. Chala, and K. Seshagiri Rao, "Rock Mass Slope Stability Analysis Under Static and Dynamic Conditions in Mumbai, India," *Lecture Notes in Civil Engineering*, vol. 15, pp. 189-198, 2019.

[14] S. Gokuldas, S. Banerjee, and S. S. Nimbalkar, "Effects of Tunneling-Induced Ground Movements on Stability of Piled Raft Foundation: Three-Dimensional Finite-Element Approach," *International Journal of Geomechanics*, vol. 20, no. 8, pp. 47-83, 2020.

[15] M. F. Yassin, S. Kato, R. Ooka, T. Takahashi, and R. Kouno, "Field and Wind-Tunnel Study of Pollutant Dispersion in a Built-up Area Under Various Meteorological Conditions," *Journal of Wind Engineering and Industrial Aerodynamics*, vol. 93, no. 5, pp. 361-382, 2005.

[16] G. Ding, J. J. Jiao, and D. Zhang, "Modelling Study on the Impact of Deep Building Foundations on the Groundwater System," *Hydrological Processes*, vol. 22, no. 12, pp. 1857-1865, 2008.

[17] V. J. Modi, and A. Akinturk, "An Efficient Liquid Sloshing Damper for Control of Wind-Induced Instabilities," *Journal of Wind Engineering and Industrial Aerodynamics*, vol. 90, no. 12-15, pp. 1907-1918, 2002.

[18] Y. Bai, A. Kawano, S. Matsuo, and K. Odawara, "Seismic Responses of High-Rise Buildings Subjected to Extreme Ground Motions of Tohoku Earthquake in Japan," *Advanced Materials Research*, vol. 374-377, pp. 2118-2125, 2012.

[19] B. Sydykhov, S. Daiyrbekov, M. Muratbekova, Z. Issayeva, M. Burkitbayeva, M. Kavakli, and L. Rizayeva, "Methodology for the Development of Search and Research Skills of Prospective Math Teachers in a Course on Mathematical Physics Equations," *Eurasia Journal of Mathematics, Science and Technology Education*, vol. 13, no. 11, pp. 7223-7236, 2017.

[20] A. Mustafin, "Coupling-Induced Oscillations in Two Intrinsically Quiescent Populations," *Communications in Nonlinear Science and Numerical Simulation*, vol. 29, no. 1-3, pp. 391-399, 2015.

[21] E. Ginters, A. Aizstrauts, and R. M. A. Chinea, "Sociotechnical Aspects of Policy Simulation," in: *Handbook of Research on Advanced ICT Integration for Governance and Policy Modeling*, Hershey: IGI Global, pp. 113-128, 2014.

[22] Y. Sapazhanov, A. Orynbassar, S. Kadyrov, and B. Sydykhov, "Factors Affecting Mathematics Achievement in Central Asian Specialized Universities," *International Journal of Emerging Technologies in Learning*, vol. 15, no. 19, pp. 143-153, 2020.

[23] A. Mustafin, "Two Mutually Loss-Coupled Lasers Featuring a Stable Multivibrator," *Physica D: Nonlinear Phenomena*, vol. 218, no. 2, pp. 167-176, 2006.

[24] A. Kantarbayeva, and A. Mustafin, "A Biologically Inspired Fluid Model of the Cyclic Service System," *Mathematical Modelling and Analysis*, vol. 25, no. 4, pp. 505-521, 2020.

[25] A. V. Babaytsev, and L. N. Rabinskiy, "Design Calculation Technique for Thick-Walled Composite Constructions Operating Under High-Speed Loading," *Periodico Tche Quimica*, vol. 16, no. 33, pp. 480-489, 2019.

[26] A. Aizstrauts, E. Ginters, M. Baltruks, and M. Gusev, "Architecture for Distributed Simulation Environment," *Procedia Computer Science*, vol. 43, no. C, pp. 18-25, 2015.

[27] R. Baembitov, T. Dokic, M. Kezunovic, Y. Hu, and Z. Obradovic, "Fast Extraction and Characterization of Fundamental Frequency Events from a Large PMU Dataset Using Big Data Analytics," *Proceedings of the Annual Hawaii International Conference on System Sciences*, vol. 2020, pp. 3195-3204, 2021.

[28] A. V. Babaytsev, Y. K. Kyaw, S. N. Vakhneev, and T. Zin Hein, "Study of the Influence of Spherical Inclusions on Mechanical Characteristics," *Periodico Tche Quimica*, vol. 17, on. 35, pp. 654-662, 2020.

[29] N. S. Severina, "Software Complex for Solving the Different Tasks of Physical Gas Dynamics," *Periodico Tche Quimica*, vol. 16, no. 32, pp. 424-436, 2019.

[30] E. Ginters, A. Aizstrauts, G. Dreija, M. Ablazevica, S. Stepucev, I. Sakne, M. Baltruks, M. A. P. Eroles, R. Buil, M. Gusev, and G. Velkoski, "Skopje Bicycle Inter-Modality Simulator – E-Involvement Through Simulation and Ticketing," in: *26th European Modeling and Simulation Symposium*, EMSS 2014, Bordeaux, France, pp. 557-562, 2014.

[31] E. Ginters, A. Cirulis, and G. Blums, "Markerless Outdoor AR-RFID Solution for Logistics," *Procedia Computer Science*, vol. 25, pp. 80-89, 2013.

[32] S. A. Montayev, N. B. Adilova, S. M. Zharylgapov, A. S. Montayeva, B. T. Shakeshev, and M. Z. Ryskaliyev, "Physical and Mechanical Properties and Structure of Heat Insulating Engineering Foam

Glass Derived from Mixed Waste Glass and Wollastonite Slags," *International Journal of Mechanical Engineering and Technology*, vol. 8, no. 12, pp. 944-954, 2017.

[33] M. Sha, O. A. Prokudin, Y. O. Solyaev, and S. N. Vakhneev, "Dependence of Glare Destruction Mechanisms on the Elongation of Samples in Tests to Three-Point Flexural," *Periodico Tche Quimica*, vol. 17, no. 35, pp. 549-558, 2020.

[34] V. A. Pogodin, A. N. Astapov, and L. N. Rabinskiy, "CCCM Specific Surface Estimation in Process of Low-Temperature Oxidation," *Periodico Tche Quimica*, vol. 17, no. 34, pp. 793-802, 2020.

[35] A. G. Getmanov, and L. N. Rabinskiy, "Assessment of Durability of Coatings in Difficult Stress Conditions," *Periodico Tche Quimica*, vol. 16, no. 33, pp. 490-497, 2019.

[36] I. Kotenko, A. Krasov, I. Ushakov, and K. Izrailov, "Detection of Stego-Insiders in Corporate Networks Based on a Hybrid NoSQL Database Model," *ACM International Conference Proceeding Series*, article number 3442612, 2020.

[37] A. V. Babaytsev, and A. A. Zotov, "Designing and Calculation of Extruded Sections of an Inhomogeneous Composition," *Russian Metallurgy (Metally)*, vol. 2019, no. 13, pp. 1452-1455, 2019.

[38] Y. Sun, M. Y. Kuprikov, and E. L. Kuznetsova, "Effect of Flight Range on the Dimension of the Main Aircraft," *INCAS Bulletin*, vol. 12, Special Issue, pp. 201-209, 2020.

[39] A. Aizstrauts, E. Ginters, I. Lauberte, M.A.P. Eroles, "Multi-Level Architecture on Web Services-Based Policy Domain Use Cases Simulator," *Lecture Notes in Business Information Processing*, vol. 153, pp. 130-145, 2013.

[40] A. Lapidus, and A. Makarov, "Statistical Learning Problem of Artificial Neural Network to Control Roofing Process," *MATEC Web of Conferences*, vol. 117, article number 00100, 2017.

[41] A. V. Babaytsev, L. N. Rabinskiy, and K. T. Aung, "Investigation of The Contact Zone of a Cylindrical Shell Located Between Two Parallel Rigid Plates with A Gap," *INCAS Bulletin*, vol. 12, Special Issue, pp. 43-52, 2020.

[42] S. A. Montayev, B. T. Shakeshev, M. Z. Ryskaliyev, N. B. Adilova, and K. A. Narikov, "Collagen Agent Technology for Foam Concrete Production," *ARPN Journal of Engineering and Applied Sciences*, vol. 12, no. 5, pp. 1674-1678, 2017.

[43] P. I. Sharikov, A. V. Krasov, A. M. Gelfand, and N. A. Kosov, "Research of The Possibility of Hidden Embedding of a Digital Watermark Using Practical Methods of Channel Steganography," *Studies in Computational Intelligence*, vol. 868, pp. 203-209, 2020.

[44] V. S. Terentieva, A. N. Astapov, and L. N. Rabinskiy, "State in The Field of Heat-Resistant Coatings for Heat-Proof Nickel Alloys and Steels," *Periodico Tche Quimica*, vol. 16, no. 33, pp. 561-572, 2019.

[45] A. V. Babaytsev, A. A. Orekhov, and L. N. Rabinskiy, "Properties and Microstructure of Alsi10mg Samples Obtained by Selective Laser Melting," *Nanoscience and Technology*, vol. 11, no. 3, pp. 213-222, 2020.

[46] A. Babaytsev, V. Dobryanskiy, and Y. Solyaev, "Optimization of Thermal Protection Panels Subjected to Intense Heating and Mechanical Loading," *Lobachevskii Journal of Mathematics*, vol. 40, no. 7, pp. 887-895, 2019.

[47] S. A. Montayev, A. B. Shinguzhiyeva, N. B. Adilova, A. S. Montayeva, and A. S. Montayeva, "Development of Effective Technological Parameters for Formation of a Porous Structure of The Raw Composition in Order to Obtain a Lightweight Granular Insulation Material," *ARPN Journal of Engineering and Applied Sciences*, vol. 11, no. 17, pp. 10454-10459, 2016.

[48] I. Suleimenov, A. Bakirov, and I. Moldakhan, "Formalization of Ternary Logic for Application to Digital Signal Processing," *Advances in Intelligent Systems and Computing*, vol. 1259, pp. 26-35, 2021.

[49] V.A. Zagovorchev, O.V. Tushavina, "The Use of Jet Penetrators for Movement in The Lunar Soil," *INCAS Bulletin*, vol. 11, pp. 121-130, 2019.

[50] O. V. Egorova, and Y. K. Kyaw, "Solution of Inverse Non-Stationary Boundary Value Problems of Diffraction of Plane Pressure Wave on Convex Surfaces Based on Analytical Solution," *Journal of Applied Engineering Science*, vol. 18, no. 4, pp. 676-680, 2020.

[51] D. Aizstrauta, A. Celmina, E. Ginters, and R. Mazza, "Validation of Integrated Acceptance and Sustainability Assessment Methodology," *Procedia Computer Science*, vol. 26, pp. 33-40, 2013.

[52] V. V. Bodryshev, A. V. Babaytsev, and L. N. Rabinskiy, "Investigation of Processes of Deformation of Plastic Materials with The Help of Digital Image Processing," *Periodico Tche Quimica*, vol. 16, no. 33, pp. 865-876, 2019.

[53] S. A. Lurie, P. A. Belov, Y. O. Solyaev, and E. C. Aifantis, "On One Class of Applied Gradient Models with Simplified Boundary Problems," *Materials Physics and Mechanics*, vol. 32, no. 3, pp. 353-369, 2017.

[54] S. A. Montayev, S. M. Zharylgapov, K. A. Bisenov, B. T. Shakeshev, and M. Z. Almagambetova, "Investigating Oil Sludges and Their Application as Energy Efficient and Modifying Component in Ceramic Pastes," *Research Journal of Pharmaceutical, Biological and Chemical Sciences*, vol. 7, no. 3, pp. 2407-2415, 2016.

[55] A. Lapidus, and A. Makarov, "Automation of Roof Construction Management by Means Artificial Neural Network," *Advances in Intelligent Systems and Computing*, vol. 692, pp. 1168-1176, 2018.

[56] A. V. Krasov, and S. I. Shterenberg, "Methods for Building a Trusted Environment in Unix Operating Systems Based on The Implementation of a Digital Watermark," *International Congress on Ultra-Modern Telecommunications and Control Systems and Workshops*, vol. 2020, pp. 253-257, 2020.

[57] V. A. Korolenko, Y. Li, V. N. Dobryanskiy, and Y. O. Solyaev, "Experimental Studies and Modelling of Fracture Toughness of The Epoxy Samples with Eccentric Cracks," *Journal of Applied Engineering Science*, vol. 18, no. 4, pp. 719-723, 2020.

[58] S. A. Montayev, N. S. Montayeva, A. B. Shinguzhiyeva, K. Zhanabaevichdosov, and M. Zhanaidarovichryskaliyev, "Possibility of Producing Sintered Fine Porous Granulated Ceramic Filler Using Ash of Thermal Power Stations in Combination with Clay Rocks," *International Journal of Mechanical and Production Engineering Research and Development*, vol. 9, no. 4, pp. 1087-1096, 2019.

[59] I. E. Suleimenov, O. A. Gabrielyan, A. S. Bakirov, and Y. S. Vitulyova, "Dialectical Understanding of Information in The Context of The Artificial Intelligence Problems," *IOP Conference Series: Materials Science and Engineering*, vol. 630, no. 1, article number 012007, 2019.

[60] I. Katranov, and A. Lapidus, "Mobile Building Life Cycle," *MATEC Web of Conferences*, vol. 193, article number 03011, 2018.

[61] V. A. Zagovorchev, and O. V. Tushavina, "Selection of Temperature and Power Parameters for Multi-Modular Lunar Jet Penetrator," *INCAS Bulletin*, vol. 11, Special Issue, pp. 231-241, 2019.

[62] D. Aizstrauta, and E. Ginters, "Integrated Acceptance and Sustainability Assessment Model Transformations into Executable System Dynamics Model," *Procedia Computer Science*, vol. 77, pp. 92-97, 2015.

[63] S. A. Mizuriaev, S. A. Montaev, and A. T. Taskaliev, "Artificial Broken Stone Production for Industrial and Civil Engineering: Technological Parameters," *Procedia Engineering*, vol. 111, pp. 534-539, 2015.

[64] A. Lapidus, A. Khubaev, and T. Bidov, "Organizational and Technological Solutions Justifying Use of Non-Destructive Methods of Control When Building Monolithic Constructions of Civil Buildings and Structures," *MATEC Web of Conferences*, vol. 251, article number 05014, 2018.

[65] O. Prentkovskis, J. Tretjakovas, A. Švedas, A. Bieliatynskyi, A. Daniūnas, and K. Krayushkina, "The Analysis of The Deformation State of The Double-Wave Guardrail Mounted on Bridges and Viaducts of The Motor Roads in Lithuania and Ukraine," *Journal of Civil Engineering and Management*, vol. 18, no. 5, pp. 761-771, 2012.

[66] S. A. Montaev, S. M. Zharylgapov, N. S. Montaeva, and B. T. Shakeshev, "Research of Possibility of Producing Ceramic Paving Stones by Vibrocompression with The Purpose of Using Them in The Improvement of Urban Areas," *IOP Conference Series: Materials Science and Engineering*, vol. 775, no. 1, article number 012118, 2020.

[67] V. S. Terentieva, and A. N. Astapov, "Conceptual Protection Model for Especially Heat-Proof Materials in Hypersonic Oxidizing Gas Flows," *Russian Journal of Non-Ferrous Metals*, vol. 59, no. 6, pp. 709-718, 2018.

[68] A. Bieliatynskyi, L. Osipa, and B. Kornienko, "Water-Saving Processes Control of An Airport," *MATEC Web of Conferences*, vol. 239, article number 05003, 2018.

[69] A. Lapidus, and T. Dmitry, "Formation of Methods for Assessing the Effectiveness of Industrial Areas' Renovation Projects," *IOP Conference Series: Materials Science and Engineering*, vol. 471, no. 2, article number 022034, 2019.

[70] V. Pukhkal, A. Bieliatynskyi, and V. Murgul, "Designing Energy Efficiency Glazed Structures with Comfortable Microclimate in Northern Region," *Journal of Applied Engineering Science*, vol. 14, no. 1, pp. 93-101, 2016.

[71] M. Starikov, A. Beljatynskij, O. Prentkovskis, and I. Klimenko, "The Use of Magnetic Coercivity Method to Diagnose Crane Metalware," *Transport*, vol. 26, no. 3, pp. 255-262, 2011.

[72] S. A. Montaev, N. B. Adilova, N. S. Montaeva, K. Z. Dosov, and A. A. Taudaeva, "Study of Raw Materials with The Aim of Obtaining Ceramic Filler and Heat-Insulating and Structural Wall Ceramics," *International Journal of Mechanical and Production Engineering Research and Development*, vol. 9, no. 5, pp. 1057-1064, 2019.

[73] Yu. G. Evtushenko, V. I. Zubov, S. A. Lurie, Yu. O. Solyaev, "Identification of Kinetic Parameters of The Model of Interphase Layer Growth in A Fibrous Composite," *Composites: Mechanics, Computations, Applications*, vol. 7, no. 3, pp. 175-187, 2016.

[74] A. Lapidus, and I. Abramov, "An Assessment Tool for Impacts of Construction Performance Indicators on The Targeted Sustainability of a Company," *IOP Conference Series: Materials Science and Engineering*, vol. 753, no. 4, article number 042089, 2020.

[75] D. Topchiy, and A. Bolotova, "Studying Specific Features of The Monolithic Construction Technology Based on Systemic Analysis," *IOP Conference Series: Materials Science and Engineering*, vol. 603, no. 5, article number 052004, 2019.

[76] A. Beljatynskij, O. Prentkovskis, and J. Krivenko, "The Experimental Study of Shallow Flows of Liquid on The Airport Runways and Automobile Roads," *Transport*, vol. 25, no. 4, pp. 394-402, 2010.

[77] D. Topchiy, and A. Bolotova, "Assessment and Inspection of The Technical Condition of Monolithic Reinforced Structures in Transportation Infrastructure Facilities," *Journal of Physics: Conference Series*, vol. 1425, no. 1, article number 012005, 2020.

[78] S. Zhang, T. N. Sakulyeva, E.A. Pitukhin, and S. M. Doguchaeva, "Neuro-Fuzzy and Soft Computing – A Computational Approach to Learning and Artificial Intelligence," *International Review of Automatic Control*, vol. 13, no. 4, pp. 191-199, 2020.

[79] A. Yu. Zhigulina, S. A. Montaev, and S. M. Zharylgapov, "Physical-Mechanical Properties and Structure of Wall Ceramics with Composite Additives Modifications," *Procedia Engineering*, vol. 111, article number 896-901, 2015.

[80] K. Krayushkina, O. Prentkovskis, A. Bieliatynskyi, J. Gigineishvili, A. Skrypchenko, A. Laurinavicius, K. Gopalakrishnan, and J. Tretjakovas, "Perspectives on Using Basalt Fiber Filaments in The Construction and Rehabilitation of Highway Pavements and Airport Runways," *Baltic Journal of Road and Bridge Engineering*, vol. 11, no. 1, pp. 77-83, 2016.

[81] K. Krayushkina, T. Khymeryk, and A. Bieliatynskyi, "Basalt Fiber Concrete as A New Construction Material for Roads and Airfields," *IOP Conference Series: Materials Science and Engineering*, vol. 708, no. 1, article number 012088, 2019.

[82] N. Kuzhel, A. Bieliatynskyi, O. Prentkovskis, I. Klymenko, S. Mikaliunas, O. Kolganova, S. Kornienko, and V. Shutko, "Methods for Numerical Calculation of Parameters Pertaining to The Microscopic Following-The-Leader Model of Traffic Flow: Using the Fast Spline Transformation," *Transport*, vol. 28, no. 4, pp. 413-419, 2013.

[83] O. Prentkovskis, A. Beljatynskij, R. Prentkovskiene, I. Dyakov, and L. Dabulevičiene, "A Study of The Deflections of Metal Road Guardrail Elements," *Transport*, vol. 24, no. 3, pp. 225-233, 2009.

[84] A. V. Bobrova, E. A. Stepanov, T. Sakulyeva, G. Z. Zhumabekova, and A. I. Yesturliyeva, "The Influence of Alternative Fuels on The Development of Large-Scale Production," *Journal of Environmental Accounting and Management*, vol. 8, no. 4, pp. 335-349, 2020.

[85] M. A. Piera, R. Buil, and E. Ginters, "State Space Analysis for Model Plausibility Validation in Multi Agent System Simulation of Urban Policies," in: *25th European Modelling and Simulation Symposium*, EMSS 2013, Athens, Greece, pp. 504-509, 2013.

[86] A. Lapidus, N. Cherednichenko, and V. Smotrov, "The Impact of Technology Installation of Injection Mortar Systems for A Long Term Behavior of Anchor Connections," *Procedia Engineering*, vol. 153, pp. 371-377, 2016.

[87] A. Bieliatynskyi, E. Krayushkina, and A. Skrypchenko, "Modern Technologies and Materials for Cement Concrete Pavement's Repair," *Procedia Engineering*, vol. 134, pp. 344-347, 2016.

[88] S. Yu. Timkina, O. V. Stepanchuk, and A. A. Bieliatynskyi, "The Design of the Length of the Route Transport Stops' Landing Pad on Streets of the City," *IOP Conference Series: Materials Science and Engineering*, vol. 708, no. 1, article number 012032, 2019.

[89] A. S. Bakirov, and I. E. Suleimenov, "On the Possibility of Implementing Artificial Intelligence Systems Based on Error-Correcting Code Algorithms," *Journal of Theoretical and Applied Information Technology*, vol. 99, no. 1, pp. 83-99, 2021.

[90] S. A. Mizuriaev, S. A. Montaev, and A. B. Shinguzhieva, "On Effectiveness of Low-Bloating Raw Materials Use (E.G., Loam) In Concrete Aggregate," *Procedia Engineering*, vol. 111, pp. 530-533, 2015.

[91] O. Stepanchuk, A. Bieliatynskyi, O. Pylypenko, and S. Stepanchuk, "Peculiarities of City Street-Road Network Modelling," *Procedia Engineering*, vol. 134, pp. 276-283, 2016.

[92] B. Kovačic, R. Kamnik, A. Bieliatynskyi, "The Different Methods of Displacement Monitoring at Loading Tests of Bridges or Different Structures," *MATEC Web of Conferences*, vol. 53, article number 01048, 2016.

[93] E. L. Kuznetsova, and A. V. Makarenko, "Mathematic Simulation of Energy-Efficient Power Supply Sources for Mechatronic Modules of Promising Mobile Objects," *Periodico Tche Quimica*, vol. 15, Special Issue 1, pp. 330-338, 2018.

[94] D. Topchiy, and A. Bolotova, "Systematization of Factors Affecting the Organizational Processes in The Conversion of Buildings," *IOP Conference Series: Materials Science and Engineering*, vol. 753, no. 3, article number 032042, 2020.

[95] D. V. Topchiy, and A. S. Bolotova, "The Monolithic Buildings Complex Renovation's Organizational and Technological Model Creation," *IOP Conference Series: Materials Science and Engineering*, vol. 913, no. 4, article number 042016, 2020.

[96] A. Lapidus, A. Khubaev, and T. Bidov, "Development of A Three-Tier System of Parameters in The Formation of The Organizational and Technological Potential of Using Non-Destructive Testing Methods," *E3S Web of Conferences*, vol. 97, article number 06037, 2019.

[97] A. A. Lapidus, and N. Yves, "Integrated Quality Index of Organizational and Technological Solutions for Implementation of Burundian Capital Master Plan," *Materials Science Forum*, vol. 931, pp. 1295-1300, 2018.

[98] A. Lapidus, T. Bidov, and A. Khubaev, "The Study of The Calibration Dependences Used When Testing the Concrete Strength by Nondestructive Methods," *MATEC Web of Conferences*, vol. 117, article number 00094, 2017.

[99] N. B. Adilova, S. A. Montayev, A. S. Montayeva, B. T. Shakeshev, K. A. Narikov, A. T. Taskaliev, S. Zharylgapov, and Z. Usenkulov, "Modifying of Ceramic Mass by Kazakhstan Bentonite for The Purpose of Improvement of Structure and Physicomechanical Properties of Front Wall Ceramics," *Life Science Journal*, vol. 11, Spec. Issue 6, pp. 83-89, 2014.

[100] A. Onishchenko, L. Stolyarova, and A. Bieliatynskyi, "Evaluation of The Durability of Asphalt Concrete on Polymer Modified Bitumen," *E3S Web of Conferences*, vol. 157, article number 06005, 2020.

[101] D. V. Topchy, "Formation of Organizational and Technological Systems for Renovation of Production Facilities Located on The Territory of Megacities," *International Journal of Civil Engineering and Technology*, vol. 9, no. 8, pp. 1452-1457, 2018.

[102] M. Y. Kuprikov, N. M. Kuprikov, and L. N. Rabinskiy, "The Dependence of The Appearance of The Aircraft on The Conditions of The Arctic Basing," *INCAS Bulletin*, vol. 11, pp. 115-123, 2019.

[103] S. Vakhneev, and E. Starovoitov, "Damping of Circular Composite Viscoelastic Plate Vibration Under Neutron Irradiation," *Journal of Applied Engineering Science*, vol. 18, no. 4, pp. 699-704, 2020.

[104] A. A. Lapidus, and N. N. Bumarskova, "Academic Sports for Construction Sector Specialist's Vocational Specialisation," *Teoriya i Praktika Fizicheskoy Kultury*, vol. 5, pp. 33-34, 2017.

[105] V. A. Pogodin, L. N. Rabinskii, and S. A. Sitnikov, "3D Printing of Components for The Gas-Discharge Chamber of Electric Rocket Engines," *Russian Engineering Research*, vol. 39, no. 9, pp. 797-799, 2019.

[106] V. Klimina, A. Yurgaitis, and D. Topchiy, "Unified Classification of Defects Detected by The Technical Examination," *E3S Web of Conferences*, vol. 110, article number 01086, 2019.

[107] Y. Li, A. M. Arutiunian, E. L. Kuznetsova, and G. V. Fedotenkov, "Method for Solving Plane Unsteady Contact Problems for Rigid Stamp and Elastic Half-Space with A Cavity of Arbitrary Geometry and Location," *INCAS Bulletin*, vol. 12, Special Issue, pp. 99-113, 2020.

[108] D. Topchiy, and A. Bolotova, "Risk Management in Monolithic Construction," *IOP Conference Series: Materials Science and Engineering*, vol. 962, no. 2, article number 022078, 2020.

[109] S. Oleksandra, K. Krayushkina, T. Khymerik, and B. Andrii, "Method of Increasing the Roughness of The Existing Road," *Procedia Engineering*, vol. 165, pp. 1766-1770, 2016.

[110] E. L. Kuznetsova, G. V. Fedotenkov, and E. I. Starovoitov, "Methods of Diagnostic of Pipe Mechanical Damage Using Functional Analysis, Neural Networks and Method of Finite Elements," *INCAS Bulletin*, vol. 12, no. Special Issue, pp. 79-90, 2020.

[111] A. Lapidus, and Y. Shesterikova, "Mathematical Model for Assessing the High-Rise Apartment Buildings Complex Quality," *E3S Web of Conferences*, vol. 91, article number 02025, 2019.

[112] A. Tokarskiy, and D. Topchiy, "State Construction Supervision During Repurposing Project Implementation in The Urban Areas," *IOP Conference Series: Materials Science and Engineering*, vol. 698, no. 6, article number 066061, 2019.

[113] Y. K. Kyaw, E. L. Kuznetsova, and A. V. Makarenko, "Complex Mathematical Modelling of Mechatronic Modules of Promising Mobile Objects," *INCAS Bulletin*, vol. 12, no. Special Issue, pp. 91-98, 2020.

[114] A. Jurgaitis, D. Topchiy, A. Kravchuk, and D. Shevchuk, "Controlling Methods of Buildings' Energy Performance Characteristics," *E3S Web of Conferences*, vol. 91, article number 02026, 2019.

[115] A. A. Lapidus, and A. N. Makarov, "Model for The Potential Manufacture of Roof Structures for Residential Multi-Storey Buildings," *Procedia Engineering*, vol. 153, no. 378-383, 2016.