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Methodology/Method

On the SEE transform and systems of ordinary differential equations

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ABSTRACT

Integral transforms, have many applications in the various diciplines of natural science and engineering to solve the problems of springs, heat transfer, electronical and electrical networks, mixing problems, carbon dating problems, bending of beams, Newton's second law of motion, signal processing, exponential growth and decay problems. Also, many phenomenon of real life (biology, neuclear physics, chemistry, and tele-communications …) can be expressed mathematically by linear or nonlinear systems and solved by using integral transform. In this paper, authers disscused the SEE (Sadiq-Emad-Eman) integral transform and systems of ordinary differential equations. SEE integral transform method is very important for the solution of the respose of a linear system governed by an ordinary differential equation in the initial conditions (data) and (or) to an external disturbance (or external input function). Also, we apply it to obtain exact solution of linear systems of ordinary differential equations.

Keywords: SEE basic change, arrangement of common differential conditions.

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1. Introduction

In this writing there are various changes and widly utilized in physical science just as in designing. The fundamental change is a proficient technique to tackle the arrangement of straight standard differential conditions [2-5]. Sadiq A. Mehdi, Emad A. Kuffi and Eman A. Mansour presented another necessary change and named as SEE essential change which is characterized by, [1]:

$$S[f(t)] = \frac{1}{v^n}\int_{t=0}^{\infty} f(t)\, e^{-vt}dt \;= T(v) \quad , \quad v \in (l_1, l_2) \quad , \quad n \in \mathbb{Z} \qquad \dots (1)$$

Furthermore, applied this fundamental change to the arrangement of arrangement of straight customary differential conditions.

SEE Integral Transform of Some Elementary Functions [1],

S.N.	$f(t)$	$S[f(t)] = T(v)$
1.	$K \equiv Constant$	$\frac{K}{v^{n+1}}$
2.	t^m	$\frac{m!}{v^{n+m+1}}$, m is a positive integer number
3.	e^{at}	$\frac{1}{v^n(v-a)}$, a is a constant
4.	$\sin(at)$	$\frac{a}{v^n(v^2+a^2)}$
5.	$\cos(at)$	$\frac{v}{v^n(v^2+a^2)}$
6.	$\sinh(at)$	$\frac{a}{v^n(v^2-a^2)}$
7.	$\cosh(at)$	$\frac{v}{v^n(v^2-a^2)}$

In this work, our motivation is to show the materialness of this intriguing new indispensable change and its productivity in tackling the straight arrangement of conventional differential conditions.

Theorem [1]: Let $T(v)$ is SEE integral transform of $[S[f(t)] = T(v)]$.
Then:

$$S[f^{(m)}(t)] = \frac{-f^{(m-1)}(0)}{v^n} - \frac{f^{(m-2)}(0)}{v^{n-1}} - \cdots - \frac{f(0)}{v^{n-m+1}} + v^m\, T(v)\,.$$

2. Material and methods

2.1 Arrangement of linear ordinary differential equations

SEE essential change technique is extremely viable for the arrangement of the reaction of a straight framework administered by a conventional differential condition to the underlying conditions and additionally to an outer unsettling influence (or outside input work). We look for the arrangement of framework for its state at resulting time $t > 0$ due to the underlying information at t equivalent to nothing and additionally to the unsettling influence applied fort > 0.

3. Results

Example (I) (System of first order ordinary differential equations)
Think about the framework:

$$\left.\begin{aligned} \frac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 + b_1(t)\,, \\ \frac{dx_2}{dt} &= a_{21}x_1 + a_{22}x_2 + b_2(t)\,. \end{aligned}\right\} \quad \ldots (2)$$

With the underlying information x_1 (0)=x_10 and x_2 (0)=x_20 where a_11 ,a_12 ,a_21 and a_22 are constants.
Presenting the grids

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \frac{dx}{dt} = \begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix}, \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$b(t) = \begin{pmatrix} b_1(t) \\ b_2(t) \end{pmatrix} \quad and \ x_0 = \begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix},$$

We can compose the above direct framework in grid differential framework as

$$\frac{dx}{dt} = Ax + b(t)\;,\; x(0) = x_0 \qquad \ldots (3)$$

We take SEE change of the direct framework with the underlying conditions, we get:

$$\begin{cases} (v - a_{11})\overline{x_1} - a_{12}\overline{x_2} = \overline{b_1(v)} + \frac{1}{v^n}x_{10}\;, \\ -a_{21}\overline{x_1} + (v - a_{22})\overline{x_2} = \overline{b_2(v)} + \frac{1}{v^n}x_{20}\;. \end{cases}$$

Where $\overline{x_1}$, $\overline{x_2}$, $\overline{b_1}$, $\overline{b_2}$ are SEE integral change of x_1, x_2, b_1, b_2 respectively.
The arrangement of these logarithmic system is:

$$\overline{x_1}(v) = \frac{\begin{vmatrix} \overline{b_1}(v) + \frac{x_{10}}{v^n} & -a_{12} \\ \overline{b_2}(v) + \frac{x_{20}}{v^n} & v - a_{22} \end{vmatrix}}{\begin{vmatrix} v - a_{11} & -a_{12} \\ -a_{21} & v - a_{22} \end{vmatrix}}\,,$$

$$\overline{x_2}(v) = \frac{\begin{vmatrix} v - a_{11} & \overline{b_1}(v) + \frac{x_{10}}{v^n} \\ -a_{21} & \overline{b_2}(v) + \frac{x_{20}}{v^n} \end{vmatrix}}{\begin{vmatrix} v - a_{11} & -a_{12} \\ -a_{21} & v - a_{22} \end{vmatrix}}\,.$$

Expanding these determinates, results for $\overline{x_1}(v)$ and $\overline{x_2}(v)$ can be rearranged and the answers for x_1 (t) and x_2 (t) can be found in shut structures.

Example (II) Settle the straight arrangement of differential conditions:

$$\begin{cases} \frac{dx}{dt} = 2x - 3y \\ \frac{dy}{dt} = y - 2x \end{cases}, \quad t > 0$$

With the underlying information: x(0)=8 , y(0)=3.
Take SEE change of the framework with the underlying conditions, to get:

$$\overline{x_1}(v)=\frac{\begin{vmatrix}\frac{8}{v^n} & 3\\ \frac{3}{v^n} & v-1\end{vmatrix}}{\begin{vmatrix}v-2 & 3\\ 2 & v-1\end{vmatrix}}=\frac{1}{v^n}\left[\frac{8v-17}{(v-4)(v+1)}\right].$$

After simple computations, we get

$$\overline{x_1}=\frac{1}{v^n}\cdot\frac{3}{(v-4)}+\frac{1}{v^n}\cdot\frac{5}{(v+1)}$$

By taking inverse SEE transform, we have

$$x_1(t)=5e^{-t}+3e^{4t}.$$

Also

$$\overline{x_2}(v)=\frac{\begin{vmatrix}v-2 & \frac{8}{v^n}\\ 2 & \frac{3}{v^n}\end{vmatrix}}{\begin{vmatrix}v-2 & 3\\ 2 & v-1\end{vmatrix}}=\frac{1}{v^n}\left[\frac{3v-22}{v^2-3v-4}\right].$$

After simple computations, we have:

$$\overline{x_2}(v)=\frac{1}{v^n}\left[\frac{-2}{v-4}\right]+\frac{1}{v^n}\left[\frac{5}{(v+1)}\right]$$

By taking inverse SEE integral transform, we have:

$$x_2(t)=-2e^{4t}+5e^{-t}.$$

Example (III) think about a direct arrangement of second request conventional differential conditions:

$$\begin{cases}\dfrac{d^2x}{dt^2}+3x-2y=0\ ,\\ \dfrac{d^2x}{dt^2}+\dfrac{d^2y}{dt^2}-3x+5y=0\,.\end{cases}\qquad \dots(4)$$

with $x(0)=\ y(0)=0\ \ and\ \ x'(0)=3\ ,\ y'(0)=2\qquad \dots(5)$

taking SEE transform of equation (4), we have

$$\begin{cases}S\left[\dfrac{d^2x}{dt^2}\right]+3S[x]-2S[y]=0\\ S\left[\dfrac{d^2x}{dt^2}\right]+S\left[\dfrac{d^2y}{dt^2}\right]-3S[x]+5S[y]=0\end{cases}\qquad \dots(6)$$

Presently, utilizing the property, SEE change of the subsidiary of the capacity in condition (6), we have

$$\begin{cases}\dfrac{-x'(0)}{v^n}-\dfrac{x(0)}{v^{n-1}}+v^2\bar{x}+3\bar{x}-2\bar{y}=0\\ \dfrac{-x'(0)}{v^n}-\dfrac{x(0)}{v^{n-1}}+v^2\bar{x}-\dfrac{y'(0)}{v^n}-\dfrac{y(0)}{v^{n-1}}+v^2\bar{y}-3\bar{x}+5\bar{y}=0\end{cases}\quad ,\ \dots(7)$$

Using equation (5) in equation (7), we have:

$$\begin{cases}\dfrac{-3}{v^n}+v^2\bar{x}+3\bar{x}-2\bar{y}=0\\ \dfrac{-3}{v^n}+v^2\bar{x}-\dfrac{2}{v^n}+v^2\bar{y}-3\bar{x}+5\bar{y}=0\end{cases}\quad ,\dots(8)$$

Equation (8) can be written as:

$$\begin{cases}(v^2+3)\bar{x}-2\bar{y}=\dfrac{3}{v^n}\\ (v^2-3)\bar{x}+(v^2+5)\bar{y}=\dfrac{5}{v^n}\end{cases}\quad ,\dots(9)$$

Solving equation (9) for $\bar{x}(v)$ and $\bar{y}(v)$, we have:

$$\bar{x}(v)=\frac{1}{v^n}\left[\frac{\frac{1}{4}}{v^2+9}+\frac{\frac{11}{4}}{v^2+1}\right]$$

The solution is:

$$x(t)=\frac{1}{12}S^{-1}\left[\frac{1}{v^n}\cdot\frac{3}{(v^2+9)}\right]+\frac{11}{4}S^{-1}\left[\frac{1}{v^{n(v^2+1)}}\right]=\sin(t)\,.$$

Then:

$$x(t)=\frac{1}{12}\sin(3t)+\frac{11}{4}\sin(t)\,.$$

And

$$\bar{y}(v)=\frac{\begin{vmatrix} v^2+3 & \frac{3}{v^n} \\ v^2-3 & \frac{5}{v^n} \end{vmatrix}}{(v^2+3)(v^2+5)+2(v^2-3)}$$

So, after simple computations, the solution is

$y(t)=\frac{11}{4}\sin(t)-\frac{1}{4}\sin(3t)$.

Example (IV) think about an arrangement of direct customary differential conditions:

$$\begin{cases} \frac{dx}{dt}+y=2\cos t \\ x+\frac{dy}{dt}=0 \end{cases} \quad \dots (10)$$

with $x(0)=0 \quad and \quad y(0)=1 \qquad \dots(11)$

taking SEE transform of equation (10), we have

$$\begin{cases} S\left[\frac{dx}{dt}\right]+S[y]=2S[\cos t] \\ S[x]+S\left[\frac{dy}{dt}\right]=0 \end{cases} \quad \dots (12)$$

Presently, utilizing the property, SEE necessary change of the subordinate of the capacity in condition (12), we have:

$\frac{-1}{v^n}x(0)+v\bar{x}+\bar{y}=\frac{2v}{v^n(v^2+1)}$

$\bar{x}-\frac{1}{v^n}y(0)+v\bar{y}=0$

Now, using equation (11), we have:

$$\left.\begin{aligned} v\bar{x}+\bar{y}&=\frac{2v}{v^n(v^2+1)} \\ \bar{x}+v\bar{y}&=\frac{1}{v^n} \end{aligned}\right\} \quad \dots (13)$$

Solving the equation (13) for $\bar{x}$ and $\bar{y}$, we get:

$\bar{x}(v)=\frac{1}{v^n}\left[\frac{1}{v^2+1}\right]$.

Then, the solution is: $x(t)=\sin(t)$.

And

$\bar{y}(v) \;=\frac{1}{v^n}\left[\frac{v}{v^2+1}\right]$

Then, the solution is: $y(t)=\cos(t)$.

4. Discussion

SEE integral transform method is very efficient to solve a linear system governed by an ordinary differential equation to the initial condition (data) and|or to an external disturbance (or external input function). Also, we seek the solution of a linear differential system for its state at sub sequence time $t>0$ due to the initial point at $t=0$ and|or to the disturbance applied for $t>0$.

5. Conclusions

In the present paper, the SEE (Sadiq- Emad- Eman) integral transform and linear systems of ordinary differential equations are established successfully. A new integral transform "SEE integral transform" is a convenant tool for solving linear system of ordinary differential equations in the time domain. In future, using this integral transform, we can easly solved many advanced systems (linear or nonlinear) of modern era such as system of drug distribution in the body, arms, race systems or (models), health problems such as detection of diabetes). Also, in this paper we introduce the solution of general linear systems of first order ordinary differential equations, as well as, solved examples of linear systems of first and second order ordinary differential equations.

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