

## Optimal HSE-PWM based on genetic algorithm for seven levels diode clamped multilevel inverter

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### ABSTRACT

In this paper, the control of seven level diode clamped inverter with selective harmonic elimination (SHE) pulse width modulation (PWM) technique based on genetic algorithm (GA) has been developed. In standard SHE-PWM, the seven level inverters allow the elimination of only two low order harmonics. To improve the total harmonic distortion (THD), and without any modification to the inverter structure, five low order harmonics can be suppressed by suitably adding holes in the staircase voltage leg. Furthermore, a hole distribution in agreement with the sin function shape is proposed. For this, a real-coded genetic algorithm is applied under the standard constraints with a proposed cost function minimization that allow a better near sin function reshape of the output voltage leg. This GA computation allow to compute the switching angles for a seven-level diode clamped inverter to produce the desired fundamental voltage and to eliminate undesirable harmonics. This developed procedure can eliminate a desired number of low harmonics and is only restricted by the maximal switching frequency of the power switches. The results of the suggested method are compared to the conventional SHE-PWM involved with a seven-level staircase wave. They reveal that the developed method is a very effective one for optimal harmonic elimination technique.

**Keywords:** Diode Clamped Multilevel Inverters, Genetic Algorithm, Objective Function, Optimization, Selective Harmonic Elimination, Total Harmonic Distortion

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## 1. Introduction

Thanks to their high-quality output waveform (weak harmonic distortion), multilevel inverters are widely applied in many industrial applications [1]. In almost cases, multilevel inverters are realized in three fundamental topologies: cascaded H-bridge inverters, diode-clamped inverters, and capacitor-clamped inverters [2].

The output waveform generated by multilevel inverters can include important low order harmonics and an effective method for eliminating them is the SHE-PWM. Besides, the fundamental control of the output voltage with the cancellation of undesirable low order harmonics content conduct to a set of nonlinear transcendental equations. Solving these SHE-PWM nonlinear equations is a challenge when optimization under constraints is involved [3]-[4]-[5]. This later can be used for example to compute the switching angles with the total harmonic distortion (THD) minimization. It is clear that the considered constraints are linked to the inverter levels and the wave symmetry of the staircase output voltage [6].

So far, for solving the set of nonlinear equations, several methods have been proposed and, they can be roughly classified in two groups. The first group regards the equations solving without optimization and the Newton-Raphson method (NRM) is naturally one of these. The major disadvantage of this technique is that the convergence depends on a proper initial guess. This difficulty is overcome by choosing the good starting point of NRM, GA [7] and Particle Swarm Optimization (PSO) [8] can be used [9]. An alternative to NRM is the mathematical theory of resultants, which is able to find all possible solutions to the problem [10]. This is done by transforming the transcendental equations into an equivalent set of polynomials. Meanwhile, the complexity of this technique (related to the equations order) grows with the increasing levels number of the output voltage. The second group has tackled this issue as an optimization task where all evolutionary search algorithms [11]-[12] has been proven efficient to solve this kind of problem. Among them, genetic algorithm [13] is regarded as a stochastic search technique that mimics the behavior of natural selection and evolution. Such algorithms where applied along with standard cost functions inspired mostly from the general Total Harmonic Distortion formula that allow a minimization of the THD factor.

The SHE-PWM technique has been widely exploited for two level inverter [14] and three-level inverters [6], in order to achieve lower THD of the output voltage waveform. When the voltage leg is considered as a symmetric staircase wave, the number of low harmonics that can be cancelled is solely related to the level number  $L$  as  $(L-1)/2$  or  $(L/2)-1$  depending on  $L$  is odd or even respectively. In this case, eliminating more harmonics is obtained at the expense of raising the level number and thus to modify the inverter structure [6]. Rather than increasing the levels number, appropriate holes can be inserted in the symmetric staircase wave representing the voltage leg without modifying the inverter structure.

For multilevel inverters, a generalized SHE-PWM formulation is developed, firstly in [15] where a half-cycle symmetry SHE-PWM is proposed, secondly in [16] for the case of quarter-wave symmetry but with introducing the rising and falling edges of the PWM waveform. This formulation relaxes the staircase output waveform constraint. However, directly applying an arbitrary amount of holes in different steps of the staircase waveform will no longer respect the shape of the sin function. Knowing that  $\sin(\cdot)$  function vary quickly in the first interval  $[0, 5\pi/12]$  and slowly in the second interval  $[5\pi/12, \pi/2]$ , the generalized staircase would reshape better the sin function if the almost solution angles are lying more in the first interval than in the second interval.

In this paper, an optimized SHE-PWM is developed for multi-level inverters. Contrary to the standard solution where the voltage leg is in a symmetric staircase wave, in our case appropriate holes are added to this signal. With this procedure and without modifying the inverter structure, a wide amount of harmonics can be cancelled and it is restricted only by the maximal power switch frequency. This formulation offers more degrees of freedom since; six angles are inserted in the instance of a seven-level inverter without changing the structure of the device, so that five low order harmonics can be eliminated. Furthermore, real coded genetic algorithm is implemented to solve equations set of SHE-PMW procedure with optimization under constraints for a proposed cost function that uses conception parameters which offers a better reshape of the output voltage fo follow the sin function shape. Obtained results, carried out in the case of seven levels diode clamped topology are compared to SHE-PWM conventional method for a seven-level staircase waveform where only two harmonics are to be suppressed.

## 2. Neutral-point clamped multilevel inverter

Figure 1 depicts the structure of one leg (phase a) related to seven-level converter of neutral-point diode clamped. Noticing that the DC input voltage bus  $E$  and the capacitor voltages are common to the three legs (leg  $k$ , with  $k=a, b, c$ ). For our study, we assume that the total DC voltage  $E$  is constant and the  $V_{dc}$  voltage across each of six capacitors ( $C1, C2, C3, C4, C5$  and  $C6$ ) is likewise constant as  $V_{dc} = E/6$ . Moreover, the reference point related to output leg voltage is the point  $n$ . In this figure,  $S1$  to  $S12$  denote switches and  $D1$  to  $D25$  stand for clamping diodes. In complementary control mode, Table 1 gives the seven states of the switches  $S1$  to  $S12$  in order to generate seven levels leg voltage  $V_{an}$  [9].

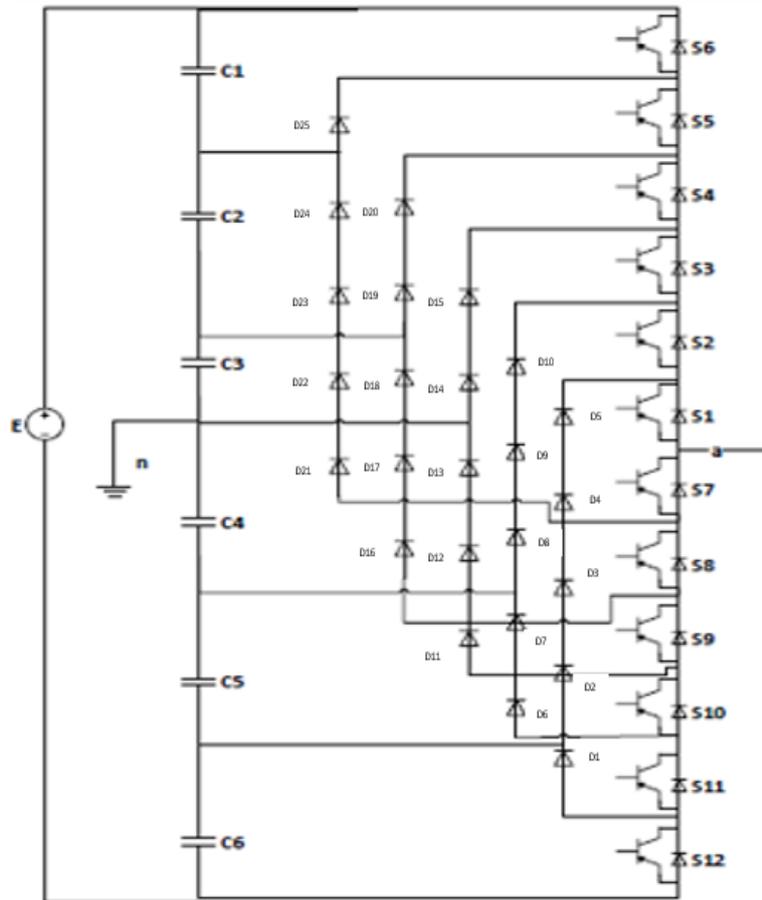


Figure 1. One leg structure related to seven-level diode clamped converter

Table 1. Switching states related to the seven levels of voltage leg  $V_{an}$

$V_{an}$	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
3Vdc	1	1	1	1	1	1	0	0	0	0	0	0
2Vdc	1	1	1	1	1	0	1	0	0	0	0	0
Vdc	1	1	1	1	0	0	1	1	0	0	0	0
0	1	1	1	0	0	0	1	1	1	0	0	0
-Vdc	1	1	0	0	0	0	1	1	1	1	0	0
-2Vdc	1	0	0	0	0	0	1	1	1	1	1	0
-3Vdc	0	0	0	0	0	0	1	1	1	1	1	1

### 3. Problem Formulation

The standard seven level SHE-PWM is presented first, where only two low order harmonics can be suppressed in addition with the control of the fundamental. Then, the developed SHE-PWM method is presented where for the same seven level structure, five low order harmonics are to be suppressed along with the control of the fundamental. This is done by suitably adding holes in the imposed voltage leg.

Recall that, the equations related to the SHE-PWM problem are non-linear. One way of solving these equations would be the Newton-Raphson method (NRM); but the major drawback of this technique is that the convergence depends on a proper initial guess. To overcome this problem a search algorithm (GA) is applied with a proposed cost function that uses conception parameters, which offers a better reshape of the output voltage is elaborated in the subsequent section.

### 3.1. Conventional SHE-PWM

The harmonic elimination procedure requires the computation of switching angles allowing the fundamental control and the abolition of unwanted low order harmonics. These switching angles are quite related to the shape of voltage leg wave. Conventionally, in multi-level inverter technique, the voltage leg has a "staircase" shape where the steps number depends directly on the levels number of the inverter structure. So, the symmetric staircase wave of  $L$  levels (or stairs) can be depicted with  $N$  independent angles given by:

$$N = \begin{cases} (L - 1)/2 & \text{if } L \text{ is odd} \\ (L/2) - 1 & \text{if } L \text{ is even} \end{cases} \quad (1)$$

With these  $N$  angles, we can eliminate  $(N-1)$  lower order harmonics with fundamental control. Figure 2 shows the case of seven levels inverter ( $L=7$ ); it is obvious that this wave can be described by using only three independent switching angles ( $N=3$ ) and thus, based on SHE-PWM procedure, only two  $(N-1)$  low order harmonics can be eliminated. In this case, to cancel more harmonics requires to raise the level number and thus to modify the inverter structure.

Recalling that an output voltage  $v(\omega t)$  of pulsation  $\omega$  can be decomposed under the series form as follows:

$$v(\omega t) = \sum_{k=0}^n b_{2k+1} \sin(2k + 1)\omega t \quad (2)$$

In the instance of the conventional SHE-PWM related to a seven-level inverter incorporating three switching angles (Figure 2), the nonlinear equations can be expressed by exploiting (2) as follows:

$$\begin{cases} \cos \alpha_1 + \cos \alpha_2 + \cos \alpha_3 - \frac{3\pi}{4} M = 0 \\ \cos 5\alpha_1 + \cos 5\alpha_2 + \cos 5\alpha_3 = 0 \\ \cos 7\alpha_1 + \cos 7\alpha_2 + \cos 7\alpha_3 = 0 \end{cases} \quad (3)$$

In addition, the switching angles must satisfy:

$$0 < \alpha_1 < \alpha_2 < \alpha_3 < \pi/2 \quad (4)$$

The optimal solutions of (3) under the constraints (4) are carried out via the minimization of the objective function  $F_1$  as in (5). Furthermore, the function part  $F_1$  is somewhat inspired from the THD expression as follows:

$$F_1 = \frac{\sqrt{V_5^2 + V_7^2}}{V_1} \quad (5)$$

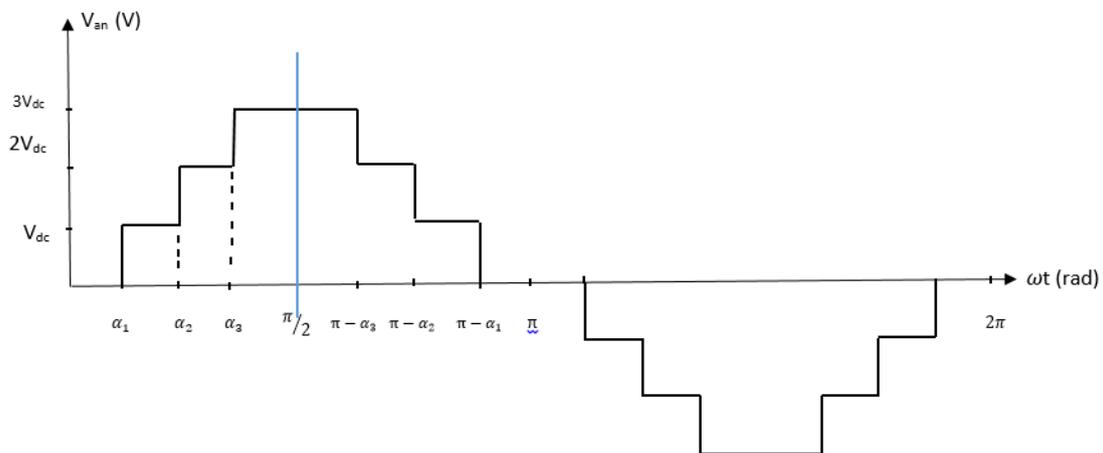


Figure 2. Staircase waveform output voltage for seven-level inverter

### 3.2. Proposed SHE-PWM

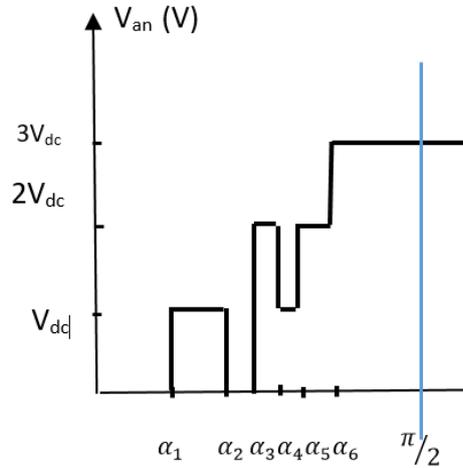


Figure 3. Proposed multilevel PWM waveform (first quarter cycle)

Rather than increasing the levels number, we propose to insert appropriate holes in symmetric staircase wave without modifying the inverter structure. Hence in our work, the imposed seven-level output voltage leg is as shown in Figure 3. The description of this later wave requires six independent angles and hence, five low order harmonics can be eliminated. Noticing that we have inserted only two holes by quarter half of the wave and it is possible to consider more holes as long as the maximal switching frequency is respected.

Furthermore, the SHE-PWM technique concerns the calculation of the appropriate switching angles ( $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_N$ ) so that the  $N-1$  non triplen odd harmonics can be cancelled and the fundamental can be controlled.

A generalized expression of  $b_{2k+1}$  for any considered  $N$  number of switching angles in a quarter cycle and any number of voltage levels is given in [16] as:

$$b_k = \frac{4V_{dc}}{\pi k} \sum_{i=1}^N p_k \cos k\alpha_i \tag{6}$$

Where  $p_k$  is determinate as that:

$$p_k = \begin{cases} 1 & \text{for rising edge} \\ -1 & \text{for falling edge} \end{cases} \tag{7}$$

And the  $k$  index is taken as follows:

$k = 5, 7, 11, \dots, 3N - 2$ , when  $N$  is odd;

$k = 5, 7, 11, \dots, 3N - 1$ , when  $N$  is even.

The SHE-PWM nonlinear equations, related to a seven-level inverter and considering the six switching angles (Figure 3), can be expressed, by exploiting (6) and (7), as that:

$$\begin{cases} \cos \alpha_1 - \cos \alpha_2 + 2 \cos \alpha_3 - \cos \alpha_4 + \cos \alpha_5 + \cos \alpha_6 - \frac{3\pi}{4}M = 0 \\ \cos 5\alpha_1 - \cos 5\alpha_2 + 2 \cos 5\alpha_3 - \cos 5\alpha_4 + \cos 5\alpha_5 + \cos 5\alpha_6 = 0 \\ \cos 7\alpha_1 - \cos 7\alpha_2 + 2 \cos 7\alpha_3 - \cos 7\alpha_4 + \cos 7\alpha_5 + \cos 7\alpha_6 = 0 \\ \cos 11\alpha_1 - \cos 11\alpha_2 + 2 \cos 11\alpha_3 - \cos 11\alpha_4 + \cos 11\alpha_5 + \cos 11\alpha_6 = 0 \\ \cos 13\alpha_1 - \cos 13\alpha_2 + 2 \cos 13\alpha_3 - \cos 13\alpha_4 + \cos 13\alpha_5 + \cos 13\alpha_6 = 0 \\ \cos 17\alpha_1 - \cos 17\alpha_2 + 2 \cos 17\alpha_3 - \cos 17\alpha_4 + \cos 17\alpha_5 + \cos 17\alpha_6 = 0 \end{cases} \tag{8}$$

Where  $M$  is the modulation index, defined as:

$$M = \frac{V_{max}^*}{(E/2)} \tag{9}$$

And  $V_{max}^*$  is the chosen voltage amplitude.

Moreover, the output voltage wave imposes the following constraint on the switching angles:

$$0 \leq \theta_1 < \theta_2 < \theta_3 < \theta_4 < \theta_5 < \theta_6 \leq \pi/2 \tag{10}$$

Hence, the desired value of the output voltage is considered via the desired corresponding M moreover, the order harmonics 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup> and 17<sup>th</sup> must be cancelled.

The optimal solution of (8) under constraints (10) is carried out with the minimization of a global objective function  $F$ . This later gives the effectiveness measure of output voltage quality and it is taken as  $F = F_2 + F_3$ .

The function part  $F_2$  is somewhat inspired from the THD expression as follows:

$$F_2 = \frac{\sqrt{V_5^2 + V_7^2 + V_{11}^2 + V_{13}^2 + V_{17}^2}}{V_1} \tag{11}$$

Beside, knowing that  $\sin(\cdot)$  function vary quickly in the first interval  $[0 \ 5\pi/12]$  and slowly in the second interval  $[5\pi/12 \ \pi/2]$  and, the generalized staircase reshapes better the sin function if the almost solution angles is laying more in the first interval than in the second interval. This strategy is performed by considering the cost function  $F_3(\cdot)$  as:

$$F_3 = \beta_1 \alpha_1^2 + \beta_2 (\alpha_2 - \alpha_1)^2 + \beta_3 (\alpha_3 - \alpha_2)^2 + \beta_4 (\alpha_4 - \alpha_3)^2 + \beta_5 (\alpha_5 - \alpha_4)^2 + \beta_6 (\alpha_6 - \alpha_5)^2 + \beta_7 (\pi/2 - \alpha_6)^2 \tag{12}$$

Positive scalar parameters  $\beta_i$  with  $i=(1,7)$  are conception parameters which influence the bias  $\alpha_i - \alpha_{i-1}$  (i.e. more  $\beta_i$  is greater more  $\alpha_i$  is nearest  $\alpha_{i-1}$ ).

A global cost function  $F$  is thus given by the following expression:

$$F = F_2 + F_3 \tag{13}$$

#### 4. Optimal solutions based on genetic algorithm

To find the best switching angles for the NPC converter, real-coded GA solves the equations set (8) under the cost function (13).

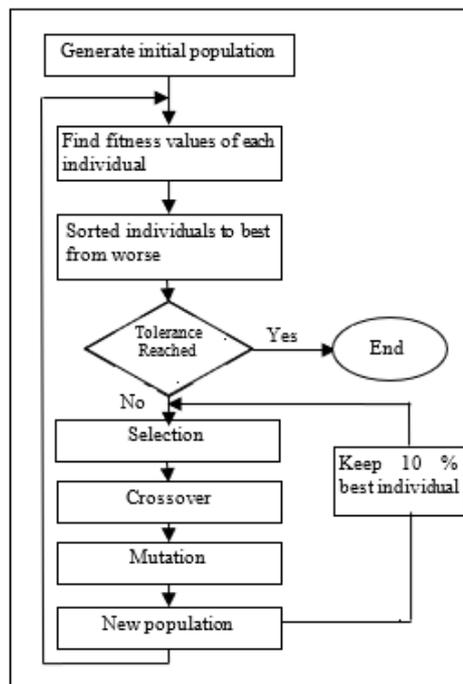


Figure 4. Flowchart of Genetic Algorithm

In this target, each possible solution is considered as an individual  $P$  constituted of six switching angles:  $P^T = [\alpha_1 \ \alpha_2 \ \alpha_3 \ \alpha_4 \ \alpha_5 \ \alpha_6]$ . The elements of this vector are called genes and all the values of the genes are taken within a range of permitted solutions. The evaluation of the fitness  $F$  is made through an objective function. Then, the individuals are classified from the best to the worst. Being at the top, those have a higher chance to be selected for reproduction. The new population is obtained through operators of crossover, mutation, selection and elitism iteratively applied to a set of candidate solution. The generation gap is fixed to 10%, meaning the assigned percentage of parent chromosomes that directly turn into the child chromosomes without undergoing the genetic operation crossover and mutation. The elitist model can guarantee not to lose the best individual in the GA optimization. The steps of GA are illustrated in Figure 4 and are repeated until the convergence criterion is fulfilled.

## 5. Simulation and results

Simulations are carried out for a seven level neutral-point clamped multilevel inverter in the case where the voltage  $E$  is imposed to 660V and thus  $V_{dc}=110V$ . By using real coded genetic algorithm, six switching angles are determinate in order to control the fundamental and to cancel five low order harmonics: 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup> and 17<sup>th</sup>. A comparison is done with the conventional SHE-PWM where three switching angles are determinate in order to control the fundamental and to cancel two low order harmonics: 5<sup>th</sup> and 7<sup>th</sup>.

The considered GA for both cases is involved with a population of 400 individuals chosen randomly in the interval  $[0 \ \pi/2]$ . Beside, control vector parameters is taken as the crossover probability = 0.75, the mutation probability = 0.06 and the generation gap is fixed to 10%.

This procedure has been repeated for the modulation index  $M$  varying in the interval  $[0.3 \ 1]$  with step of 0.0125.

### 5.1. Conventional SHE-PWM

The Fitness function  $F_1$  is used, based on this considered GA and in the case of desired  $M=0.8125$  and  $M=0.775$ , the solution angles are gathered in table 2. The corresponding wave of voltage leg for  $M= 0.8125$  is illustrated in Figure 5 and spectrums are given in Figure 6. An inspection of these later reveals that the 5<sup>th</sup> and 7<sup>th</sup> harmonics have been effectively cancelled and only triplen harmonics remain. Figure 7 depicts the THD variation versus modulation index  $M$ . This latter is varying between 11 and 19%.

Table 2. Calculated Switching Angles

Modulation Index $M$	$\alpha_1$ (°)	$\alpha_2$ (°)	$\alpha_3$ (°)
0.8125	15.84	40.63	63.41
0.7550	17.06	17.37	52.24

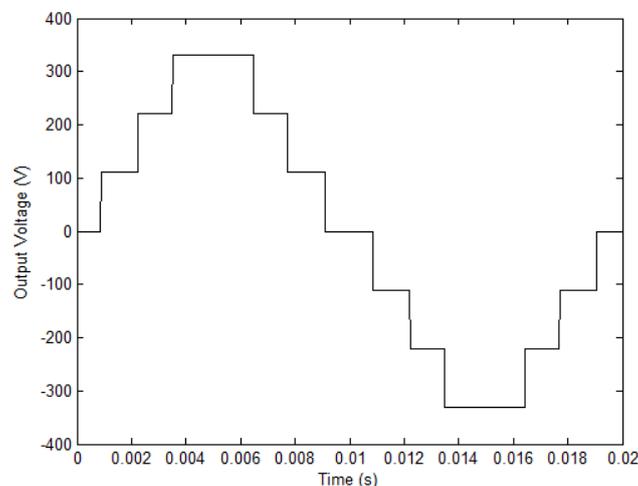


Figure 5. Seven-level line to neutral output waveform

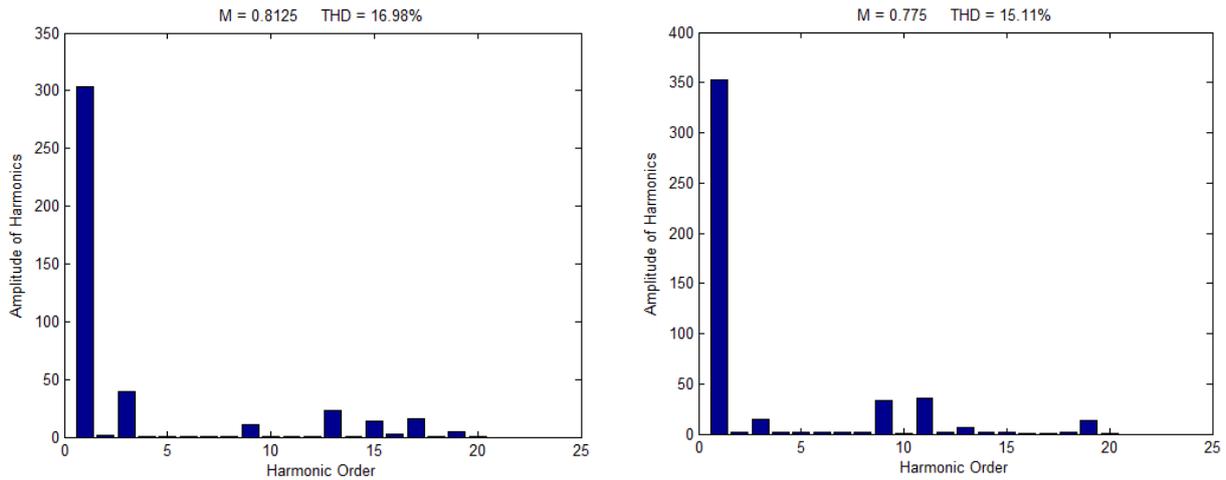


Figure 6. Harmonic Spectra for seven-level inverter with three switching angles

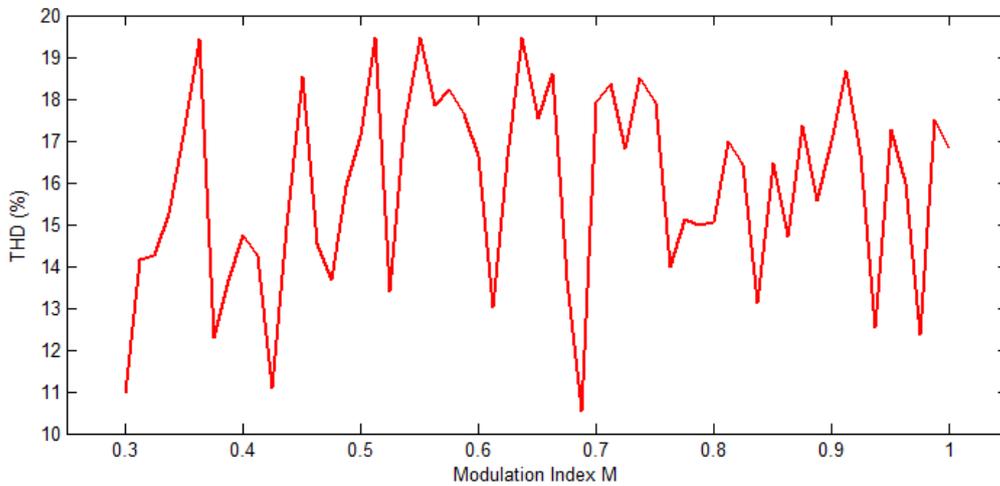


Figure 7. THD versus modulation index M

### 5.2. Proposed SHE-PWM

The Fitness function  $F$  is used, noticing that the conception parameters  $\beta_i$  ( $i=1, 7$ ) are selected between [5 10]:  $\beta_1 = 8, \beta_2 = 7, \beta_3 = 6, \beta_4 = 5, \beta_5 = 4, \beta_6 = 3$  and  $\beta_7 = 2$ .

Based on this considered GA and in the case of desired  $M=0.825$  and  $M=0.775$ , the solution angles are gathered in table 3 where thanks to the choice of  $\beta_i$  all angles are lying between  $[0 \ 5\pi/12]$ . The corresponding wave of voltage leg of  $M=0.825$  is illustrated in Figure 8 and spectrums are given in Figure 9. An inspection of this later reveals that the selected harmonics have been effectively cancelled and only triplen harmonics remain. Furthermore, the next significant harmonic appearing in the line-to neutral output voltage is the 19<sup>th</sup>.

The obtained optimal switching angles are gathered in Figure 10. Figure 11 depicts the THD variation versus modulation index  $M$ . This latter is varying between 9.5 and 11.5%.

Table 3. Calculated Switching Angles

Modulation Index M	$\alpha_1$ (°)	$\alpha_2$ (°)	$\alpha_3$ (°)	$\alpha_4$ (°)	$\alpha_5$ (°)	$\alpha_6$ (°)
0.825	8.25	10.82	18.41	23.80	32.75	58.48
0.755	10.14	13.49	19.26	24.68	32.99	58.53

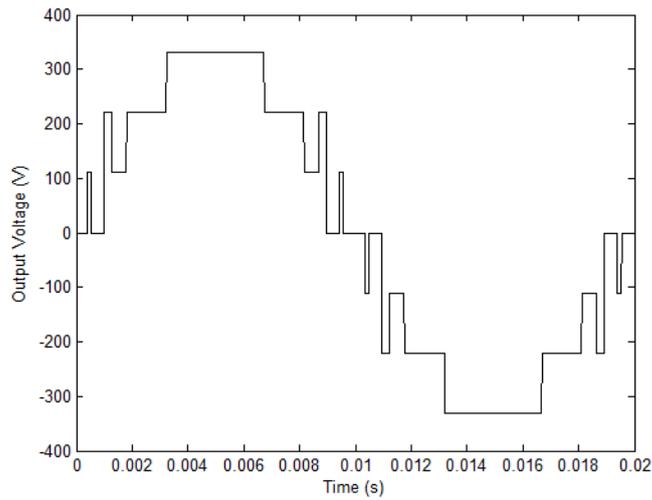


Figure 8. Seven-level line to neutral output waveform

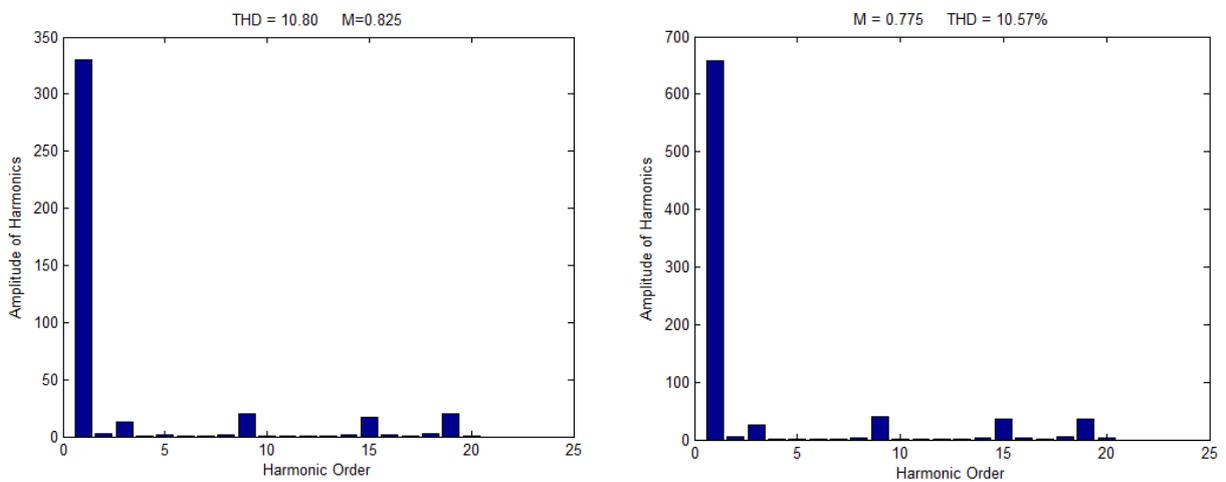


Figure 9. Harmonic Spectra for seven-level inverter with six switching angles

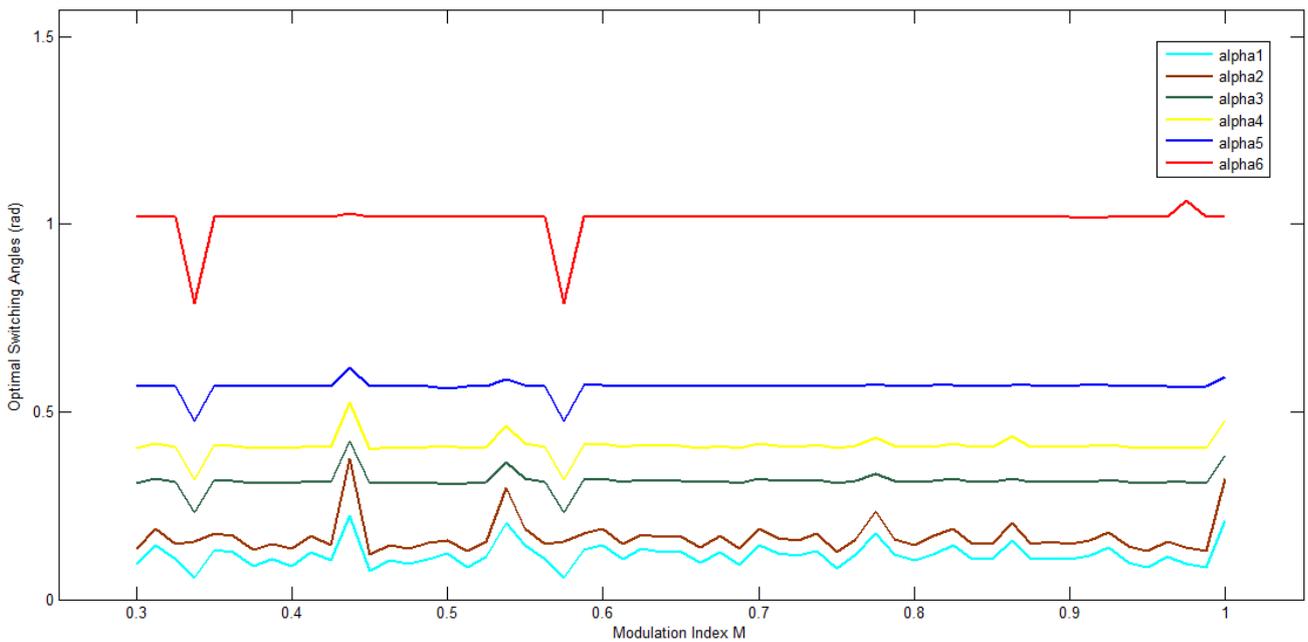


Figure 10. Optimal Switching angles versus modulation index M

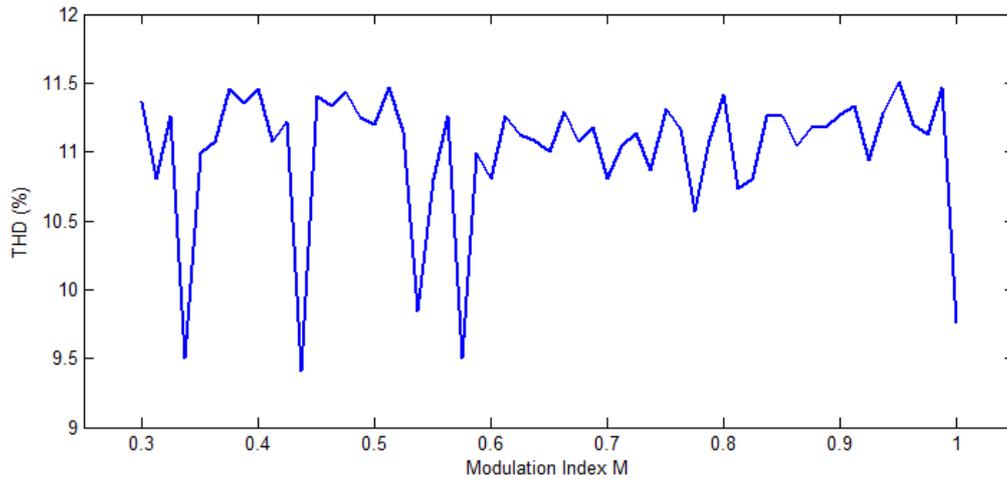


Figure 11. THD versus modulation index

### 5.3. Comparison

Figure 12 depicts the THD variation versus modulation index  $M$  for two cases. THD1 curve is concerned with the developed optimal SHE-PWM and, whereas THD2 curve is related to the conventional SHE-PWM. The comparison between the two cases, clearly shows that the global THD1 is lower than the global THD2 for the whole range of the modulation index and furthermore the THD1 is globally around 11%.

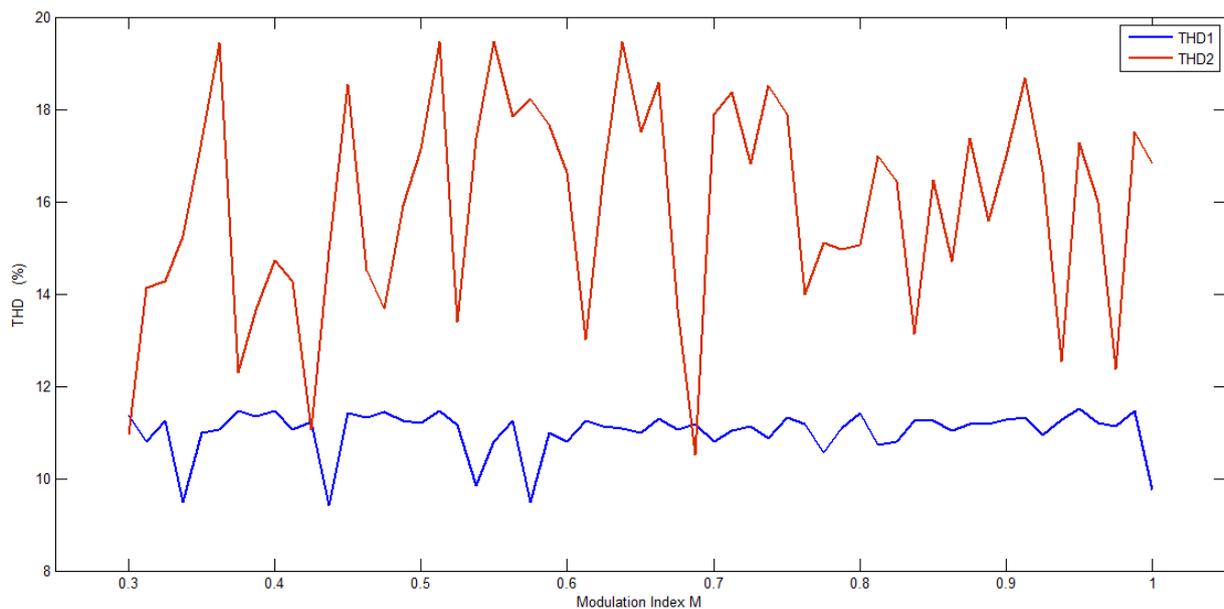


Figure 12. THD versus modulation index

## 6. Conclusion

This work describes an optimized formulation of the SHE-PWM procedure in the context of multilevel inverter. Moreover, the voltage leg is considered in staircase waveform including appropriate holes. It is worth noting that this procedure allows to cancel a large amount of low-order harmonics without modifying the inverter structure, as long as the maximum switching frequency of power switches is undergoes. Recall that the proposed analytical formulation problem is involved with standard constraints and a global cost function that enhances the output waveform quality by allowing a better near sin function reshape of the voltage leg. A Real-Coded genetic algorithm with constraints consideration is utilized to compute the optimal switching angles for the Seven Level Diode Clamped Multilevel Inverter.

The obtained results highlight the approach validity, a better reshape of the output voltage quality and the enhancement of global THD rate for the modulation index, which varies in the entire range in comparison with the conventional method. Thus, the performance of the converter is greatly enhanced.

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