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# An integrated model for solving production planning and production capacity problems using an improved fuzzy model for multiple linear programming according to Angelov's method

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**ABSTRACT**

Decision making has become a part of our everyday lives. The main apprehension is that almost all decision difficulties include certain criteria, which usually can be multiple or conflicting. Certainly, the production planning and production capacity development includes several parameters uncertainty such as fuzzy resource capacity, fuzzy demand and fuzzy production rate. This situation makes decision maker challenging to describe the objective crisply and at the end the real optimum solution cannot attained correctly. The Fuzzy model for multi-objective linear programming should be an suitable approach for dealing with the production planning and production capacity (PP& PC) problems. The PP& PC problem based on the fuzzy environment becomes even more sophisticated as decision makers try to consider multi-objectives, Therefore, this study attempts to propose a novel scheme which is capable of dealing with these obstacles in PP& PC problem. Intuitionistic Fuzzy Optimization (1FO) by implementing the optimization problem in an Intuitionistic Fuzzy Set (IFS) environment and considered the degrees of rejection of objective(s) and of constraints as the complement of satisfaction degrees. The aim of the research is to propose a new method capable of dealing with these obstacles in the PP & PC problem. It takes into account uncertainty and makes trade-offs between multiple conflicting goals simultaneously .To verify the validity of the proposed method, a case study of the fuzzy multi-objective model of the PP&PC is used. This research takes into account uncertainty and makes a comparison between multiple conflicting goals at the same time. Therefore, this study attempts to propose a new scheme which is the modified Angelov's approach.

**Keywords**: Production planning, production capacity, Intuitionistic Fuzzy Set (IFS), Angelov's approach, Multiple-objective linear programming

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## 1. Introduction

The production planning and production capacity (PP& PC) is considered as significant for efficient production systems [1]. There are considerably important several manufacturing concerns [2]. In actual PP& PC problems, Parameters input: including forecasting demand, resource, and cost and objective functions, may be inaccurate [3]. On the other hand, consideration of all parameters in an aggregate production planning (APP) model makes the generation of a master production schedule deeply complicated especially in real-world to solve the problems PP& PC [4], where data input are frequently (fuzzy) due to incomplete or unobtainable information and daily changes patterns of demand and manufacturers capacity, in addition, the PP& PC problem based on the fuzzy environment becomes even more sophisticated as decision makers try to consider multi- objective [5]. Fuzzy set theory has been extensively used to capture uncertainty and fuzzy decision-making problems [6].

In addition, fuzzy set theory has been widely developed and various modifications and generalizations have appeared. One of these modified is intuitionistic fuzzy set (IFS). Angelov's considered membership and non-membership in optimization problem and gave intuitionistic fuzzy approach (IFO) to solve optimization problems [7]. Therefore, to solve fuzzy multi-objective linear programming PP& PC problems, Angelov's approach based on IFO was considered. However, when using this approach, most researchers have relied on the decision maker experiences by determining the rejection level in the Angelo's method to handle ambiguity parameters. Hence, this weakness in a scientific approach led to the development of several solutions to the same problem [8].

Conversely, in the real world, many of the decision-making difficulties occur in a situation where the consequences of possible actions, constraints, and the goals are not specifically known [9]. Fuzzy set theory has been widely developed and various modification and generalization have appeared. One of these is IFS. Introduced the concept of IFS as an extension of a fuzzy set [10]. Since it's characterized by a membership function and a non-membership function, therefore generalizing Zadeh′s fuzzy sets which only assign a membership degree to each element. Then after, suggested a new concept intuitionistic fuzzy optimization (IFO) by implementing the optimization problem in an IFS environment and considered the degrees of rejection of objective(s) and of constraints as the complement of satisfaction degrees. The degrees of acceptance and of rejection can be arbitrary (the sum of both have to be less than or equal to [11]. On the other hand, the rejection level for Angelov's approach based on intuitionistic fuzzy optimization technique was chosen subjectively by the decision-makers. Regarding this issue, a modified Angelov's approach to find rejection levels was proposed to find a solution the fuzzy multi-objective model for production planning and production capacity [12][13] .

## 2. Modified Angelov's approach to solve the problem

The advantages of Angelov's technique based on IFO are twofold: (1) It creates the best method for identifying and formulating improvement problems. (2) IFO solutions can meet target to a greater degree as compared to fuzzy optimization problem outcome [14]. In addressing the optimization problem, the procedure must identify the rejection function or the non-membership function, which is expressed as:

$$\nu_k(Z_k) = \begin{cases} 1 & Z_k \le Z_k^* \\ 1 - \frac{Z_k - Z_k^*}{r_k} & Z_k^* \le Z_k \le Z_k^* + r_k \\ 0 & Z_k > Z_k^* + r_k \end{cases} \quad (1)$$

Where $r_k$ is rejected level value for each objective. However, this value is determined by the DM based on their experience, which leads to existence more than one solution to the same problem. Therefore, a modified Angelov's method was used as the second technique to solve the fuzzy multi-objective PP& PC problem. Consequently, the following steps describe the modified Angelov's approach to solve multi-objective APP problem under fuzzy environment.

**Step 1**: Compose multiple-objective linear programming model for the PP&PC.

Where i = 1, 2,..,I and j = 1, 2, ..., m while k is a number of objectives.

Thereafter, denoting aspiration levels, *($Z_k$*)* to find a solution individually for each fogging process to generate for each goal optimal solutions

$$\begin{aligned}
&Min\ Z_1 = C_1^1 X_1^1 + C_2^1 X_2^1 + \ldots + C_m^1 X_m^1 \\
&Min\ Z_2 = C_1^2 X_1^2 + C_2^2 X_2^2 + \ldots + C_m^2 X_m^2 \\
&. \\
&. \\
&. \\
&Min\ Z_k = C_1^k X_1^k + C_2^k X_2^k + \ldots + C_m^k X_m^k \\
&\text{Subject to:} \\
&\sum_{j=1}^{m} A_{i,j}\ X_j \le\ b_i \\
&X_j \ge 0
\end{aligned}$$

**Step 2**: Find the tolerance level ( $T'_k$ ) from the solution in step 1, taking the last two smaller decision variable values for each objective. Due tolerance levels are limit of the admissible violation of inequality, we chose last two smaller decision variable values which have less effect on contribution for decision making. Then we subtract the minimum number for each objective function of highest number identified in this way, any DM to achieve the desired results, the values are similar. It can be written as follows:

$$Z_1^* = C_1^1 X_1^1 + C_2^1 X_2^1 + \ldots + C_m^1 X_m^1$$
$$Z_2^* = C_1^2 X_1 + C_2^2 X_2^2 + \ldots + C_m^2 X_m^2$$
.
.
.
$$Z_k^* = C_1^k X_1^k + C_2^k X_2^k + \ldots + C_m^k X_m^k$$

After we solve each objective function individual, order all decision variables in ascending or descending order for each objective function as:

$$k = 1, X_1^1 \geq X_2^1 \geq \ldots . \geq X_{m-1}^1 \geq X_m^1 \Longrightarrow X_{m-1}^1 - X_m^1 = T_1'$$
$$k = 2,\ X_1^2 \geq X_2^2 \geq \ldots . \geq X_{m-1}^2 \geq X_m^2 \Longrightarrow X_{m-1}^2 - X_m^2 = T_2'$$
.
.
.
$$k = K,\ X_1^K \geq X_2^K \geq \ldots . \geq X_{m-1}^K \geq X_m^K \Longrightarrow X_{m-1}^K - X_m^K = T_K'.$$

*T1, T2*… ( $T'_k$ ) are a tolerance levels for $Z_1$, $Z_2$, ..., $Z_k$, respectively.
Where ( $T'_k$ ) for each objective are constant and not subjectively chosen by any decision maker.

**Step 3:** To find the value of the rejection level *($r_k$ )* as a general way, the two largest values of the decision variable for each objective function are specified then find the difference between these two values as the following describe:

$$k = 1,\ X_1^1 \geq X_2^1 \geq \ldots . \geq X_{m-1}^1 \geq X_m^1 \Longrightarrow X_1^1 - X_2^1 = r_1$$
$$k = 2,\ X_1^2 \geq X_2^2 \geq \ldots . \geq X_{m-1}^2 \geq X_m^2 \Longrightarrow X_1^2 - X_2^2 = r_2$$
.
.
.
$$k = K,\ X_1^K \geq X_2^K \geq \ldots . \geq X_{m-1}^K \geq X_m^K \Longrightarrow X_1^K - X_2^K = r_k.$$

Where $r_1$, $r_2$, ... , $r_k$ are a rejection levels for $Z_1$, $Z_2$, ..., $Z_k$, respectively.
Thus, not need to be chosen subjectively by any decision maker.

Step 4: Apply the membership function for each objective function on Zimmermann’s approach after aspiration and tolerance levels were found:

$$\mu_k(Z_k) = \begin{cases} 1 & Z_k \leq Z_k^* \\ 1 - \frac{Z_k - Z_k^*}{T_k'} & Z_k^* \leq Z_k \leq Z_k^* + T_k' \\ 0 & Z_k > Z_k^* + T_k' \end{cases} \qquad (2)$$

The membership function rewritten as :
$\mu_k(Z_k) = 1- ((Z_k - Z^*_k) / T'_k)$

After that, apply the suggest non-membership function (1) to get;

$V_k (Zk) = 1- ((Z_k - Z^*_k) / r_k)$

**Step 4**: To derive the compromise solution of the above system, suggested symmetric decision procedure to solve problems with several objective functions.
Suppose that $\mu_D(x)$ is the membership function of the fuzzy set 'decision' of the model. Then,

$\mu_D(x) = min \{\mu_1(Z_1), \mu_2(Z_2), ..., \mu_k(Z_k)\}$.

Since these membership functions are the satisfaction of the DM they must be maximized. As a result, the objective function becomes:

Maximize $\mu_D(x)$= Maximize min $\{\mu_1(Z_1), \mu_2(Z_2), ..., \mu_k(Z_k)\}$ , replacing $\mu_D(x)$ by α.

On the other hand, find decision set $v_D(x)$ for non-membership functions.

$$v_D(x) = Max\{v1(Z_1), v_2(Z_2), ..., v_k(Z_k)\} \quad (3)$$

Thus, the above Equivalent 3 can be transformed to the following equation of inequalities:

$\beta = v_D(x) = Max\{v_1(Z_1), v_2(Z_2), ..., v_k(Z_k)\}$,

where $\beta$ denotes the maximum degree of rejection objectives and constraints.

Therefore, IFO is transformed to the linear programming problem given as:

$Max\ \alpha - \beta$  Subject to :

$\alpha \le 1- ((Z_1 - Z^*_1) / T'_1)$

$\alpha \le 1- ((Z_2 - Z^*_2) / T'_2)$

$:$

$\alpha \le 1- ((Z_k - Z^*_k) / T'_k)$

$\beta \le 1- ((Z_1 - Z^*) / r_1)$

$\beta \le 1- ((Z_2 - Z^*_2) / r_2)$

$:$

$\beta \le 1- ((Z_k - Z^*_k) / r_k)$

$$\sum_{j=1}^{m} Aij\ Xj \le bi\ \forall i$$

$Xj \ge 0$

$\alpha + \beta \ge 1$

$\alpha \ge \beta$

$\alpha , \beta \ge 0$

However, if $r_k < T_k$ then we must take the largest value for the decision variable with the value that follows in the third order, i.e: $x_1 \ge x_2 \ge .... \ge x_n$, then $r_k = x_1 - x_3$, we do this until reach $r_k > T_k$.

### 2.1. Mathematical Model for production planning and production capacity problem

Proposed mathematical model with two objective functions to reduce overall production costs and identify changes in workforce level. All indices, parameters, variables, objective functions and constants are presented as follows:

#### 2.1.1. Objective functions

$$\min Z_1 = \sum_{n=1}^{N}\sum_{t=1}^{T}[a_{nt}Q_(1+i_a)^t + b_{nt}O_{nt}(1+i_b)^t + c_{nt}S_{nt}(1+i_c)^t + d_{nt}I(1+i_d)^t + e_{nt}B_{nt}(1+i_e)^t] + \sum_{t=1}^{T} k_tH_t + m_tF_t(1+i_f)^t$$

$\min Z2 = \sum_{t=1}^{T} \mathrm{Ht} + Ft$

Constraints

$I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} = D_{nt} \quad \forall n, t$

$$\sum_{n=1}^{N} l_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - F_t - \sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) = 0 \forall t$$

$$\sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) \leq W_{tmax} \forall t$$

$$\sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) \leq M_{tmax} \forall t$$

$$\sum_{n=1}^{N} v_{nt} I_{nt} \leq V_{tmax} \forall t$$

$S_{nt} \geq S_{ntmax} \quad \forall n, t$

$Q_{nt}; I_{nt}; O_{nt}; H_t; F_t; B_{nt}; S_{nt} \geq 0 \quad \forall n, t$ (4)

### 2.2 . Case study

By reviewing the literature and considering practical situations, the linear programming (i-PLP) approach, interactive possibility for was used to investigate the novelty proposed approaches in this section.

Daya Technology Corporation served is used applied research to demonstrate the proposed method of Angelov's. This Company produces two types of products (A & B). The time horizon of Production and capacity planning the decision includes four months (May, Jun, July, and Aug). Tables 1 and 2 show the operating costs and projected demand from the raw production marketing data used in Daya Company.

In addition, the relevant data are as follows:

(1) Initial labor level is 300 man-hours. The costs of hiring and layoff are $10 and $ 2.5 man-hour, respectively.
(2) The inventory at the end of the period for the first batch in the first time period was 400 units of product A and 200 units of product B. The end-of-period stock in period 4 was 300 units of product A and 200 units of product B.
(3) For each period, the maximum subcontracting volumes is 500 units for product 1 and 400 units for product 2.
(4) Hours, Working hours per unit are 0.05 man-hours for product 1 and 0.07 man-hours for product 2. Hours of machine usage per unit for each of the four planning periods are 0.10 machine-hours for product 1 and 0.08 machinehours for product 2.
(5) Warehouse spaces required per unit is two square feet for product 1 and three square feet for product 2. Also, the expected escalating factor for each of the operating cost categories is fixed to 5 % in each period.

Table 1: Related cost coefficients data

| Item | A | b | C | d | e |
|---|---|---|---|---|---|
| Product1 | 20 | 30 | 25 | 0.3 | 40 |
| Product2 | 10 | 15 | 12 | 0.15 | 20 |

Table 2: Forecast demand data with maximum machine capacity, workforce levels, and warehouse space data.

| Item | $D1$ | $D2$ | $W_{max}$ | $M_{max}$ | $V_{max}$ |
|---|---|---|---|---|---|
| Period1 | 1000 | 1000 | 300 | 400 | 10000 |
| Period2 | 3000 | 500 | 300 | 500 | 10000 |
| Period3 | 5000 | 3000 | 300 | 600 | 10000 |
| Period4 | 2000 | 2500 | 300 | 500 | 10000 |

## 3. Modified Angelov's Approach to solve the problem

A new method to modify Angelov's method depend on IFO is used for solving fuzzy multi objective linear programming of problem All inaccurate data were first follow back with a new method based on Angelov's grandfather's method in the functions membership to derive DMs level should be inside [0,1]. Then, the tolerance level is by using the new method in Section

The steps of modified Zimmermann's approach for solving this case study are given as follows: as the following steps:

**Step 1:** Rewrite the FMOLP model for APP problem. Then, solved to find aspiration levels *($Z^*_k$ )* for each objective function.

$$\min Z_1 = \sum_{n=1}^{N}\sum_{t=1}^{T}[a_{nt}Q(1+i_a)^t + b_{nt}O_{nt}(1+i_b)^t + c_{nt}S_{nt}(1+i_c)^t + d_{nt}(1+i_d)^t + e_{nt}B_{nt}(1+i_e)^t] + \sum_{t=1}^{T} k_t H_t + m_t F_t (1+i_f)^t$$

s.t

$$I_{nt-1} - B_{nt-1} + Qnt + O_{nt} + S_{nt} - I_{nt} + B_{nt} = D_{nt} \forall n,t$$
$$\sum_{n=1}^{N} l_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - F_t - \sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) = 0 \forall t$$
$$\sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) \le W_{tmax} \forall t$$
$$\sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) \le M_{tmax} \forall t$$
$$\sum_{n=1}^{N} v_{nt} I_{nt} \le V_{tmax} \forall t$$
$$S_{nt} \le S_{ntmax} \forall n,t$$
$$Q_{nt}, I_{nt}, O_{nt}, H_t, F_t, B_{nt}, S_{nt} \ge 0 \qquad \forall n,t$$

$$\min Z_2 = \sum_{t=1}^{T} H_t + F_t$$

s.t

$$I_{nt-1} - B_{nt-1} + Qnt + O_{nt} + S_{nt} - I_{nt} + B_{nt} = D_{nt} \forall n,t$$
$$\sum_{n=1}^{N} l_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - F_t - \sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) = 0 \forall t$$
$$\sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) \le W_{tmax} \forall t$$
$$\sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) \le M_{tmax} \forall t$$
$$\sum_{n=1}^{N} v_{nt} I_{nt} \le V_{tmax} \forall t$$
$$S_{nt} \le S_{ntmax} \forall n,t$$
$$Q_{nt}, I_{nt}, O_{nt}, H_t, F_t, B_{nt}, S_{nt} \ge 0 \qquad \forall n,t$$

**Step 2**

The level (*T′*) tolerance was determined by using for each objective the two small last values from the solution of the decision variable. Then, the lower bound of each function present subtracted from the higher number then find rejection levels value for each objective function by using the proposed modify Angelov's approach by taking the first two greatest value and subtract the minimum value from maximum one.

The following table (3) illustrated the aspiration, tolerance, and rejection levels for each objective function.

Table 3: Aspiration and tolerance levels for each objective

| Obj. No. | Aspiration level $Z^*$ | Tolerance level $T'$ | Rejection level $r^*$ |
|---|---|---|---|
| 1 | 330277.8 | 135 | 300 |
| 2 | 150 | 100 | 100 |

**Step 3**: Apply the membership and non-membership function for each objective function depend on Equation:

$$\mu_1(Z_1)=\begin{cases} 1 & Z_1 \le 330277.8 \\ 1- \ (Z_1 - 330277.8 \ / 135 & 330277.8 \le Z_1 \le 330412.8 \\ 0 & Z_1 \le 330412.8 \end{cases} \qquad (5)$$

$$\mu_2(Z_2)=\begin{cases} 1 & Z_2 \le 150 \\ 1- \ (Z_2 - 150 \ / 100 & 150 \le Z_2 \le 250 \\ 0 & Z_2 \le 250 \end{cases} \qquad (6)$$

$$\begin{cases} 1 & Z_1 \le 330277.8 \end{cases}$$

$V_1(Z_1)=$ 1- $(Z_1 - 330277.8\ /\ 300$ $\quad 330277.8 \le Z_1 \le 330412.8$ (7)
0 $\quad Z_1 \le 330412.8$

$$V_2(Z_2)=\begin{cases} 1 & Z_2 \le 150 \\ 1-\ (Z_2 - 150\ /\ 100 & 150 \le Z_2 \le 250 \\ 0 & Z_2 \le 250 \end{cases} \quad (8)$$

**Step 4:** By introducing the auxiliary variables $\alpha = \mu_D(Z_D)$ , and $\beta = \nu_D(Z_D)$
$\forall_k$, where $\mu_D(Z_D) = \min\{\mu_1(Z_1), \mu_2(Z_2)\}$ and $\nu_D(Z_D) = \max\{\nu_1(Z_1), \nu_2(Z_2)\}$
. $\alpha \le 1 - ((Z_1 - 330277.8)/135)$ , $\alpha \le 1 - ((Z_2 - 150)/100)$
$\beta \ge 1 - ((Z_1 - 330277.8)/300)$ $\beta \ge 1 - ((Z_2 - 150)/100)$
The IFO model can be changed into the following crisp (non-fuzzy) optimization model as: max α - β Subject to:
$\alpha \le 1 - ((Z_1 - 330277.8)/\ 135)$
$\alpha \le 1 - ((Z_2 - 150)/\ 100)$
$\beta \le 1 - ((Z_1 - 330277.8)/\ 300)$
$\beta \le 1 - ((Z_2 - 150)/\ 100)$
$I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} = D_{nt} \quad \forall_{n,t}$

$$\sum_{n=1}^{N} l_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - F_t - \sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) = 0 \qquad \forall t$$
$$\sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) \le W_{tmax} \qquad \forall t$$
$$\sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) \le M_{tmax} \qquad \forall t$$
$$\sum_{n=1}^{N} v_{nt} I_{nt} \le V_{tmax} \qquad \forall t$$

$S_{nt} \ge S_{ntmax} \quad \forall_{n,t}$
$0 \le \alpha + \beta \le 1$
$\alpha \ge \beta$
$\alpha, \beta \ge 0$
$Q_{nt}, I_{nt}, O_{nt}, H_t, F_t, B_{nt}, S_{nt} \ge 0 \quad \forall_{n}, t$

### 3.2 . Results and Discussion for case study

In order to evaluate and compare the performance of the proposed approach for solving APP problems, a mathematical model and case study data in the Daye Technology Corporation presented by were used. They are inaccurate numbers determined by expected demand, cost data, machine capacity, and level of employment with triangular potential distributions. For better comparison, to carry out our proposed approach, middle values was adopted to track all the imprecise data. On the other hand, the aspiration levels for each objective were found by using the branch and bound in WINQSP solver. These values represent the optimal solution for each goal assuming that their data is ordinary (not fuzzy) as well as non-compatibility with other goals. The optimal solution for each objective function provided as shown in table 3.
By using modified Angelov's method, the overall satisfaction degree and the degree of rejection are 0.9792 and 0.03. This means that the solution converge to optimality. Since the result for each approach which are (α) is greater than 0.5 and value of ($\beta$) approaching to the zero.

Table 4. Comparison of solution

| Methods | Z1 | Z2 |
|---|---|---|
| Modify Angelov's approach | 334987 | 155 |
| i-PLP | 336605 | 163 |

Table 5. Comparative results of decision variables for the proposed method and existing method (i-PLP)

| item | Modified Angelov's approach | | | | i-PLP | | | |
|---|---|---|---|---|---|---|---|---|
| Product1 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| Q1t(unit) | 402 | 2500 | 4198 | 2000 | 598 | 2996 | 4992 | 2138 |
| O1t(unit) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| S1t(unit) | 500 | 500 | 500 | 0 | 0 | 0 | 0 | 158 |
| I1t(unit) | 302 | 302 | 0 | 0 | 0 | 0 | 0 | 300 |
| B1t(unit) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Product2 | | | | | | | | |
| Q2t(unit) | 1498 | 0 | 1239 | 2836 | 2972 | 1259 | 0 | 1376 |
| O2t(unit) | 0 | 0 | 26 | 0 | 0 | 0 | 0 | 182 |
| S2t(unit) | 400 | 400 | 400 | 0 | 0 | 400 | 400 | 400 |
| I2t(unit) | 1098 | 998 | 0 | 0 | 2173 | 3333 | 818 | 200 |
| B2t(unit) | 0 | 0 | 0 | 0 | 0 | 0 | 80 | 0 |
| Ht(man-hours) | 125 | 0 | 5 | 0 | 0 | 0 | 12 | 28 |
| Ft(man-hours) | 0 | 25 | 0 | 0 | 62 | 0 | 0 | 61 |

To investigate the performance of the proposed method, the results of i-PLP approach was used to compare with the results of the proposed methods for each objective functions as shown in Table 4. This table indicates that, the modified Angelov's approach for determining total production cost ($Z_1$=334987) and change in workforce rate ($Z_2$=155) provides better results than i-PLP method; ($Z_1$=336605) and ($Z_2$ = 163) for total production cost and change in workforce rate respectively. Furthermore, Table 5 presents detail comparison of the Angelov's approaches, and the existing i-PLP method for the decision variables; production (Q), overtime (O), subcontract (S), inventory level (I) backorder (B), hiring level (H) and firing level (F) for two products (1 and 2) at different production periods. The result shows that, the Angelov′s approaches are better than the existing i-PLP method for Aggregate production planning. In addition, table 6 illustrated the results of left and right hand side for the proposed methods, the first part refer to equal constraints and .the bellow part refer to inequality of consternates. Therefore, the proposed method minimizes total costs of production and the rates of change in labor levels than (i-PLP) approach.

Table 6. The results of left- and right-hand side for the proposed method

| Left hand side | Right hand side |
|---|---|
| Q11 + O11 + S11 − I11 + B11 = 600 | 600 |
| Q12 + O12 + S12 − I12 + I11 − B11 + B12= 3000 | 3000 |
| Q13 + O13 + S13 − I13 + I12 − B12 + B13 = 5000 | 5000 |
| Q14 + O14 + S14 − I14 + I13 − B13 + B14 = 2000 | 2000 |
| Q21 + O21 + S21 − I21 + B21 = 800 | 800 |
| Q22 + O22 + S22 − I22 + I21 − B21 + B22 = 500 | 500 |
| Q23 + O23 + S23 − I23 + I22 − B22 + B23 = 3000 | 3000 |
| Q24 + O24 + S24 − I24 + I23 − B23 + B24 = 2500 | 2500 |
| H1 − F1 − (0.05Q11 + 0.05O11 + 0.07Q21 + 0.07O21) = 0 | 0 |
| (0.05Q11 + 0.05O11 + 0.07Q21 + 0.07O21) + H2 − F2 − (0.05Q12 + 0.05O12 + 0.07Q22 + 0.07O22) = 0 | 0 |
| (0.05Q12 + 0.05O12 + 0.07Q22 + 0.07O22) + H3 − F3 − (0.05Q13 + 0.05O13 + 0.07Q23 + 0.07O23) = 0 | 0 |
| (0.05Q13 + 0.05O13 + 0.07Q23 + 0.07O23) + H4 − F4 − (0.05Q14 + 0.05O14 + 0.07Q24 + 0.07O24) =0 | |
| 0.05Q11 + 0.05O11 + 0.07Q21 + 0.07O21 = 124.96 | 300 |
| (0.05Q12 + 0.05O12 + 0.07Q22 + 0.07O22) = 125 | 300 |
| (0.05Q13 + 0.05O13 + 0.07Q23 + 0.07O23) = 298.45 | 300 |
| (0.05Q14 + 0.05O14 + 0.07Q24 + 0.07O24) =298.9 | 300 |
| 0.1Q11 + 0.1O11 + 0.08Q21 + 0.08O21 =160.04 | 400 |
| (0.1Q12 + 0.1O12 + 0.08Q22 + 0.08O22) =250 | 500 |
| (0.1Q13 + 0.1O13 + 0.08Q23 + 0.08O23) = 600 | 700 |
| 0.1Q14 + 0.1O14 + 0.08Q24 + 0.08O24 = 426.88 | 500 |
| 2 I11 + 3 I 21 =3898 | 10000 |
| 2I12 + 3I22 =3598 | 10000 |
| 2I13 + 3I23 = 0 | 10000 |
| 2I14 + 3I24 =0 | 10000 |

## 4. Conclusions

A numerical example from the literature was used as benchmarking. The quality of solutions obtained by modified Angelov's approach gave the best outcome as compared to case study. The proposed approaches for determining tolerance ($T'_k$) and rejection levels ($r_k$) are much flexible and these allow finding solutions.

1. These methods can solve almost all multi-objective PP&PC problems in a scientific manner. Hence, similar results can be achieved for any DM by making use of the values.
2. This is unlike Previously, DM estimates were assumed to find values of ($T'_k$) and ($r_k$) by their individual experiences, and then worked out several solutions to the problem to depend on the number of DM.
3. The results obtained by the modified Angelov's approach yield an efficient solution in term of cost.
4. In proposed approach, in providing solution to the APP problem, whenever the value of ($\alpha$) is closer to one, and the value of ($\beta$) approaching to the zero, the solution converge to the optimality solution.
5. More so, Modified Angelov's is better choice for handling all imprecise data, than the triangular (trapezoidal) distributions, because in the triangular (trapezoidal) DM distribution so create and obtain appropriate distributions based on historical resources and autonomy for all fuzzy data.
6. The proposed approach provided a better result than those obtained by solving each objective function individually.

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