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The new integral transform "SEE transform" and its applications

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ABSTRACT

In this paper another fundamental change in particular SEE change was applied to address straight normal deferential conditions with consistent coefficients and SEE change of incomplete derivative is inferred and its appropriateness showed utilizing three is inferred and its appropriateness showed utilizing: wave equation, heat equation and Laplace equation, we find the particular solutions of these equations.

Keywords: Integral, Transform and Applications.

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1. Introduction

Sadiq, Emad and Eman (SEE) are gotten from the old-style Laplace indispensable. In light of the numerical effortlessness of the SEE and its principal properties.
SEE integral was introduced by Sadiq, Emad A. Kuffi, Eman M. to work with the way toward addressing common and halfway differential conditions in the time area. Regularly, Fourier, Laplace, Elzaki, Aboodh, Mohanad, Al-Zughair, Kamal and Mahgoub changes are the helpful numerical instruments for addressing differential conditions, likewise SEE change and a portion of its crucial properties are utilized to tackle differential conditions, [1-12].
A new integral transform said to be SEE change characterized for capacity of outstanding request we think about capacities in the set A characterized by:

$$A = \{f(t): there\ exist\ M, \ell_1, \ell_2 > 0\,.\, |f(t)| < M\, e^{\ell_i |t|}, if\ t \in (-1)^i x[0,\infty)\} \qquad \ldots (1)$$

For a given capacity in the arrangement ofA, the constant M should be limited number, ℓ_1, ℓ_2 might be limited or boundless.
SEE fundamental change signified by the administrator $S(.)$ characterized by the vital condition:

$$S[f(t)] = T(v) = \frac{1}{v^n}\int_0^\infty f(t)e^{-vt}dt\ ,\ n \in \mathbb{Z}, t \geq 0,\ \ell_1 \leq v \leq \ell_2 \qquad \ldots (2)$$

The variable v in this vital change is utilized to figure the variable t the contention of the capacity f. This necessary change has further association with the Laplace, Aboodh, and Mohanad changes.
The reason for this examination is to show the pertinence of this intriguing new change and it productivity in tackling the direct differential conditions.

Notes:

(1) If $n = 1$ then eq. (2) becomes:

$$S[f(t)] = T(v) = \frac{1}{v}\int_0^\infty f(t)e^{-vt}dt\ ,\ t \geq 0,\ \ell_1 \leq v \leq \ell_2$$

This integral transform is called “Aboodh Transform”, [1].

(2) If $n = -2$ then eq. (2) becomes:

$$S[f(t)] = T(v) = v^2 \int_0^\infty f(t)e^{-vt}dt \ , \ t \geq 0, \ \ell_1 \leq v \leq \ell_2$$

This integral transform is called “Mohanad Transform”, [9].

(3) If $n = 0$ then eq. (2) becomes:

$$S[f(t)] = T(v) = \int_0^\infty f(t)e^{-vt}dt \ , \ t \geq 0, \ \ell_1 \leq v \leq \ell_2$$

This integral transform is called “Laplace Transform”, [10].

(4) If $n = -1$ then eq. (2) becomes:

$$S[f(t)] = T(v) = v \int_0^\infty f(t)e^{-vt}dt \ , \ t \geq 0, \ \ell_1 \leq v \leq \ell_2$$

This integral transform is called “Mahgoub Transform”, [11].

2. Methodology

2.1 SEE integral change of some functions

For any capacity$f(t)$, we accept that the fundamental condition (2) exists. The adequate conditions for the presence of SEE integral change are that$f(t)$ for $t \geq 0$ be piecewise ceaseless and of remarkable request, in any case SEE integral change could conceivably exists.
In this segment we find SEE integral change of basic capacities:

(1) If $f(t) = k$, where k is a constant function, then by the definition we have:

$$S[k] = T(v) = \frac{1}{v^n}\int_0^\infty e^{-vt} \, k \, dt \ = \frac{1}{v^n}.\frac{-k}{v}\int_0^\infty (-v) \, e^{-vt} \, dt \ = \frac{-k}{v^{n+1}}[e^{-vt}|_0^\infty = \frac{k}{v^{n+1}} \quad .$$

(2) If $f(t) = t$, then:

$$S[t] = T(v) = \frac{1}{v^n}\int_0^\infty e^{-vt} \, t \, dt \quad .$$

Integration by parts, we get $S[t] = \frac{1}{v^{n+2}}$.

Also:

(i) $S[t^2] = \frac{2}{v^{n+3}}$,

(ii) $S[t^3] = \frac{3!}{v^{n+4}}$,

(iii) In general case if m is a positive integer number, then $S[t^m] = \frac{m!}{v^{n+m+1}}$.

(3) If $f(t) = e^{at}$, where a is a constant number, then

$$S[e^{at}] = T(v) = \frac{1}{v^n}\int_0^\infty e^{-vt}e^{at} \, dt \ = \frac{1}{v^n}\int_0^\infty e^{-(v-a)t} \, dt = \frac{-1}{v^n(v-a)}[e^{-(v-a)t}]_0^\infty = \frac{1}{v^n(v-a)} \, .$$

(4) If $f(t) = \sin(at)$, where a is a constant number, then

$S[\sin(at)] = T(v) = \frac{1}{v^n}\int_0^\infty e^{-vt}\sin(at)\,dt \ = \frac{1}{v^n}\int_0^\infty e^{-vt}\left[\frac{e^{iat}-e^{-iat}}{2i}\right]dt =$
$\frac{1}{2i\,v^n}[\frac{-1}{(v-ai)}\int_0^\infty e^{-(v-ai)t}\,dt - \frac{1}{-(v+ai)}\int_0^\infty e^{-(v+ai)t}\,dt = \frac{1}{2i\,v^n}[\frac{-1}{v-ai}e^{-(v-ai)t}|_0^\infty + \frac{1}{(v+ai)}[e^{-(v+ai)t}|_0^\infty =$
$\frac{1}{2i\,v^n}[\frac{1}{v-ai} - \frac{1}{v+ai}] = \frac{1}{2i\,v^n}\left[\frac{2ai}{(v^2+a^2)}\right] = \frac{a}{v^n(v^2+a^2)}$.

(5) If $f(t) = \cos(at)$, where a is a constant number, then

$$S[\cos(at)] = T(v) = \frac{1}{v^n}\int_0^\infty e^{-vt}\cos(at)\,dt \ = \frac{1}{v^n}\int_0^\infty e^{-vt}\left[\frac{e^{iat}+e^{-iat}}{2}\right]dt\ .$$

After simple computations, we get:

$$S[\cos(at)] = \frac{1}{v^{n-1}(v^2+a^2)}\ .$$

(6) If $f(t) = \sinh(at)$, where a is a constant number, then

$$S[\sinh(at)] = \frac{1}{v^n}\int_0^\infty e^{-vt}\sinh(at)\,dt \ = \frac{1}{v^n}\int_0^\infty e^{-vt}\left[\frac{e^{at}-e^{-at}}{2}\right]dt\ .$$

After simple computations, we get:

$$S[\sinh(at)] = \frac{a}{v^n(v^2-a^2)}\ .$$

(7) If $f(t) = \cosh(at)$, where a is a constant number, then

$$S[\cosh(at)] = \frac{1}{v^{n-1}(v^2-a^2)}\ .$$

Theorem (2.1):

Let $T(v)$ is the SEE transform of $[S[f(t)] = T(v)]$ then:

(i) $S[f'(t)] = \frac{-1}{v^n}f(0) + v\,T(v)$

(ii) $S[f''(t)] = \frac{-f'(0)}{v^n} - \frac{f(0)}{v^{n-1}} + v^2\,T(v)$

(iii) $S[f'''(t)] = \frac{-f''(0)}{v^n} - \frac{f'(0)}{v^{n-1}} - \frac{f(0)}{v^{n-2}} + v^3\,T(v)$

(iv) $S\left[f^{(4)}(t)\right] = \frac{-f'''(0)}{v^n} - \frac{f''(0)}{v^{n-1}} - \frac{f'(0)}{v^{n-2}} - \frac{f(0)}{v^{n-3}} + v^4\,T(v)$

(iiv) $S\left[f^{(m)}(t)\right] = \frac{-f^{(m-1)}(0)}{v^n} - \frac{f^{(m-2)}(0)}{v^{n-1}} - \cdots - \frac{f(0)}{v^{n-m+1}} + v^m\,T(v)$

Proof

(i) by the definition we get:

$$S[f'(t)] = \frac{1}{v^n}\int_0^\infty f'(t)e^{-vt}\,dt\ ,$$

Integrating by parts, we get:

$S[f'(t)] = \frac{-1}{v^n} f(0) + vT(v)$,

(ii) Also, by the definition, we have:

$S[f''(t)] = \frac{1}{v^n} \int_0^{\infty} f''(t) e^{-vt}\, dt$,

Also, integrating by parts, we get:

$S[f''(t)] = \frac{-f'(0)}{v^n} - \frac{vf(0)}{v^n} + v^2 T(v)$,

$= \frac{-f'(0)}{v^n} - \frac{f(0)}{v^{n-1}} + v^2 T(v)$.

Similarly, the proof of (iii) and (iv).

(iiv) can be confirmation by numerical enlistment.

2.2 The inverse of SEE Integral Transform

If $S[f(t)] = T(v)$ is the SEE integral transform, then $[f(t)] = S^{-1}[T(v)]$ is called an inverse of the SEE integral transform.

In this section, we introduce the inverse of SEE integral transform of simple functions:

(1) $S^{-1}\left[\frac{1}{v^{n+1}}\right] = 1$.

(2) $S^{-1}\left[\frac{1}{v^{n+2}}\right] = t$.

(3) $S^{-1}\left[\frac{m!}{v^{n+m-1}}\right] = t^m\ , where\ n, m > 0$ integer numbers.

(4) $S^{-1}\left[\frac{1}{v^n(v+a)}\right] = e^{-at}$, where a is a constant number.

(5) $S^{-1}\left[\frac{a}{v^n(v^2+a^2)}\right] = \sin(at)$.

(6) $S^{-1}\left[\frac{1}{v^{n-1}(v^2+a^2)}\right] = \cos(at)$.

(7) $S^{-1}\left[\frac{a}{v^n(v^2-a^2)}\right] = \sinh(at)$.

(8) $S^{-1}\left[\frac{v}{v^n(v^2-a^2)}\right] = \cosh(at)$.

2.3 Applications of SEE integral change of ordinary differential equations (ODEs)

As expressed in the presentation of this work, the SEE necessary change can be utilized as a successful device. For investigating the essential attributes of a direct framework represented by the differential condition in light of introductory conditions. The accompanying models represent the utilization of the SEE necessary change in taking care of certain underlying worth issue depicted by common differential conditions.

Consider the 1st order linear conventional differential condition:

$$\frac{dx}{dt} + px = f(t)\ , where\ t > 0 \qquad ...(3)$$

With beginning condition $x(0) = a$ ... (4)

Where p and a are constants and $f(t)$ is an outside input work with the goal that its SEE integral transform exists.

Take the SEE integral transform for equation (3), we have

$$S\left[\frac{dx}{dt}\right] + S[px] = S[f(t)]\ , t > 0\ .$$

So,

$$\frac{-x(0)}{v^n} + vT(v) + pT(v) = \bar{f}(v),$$

$$T(v)(v+p) = \frac{x(0)}{v^n} + \bar{f}(v)\ ,$$

$$T(v) = \frac{\bar{f}(v)}{(v+p)} + \frac{a}{v^n(v+p)}\ .$$

The inverse SEE integral change prompts the arrangement. Now the 2nd request straight normal differential condition has the structure:

$$\frac{d^2y}{dx^2} + 2p\frac{dy}{dx} + qy = f(x)\ , x > 0 \qquad ...(5)$$

The initial conditions are:

$$y(0) = a\ ,\ \frac{dy}{dx}(0) = b \qquad ...(6)$$

Where p, a and b are constants.

Applying SEE integral change to this overall beginning worth issue gives:

$$\frac{y'(0)}{v^n} - \frac{y(0)}{v^{n-1}} + v^2\,T(v) + 2p\left[\frac{-y(0)}{v^n} + vT(v)\right] + qT(v) = \bar{f}(v)\ ,$$

$$\frac{-b}{v^n} - \frac{a}{v^{n-1}} + v^2\,T(v) - \frac{2pa}{v^n} + 2pv\,T(v) + qT(v) = \bar{f}(v)\ .$$

$$T(v)(v^2 + 2pv + q) = \bar{f}(v) + \frac{b+2ap}{v^n} + \frac{a}{v^{n-1}}$$

$$T(v) = \frac{\bar{f}(v)}{v^2+2av+q} + \frac{b+2ap}{v^n(v^2+2av+q)} + \frac{a}{v^{n-1}(v^2+2av+q)}\ .$$

Take the inverse of SEE integral transform to above equation gives the arrangement.

Example (1): Consider the differential equation: $\frac{dy}{dx} + y = 0\ , y(0) = 1$.

Solution: take SEE integral change to this condition, we get:

$$S\left[\frac{dy}{dx}\right] + S[y] = 0\ ,$$

So

$$\frac{-y(0)}{v^n} + vT(v) + T(v) = 0\ ,$$

$T(v) = \frac{y(0)}{v^n(v+1)} = \frac{1}{v^n(v+1)}$.

Take inverse to both sides, we get:

The solution: $y(x) = e^{-x}$.

Example (2): Solve the differential equation $y' + 2y = x \;\; , y(0) = 1$.

Solution: take SEE integral transform to this equation gives

$\frac{-y(0)}{v^n} + vT(v) + 2T(v) = \frac{1}{v^{n+2}}$,

$\frac{-1}{v^n} + (2+v)T(v) = \frac{1}{v^{n+2}}$,

So

$T(v) = \frac{1}{v^{n+2}(2+v)} + \frac{1}{v^n(2+v)}$.

Now,

$\frac{1}{v^{n+2}(2+v)} = \frac{1}{v^n v^2(2+v)} = \frac{1}{v^n}\left[\frac{A}{v} + \frac{B}{v^2} + \frac{C}{v+2}\right]$

After simple computations, we get:

$A = \frac{1}{2}$, $B = -\frac{1}{4}$ $\;\; and \; C = \frac{1}{4}$.

Then $T(v) = \frac{1}{v^n}\left[\frac{\frac{1}{2}}{v^2} + \frac{-\frac{1}{4}}{v} + \frac{\frac{1}{4}}{v+2}\right] + \frac{1}{v^n(2+v)}$

$T(v) = \frac{1}{2}.\frac{1}{v^{n+2}} - \frac{1}{4}.\frac{1}{v^{n+1}} + \frac{5}{4}.\frac{1}{v^n(2+v)}$.

Take inverse SEE change of this condition is basically acquired as:

$y(x) = \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x}$.

Example (3): Find the solution of differential condition: $y'' + y = 0 \;\; , y(0) = y'(0) = 1$.

Solution: take SEE transform to above differential condition gives:

$\frac{-y'(0)}{v^n} - \frac{y(0)}{v^{n-1}} + v^2T(v) + T(v) = 0$,

$\frac{-1}{v^n} - \frac{1}{v^{n-1}} + (v^2+1)T(v) = 0$,

So $T(v) = \frac{1}{v^n(v^2+1)} + \frac{1}{v^{n-1}(v^2+1)}$.

The inverse SEE transform of this equation is simply obtained as: $y(x) = \sin(x) + \cos(x)$.

Example (4): Consider the 2nd -request differential condition$y'' - 3y' + 2y = 0 \;\; , y(0) = 1, \;\; y'(0) = 4$.

Solution: take SEE transform to above differential equation, we get:

$\frac{-y'(0)}{v^n} - \frac{y(0)}{v^{n-1}} + v^2T(v) - 3\left[\frac{-y(0)}{v^n} + vT(v)\right] + 2T(v) = 0$,

$\frac{-4}{v^n} - \frac{1}{v^{n+1}} + \frac{3}{v^n} + (v^2 - 3v + 2)T(v) = 0$,

$\frac{-1}{v^n} - \frac{1}{v^{n-1}} + \frac{3}{v^n} + (v^2 - 3v + 2)T(v) = 0$,

$\frac{-(1+v)}{v^n} + (v^2 - 3v + 2)T(v) = 0$,

$T(v) = \frac{1+v}{v^n(v^2-3v+2)}$,

So, $T(v) = \frac{1}{v^n}\left[\frac{(1+v)}{(v-2)(v-1)}\right]$.

Now

$\frac{1+v}{(v-2)(v-1)} = \frac{A}{v-2} + \frac{B}{v-1}$.

After simple computations, we get: $A = 3$ and $B = -2$.

So,

$T(v) = \frac{1}{v^n}\left[\frac{3}{v-2} + \frac{-2}{v-1}\right]$,

$T(v) = \frac{3}{v^n(v-2)} - \frac{2}{v^n(v-1)}$.

The general solution is: $y(x) = 3e^{2x} - 2e^x$.

<u>Example (5):</u> Consider the second-order linear nonhomogeneous request differential condition: $y'' + 9y = \cos(2x)$, $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = -1$.

Solution: since $y'(0)$ is unknown, let $y'(0) = a$.

Take SEE change of this condition and utilizing beginning conditions, we have:

$\frac{-y'(0)}{v^n} - \frac{y(0)}{v^{n-1}} + v^2T(v) + 9T(v) = \frac{1}{v^{n-1}(v^2+4)}$,

$\frac{-a}{v^n} - \frac{1}{v^{n-1}} + (v^2 + 9)T(v) = \frac{1}{v^{n-1}(v^2+4)}$,

So, $T(v) = \frac{a}{v^n(v^2+9)} + \frac{1}{v^{n-1}(v^2+9)} + \frac{1}{v^{n-1}(v^2+9)(v^2+4)}$,

$T(v) = \frac{a}{v^n(v^2+9)} + \frac{v^2+5}{v^{n-1}(v^2+9)(v^2+4)}$.

Now

$\frac{v^2+5}{v^{n-1}(v^2+9)(v^2+4)} = \frac{1}{v^{n-1}}\left[\frac{Av+B}{v^2+9} + \frac{Cv+D}{v^2+4}\right]$.

After simple computations, we get: $A = 0$, $B = \frac{4}{5}$, $C = 0$, $D = \frac{1}{5}$.

$\frac{1}{v^{n-1}}\left[\frac{v^2+5}{(v^2+9)(v^2+4)}\right] = \frac{\frac{4}{5}}{v^{n-1}(v^2+9)} + \frac{\frac{1}{5}}{v^{n-1}(v^2+4)}$.

So $T(v) = \frac{3a}{3v^n(v^2+9)} + \frac{\frac{1}{5}}{v^{n-1}(v^2+4)} + \frac{\frac{4}{5}}{v^{n-1}(v^2+9)}$.

Take inverse SEE transform, then the solution is:

$y(x) = \frac{a}{3}\sin(3x) + \frac{1}{5}\cos(2x) + \frac{4}{5}\cos(3x)$

To find a note that , $y\left(\frac{\pi}{2}\right) = -1$, then we find $a = \frac{12}{5}$, then

$y(x) = \frac{4}{5}\sin(3x) + \frac{1}{5}\cos(2x) + \frac{4}{5}\cos(3x)$.

Example (6): Tackle the differential condition: $y'' - 3y' + 2y = 4e^{3x}$, $y(0) = -3$, $y'(0) = 5$.

Solution: take SEE integral transform of this differential equation and using the initial conditions, gives:

$\frac{-y'(0)}{v^n} - \frac{y(0)}{v^{n-1}} + v^2T(v) - 3\left[\frac{-y(0)}{v^{n-1}} + vT(v)\right] + 2T(v) = \frac{4}{v^n(v-3)}$,

$\frac{-5}{v^n} + \frac{3}{v^{n-1}} + v^2T(v) - \frac{9}{v^n} - 3vT(v) + 2T(v) = \frac{4}{v^n(v-3)}$,

So,

$T(v)(v^2 - 3v + 2) = \frac{14}{v^n} - \frac{3}{v^{n-1}} + \frac{4}{v^n(v-3)}$.

$T(v) = \frac{-3v^2+23v-38}{v^n(v-3)(v-2)(v-1)} = \frac{1}{v^n}\left[\frac{A}{(v-3)} + \frac{B}{(v-2)} + \frac{C}{(v-1)}\right]$

After simple computations, we have: $A = 2$, $B = 4$ and $C = -9$.

Then

$T(v) = \frac{2}{v^n(v-3)} + \frac{4}{v^n(v-2)} - \frac{9}{v^n(v-1)}$

Take inverse transform, we get: $y(x) = 2e^{3x} + 4e^{2x} - 9e^x$.

Notes:

(1) (Shifting property of SEE integral transform)

If SEE integral transform of $f(t)$ is $T(v)$, then SEE transform of function $e^{at}f(t)$ is given by $\frac{(v-a)^n}{v^n} . T(v-a)$.

Proof: $S[e^{at}f(t)] = \frac{1}{v^n}\int_0^\infty e^{at}f(t)e^{-vt}dt = \frac{1}{v^n}\int_0^\infty f(t)e^{-(v-a)t}dt = \frac{(v-a)^n}{v^n} . \frac{1}{(v-a)^n}\int_0^\infty f(t)e^{-(v-a)t}dt = \frac{(v-a)^n}{v^n} T(v-a)$.

(2) (SEE integral transform of function $tf(t)$)

If $S[f(t)] = T(v)$, then $S[tf(t)] = \left[-\frac{n}{v} - \frac{d}{dv}\right]T(v)$.

Proof: by the definition of SEE integral transform

$S[f(t)] = \frac{1}{v^n}\int_0^\infty e^{-vt}f(t)dt = T(v)$

$\frac{d}{dv}T(v) = -n\, v^{-n-1}\int_0^\infty e^{-vt}f(t)dt + \frac{1}{v^n}\int_0^\infty (-t)f(t)\, e^{-vt}dt$

$\frac{d}{dv}T(v) = \frac{-n}{v^{n+1}}\int_0^\infty e^{-vt}f(t)dt - \frac{1}{v^n}\int_0^\infty tf(t)\, e^{-vt}dt$

$\frac{d}{dv}\big(T(v)\big) = \frac{-n}{v}\frac{1}{v^n}\int_0^\infty e^{-vt}f(t)dt - S[tf(t)]$

$\frac{d}{dv}\big(T(v)\big) = \frac{-n}{v}S[f(t)] - S[tf(t)]$

$S[tf(t)] = \frac{-n}{v}T(v) - \frac{d}{dv}T(v) = \left[\frac{-n}{v} - \frac{d}{dv}\right]T(v)$.

Example (7): Solve the differential equation: $y'' + 8y' + 25y = 150 \quad , y'(0) = y(0) = 0$.

Solution: take SEE integral transform of this equation gives:

$\frac{-y'(0)}{v^n} - \frac{y(0)}{v^{n-1}} + v^2T(v) + 8\left[\frac{-y(0)}{v^n} + vT(v)\right] + 25T(v) = \frac{150}{v^{n+1}}$,

$(v^2 + 8v + 25)T(v) = \frac{150}{v^{n+1}}$,

So $T(v) = \frac{1}{v^n+1}\left[\frac{150}{v^2+8v+25}\right]$,

$T(v) = \frac{1}{v^n}\left[\frac{150}{v(v^2+8v+25)}\right]$,

$T(v) = \frac{1}{v^n}\left[\frac{A}{v} + \frac{Bv+C}{v^2+8v+25}\right]$.

After simple computations, we get: $A = 6$, $B = -6$ $and\ C = -48$.

Then, $T(v) = \frac{1}{v^n}\left[\frac{6}{v} + \frac{-6v-48}{v^2+8v+25}\right] = \frac{6}{v^{n+1}} - \frac{1}{v^n}\left[\frac{6(v+4)}{(v+4)^2+9}\right] - \left[\frac{24}{v^n(v+4)^2+9}\right]$,

Take inverse SEE transform leads to the solution: $y(t) = 6 - 6e^{-4t}\cos(3t) - 8e^{-4t}\sin(3t)$.

2.4 Definition and Derivations the SEE Integral Transform of Derivatives

The SEE transform of the function $f(t)$ is defined as:

$S[f(t)] = \frac{1}{v^n}\int_0^\infty f(t)e^{-vt}dt \quad , n \in \mathbb{Z} \ , \ t \geq 0$ ... (1).

To acquire the SEE change of incomplete subordinate we use reconciliation by parts as follows:

$S\left[\frac{\partial f}{\partial t}(x,t)\right] = \int_0^\infty \frac{1}{v^n}\frac{\partial f}{\partial t}e^{-vt}dt = \lim_{p\to\infty}\int_0^p \frac{1}{v^n}\frac{\partial f}{\partial t}e^{-vt}dt$,

$= \lim_{p\to\infty}\{[\frac{1}{v^n}e^{-vt}f(x,t)\,|_0^p + \frac{1}{v^{n-1}}\int_0^p e^{-vt}f(x,t)dt\}$,

$= vT(x,v) - \frac{1}{v^n}f(x,0)$ ... (2) .

We assume that f is a piecewise continues and is of exponential order.

Now, $S\left[\frac{\partial f}{\partial x}\right] = \int_0^\infty \frac{1}{v^n}e^{-vt}\frac{\partial f(x,t)}{\partial x}dt$

$= \frac{\partial}{\partial x}\int_0^\infty \frac{1}{v^n} e^{-vt} f(x,t)dt$ (using Leibnitz rule).

$= \frac{\partial}{\partial x}\left[\frac{1}{v^n}\int_0^\infty e^{-vt} f(x,t)dt\right] = \frac{\partial}{\partial x}[T(x,v)]$.

And $S\left[\frac{\partial f}{\partial x}\right] = \frac{d}{dx}[T(x,v)]$ ... (3) .

Also, we can find:

$S\left[\frac{\partial^2 f}{\partial x^2}\right] = \frac{d}{dx}[T(x,v)]$ ... (4) .

To find $S\left[\frac{\partial^2 f}{\partial t^2}(x,t)\right]$,

Let $\frac{\partial f}{\partial t} = h$, then

By using equation (2), wee have

$S\left[\frac{\partial^2 f}{\partial t^2}(x,t)\right] = S\left[\frac{\partial h(x,t)}{\partial t}\right] = vS[h(x,t)] - \frac{h(x,0)}{v^n}$,

$S\left[\frac{\partial^2 f}{\partial t^2}(x,t)\right] = v^2 T(x,v) - \frac{1}{v^n}\frac{\partial f}{\partial t}(x,0) - \frac{f(x,0)}{v^{n-1}}$ ... (5) .

We can without much of a stretch out this outcome to the n^{th} incomplete subsidiary by utilizing numerical acceptance.

3. Results and discussions

In this paper, we settle first and second request halfway differential conditions, wave condition, heat condition and Laplace's condition, which are known as three basic conditions in numerical physical science and happen in numerous parts of physical science, in applied arithmetic just as designing.

Example (1): find the solution of the 1^{st} request introductory worth issue:

$y_x = 2y_t + y$, $y(x,0) = 6e^{-3x}$ ... (6)

and y is bounded for $x, t > 0$.

Solution: let T be the SEE transform of y. Then, taking SEE change of condition (6) we have:

$\frac{dT(x,v)}{dx} - 2\left[vT(x,v) - \frac{1}{v^n}y(x,0)\right] = T(x,v)$,

$\frac{dT(x,v)}{dx} - 2vT(x,v) + \frac{2}{v^n}(6e^{-3x}) = T(x,v)$,

$\frac{dT(x,v)}{dx} - (2v+1)T(x,v) = \frac{-12}{v^n}e^{-3x}$.

This is the linear ordinary differential equation:

$p(x) = -(2v+1)$ and , $Q(x) = \frac{-12}{v^n}e^{-3x}$.

The integration factor is $\rho(x) = e^{\int p(x)dx} = e^{\int -(2v+1)dx} = e^{-(2v+1)x}$.

Therefore

$$T(x,v) = \frac{1}{\rho(x)} . \int \rho(x). Q(x)dx ,$$

$$T(x,v) = e^{(2v+1)x} . \int e^{-(2v+1)x} . \left(\frac{-12}{v^n} e^{-3x}\right) dx ,$$

$$T(x,v) = e^{(2v+1)x} . \left[\frac{-12}{v^n} \int e^{(-2v-4)x} dx \right] .$$

$$T(x,v) = e^{(2v+1)x} . \left[\frac{-12}{v^n(-2v-4)} e^{(-2v-4)x} + C \right]$$

$$T(x,v) = \frac{6}{v^{n+1}+2v^n} e^{-3x} + Ce^{(2v+1)x}$$

$$T(x,v) = \frac{6e^{-3x}}{v^n(v+2)} + Ce^{(2v+1)x} .$$

Since T is bounded, C ought to be zero. Taking the reverse SEE change, we have:

$$y(x,t) = 6e^{-3x}. e^{-2t} = 6e^{-2t-3x} .$$

Example (2): Consider the Laplace's equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 0 \quad u(x,0) = 0, \quad u_t(x,0) = \cos x \quad , x,t > 0 \quad \ldots (7) .$$

Let $T(v)$ be the SEE integral transform of u. Then, taking the SEE change of condition (7), we get:

$$v^2 T(x,v) - \frac{1}{v^n} u_t(x,0) - \frac{u(x,0)}{v^{n-1}} + T''(x,v) = 0 ,$$

$$T(x,v) + \frac{T''(x,v)}{v^2} - \frac{1}{v^{n+2}} \cos(x) = 0 ,$$

$$T(x,v) + \frac{1}{v^2} T''(x,v) = \frac{1}{v^{n+2}} \cos(x) ,$$

This is the second request differential condition have the specific, arrangement in the structure:

$$T(x,v) = \frac{\frac{1}{v^{n+2}} \cos(x)}{\frac{1}{v^2} D^2 + 1} = \frac{\frac{1}{v^{n+2}} \cos(x)}{\frac{1}{v^2}(-1)+1} = \frac{\frac{1}{v^{n+2}} \cos(x)}{1-\frac{1}{v^2}} = \frac{\frac{v^2}{v^{n+2}} \cos(x)}{v^2-1} = \frac{\frac{1}{v^n} \cos(x)}{v^2-1} = \frac{\cos(x)}{v^n(v^2-1)} \quad \ldots (8) .$$

Where $D^2 \equiv \frac{d^2}{dx^2}$.

If we take the inverse SEE integral transform for equation (8), we get arrangement of equation (7) in the form: $u(x,t) = \sinh(t) . \cos(x)$.

Example (3): Solve the wave equation: $u_{tt} = 4u_{xx}$, $u(x,0) = \sin(\pi x)$, $u_t(x,0) = 0$ $t,x > 0$ ... (9) .

Solution: taking the SEE integral transform for equation (9) and utilizing conditions, we have:

$$T''(x,v) - 4\left[v^2 T(x,v) - \frac{1}{v^n} u_t(x,0) - \frac{1}{v^{n-1}} u(x,0)\right] = 0 ,$$

$T''(x,v) - 4v^2 T(x,v) + \frac{4}{v^{n-1}} \sin(\pi x) = 0$,

$\frac{1}{4v^2} T''(x,v) - T(x,v) = \frac{-1}{v^{n+1}} \sin(\pi x)$.

So, $T(x,v) = \frac{\frac{-1}{v^{n+1}}\sin(\pi x)}{\frac{1}{4v^2}D^2 - 1} = \frac{\frac{-1}{v^{n+1}}\sin(\pi x)}{\frac{1}{4v^2}(-\pi^2) - 1} = \frac{\frac{-1}{v^{n+1}}\sin(\pi x)}{\frac{-\pi^2}{4v^2} - 1}$,

$T(x,v) = \frac{1}{v^{n+1}} \frac{4v^2}{\pi^2 + 4v^2} \sin(\pi x)$,

$T(x,v) = \frac{1}{v^n} \frac{v}{v^2 + \left(\frac{\pi}{2}\right)^2} \sin(\pi x)$.

Presently, we take the SEE change to track down the specific arrangement of condition (9) in the structure:

$u(x,t) = \cos\left(\frac{\pi}{2} t\right) . \sin(\pi x)$.

<u>Example (4):</u> Consider the homogeneous warmth condition in one measurement in a standardized structure:

$u_{xx} - 4u_t = 0$, $u(x,0) = \sin\left(\frac{\pi}{2} x\right)$, $x, t > 0$ ... (10) .

By using the SEE integral transform for equation (10), we have:

$T''(x,v) - 4\left[vT(x,v) - \frac{1}{v^n} u(x,0)\right] = 0$,

$T''(x,v) - 4vT(x,v) + \frac{4}{v^n} \sin\left(\frac{\pi}{2} x\right) = 0$,

$T''(x,v) - 4vT(x,v) = \frac{-4}{v^n} \sin\left(\frac{\pi}{2} x\right)$,

$T''(x,v) = \frac{\frac{-4}{v^n}}{\frac{-\pi^2 - 16v}{4}} . \sin\left(\frac{\pi}{2} x\right)$.

Solve for $T(x,v)$ we find the particular solution is:

$T(x,v) = \frac{16}{v^n(\pi^2 + 16v)} \sin\left(\frac{\pi}{2} x\right)$,

$T(x,v) = \frac{1}{v^n\left(\frac{\pi^2}{16} + v\right)} \sin\left(\frac{\pi}{2} x\right)$ ... (11) .

What's more, also, in the event that we take the converse SEE change for condition (11) we get the arrangement of condition (10) in the structure: $u(x,t) = e^{\frac{-\pi^2}{16} t} \sin\left(\frac{\pi}{2} x\right)$.

<u>Example (5):</u> Consider the linear telegraph equation: $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} + u$

Subject to initial conditions

$u(x,0) = e^x$

$u_t(x,0) = -2e^x$

Solution: Applying SEE transform we obtain the following

$$T''(x,v) - v^2T(x,v) + \frac{1}{v^n}u_t(x,0) + \frac{1}{v^{n-1}}u(x,0) - 2vT(x,v) + \frac{2}{v^n}u(x,0) - T(x,v) = 0$$

Applying the SEE transform to the initial conditions we have

$$T''(x,v) - v^2T(x,v) + \frac{1}{v^n}(-2e^x) + \frac{e^x}{v^{n-1}} - 2vT(x,v) + \frac{2e^x}{v^n} - T(x,v) = 0$$

We get

$$T''(x,v) - (v^2 + 2v + 1)T(x,v) = \frac{-e^x}{v^{n-1}}$$

$$\frac{1}{v^2+2v+1}T''(x,v) - T(x,v) = \frac{-e^x}{v^{n-1}\,(v^2+2v+1)}$$

$$\frac{1}{(v+1)^2}T''(x,v) - T(x,v) = \frac{-e^x}{v^{n-1}\,(v+1)^2}$$

$$T(x,v) = \frac{\frac{-e^x}{v^{n-1}\,(v+1)^2}}{\frac{1}{(v+1)^2}D^2-1}$$

$$T(x,v) = \frac{\frac{-e^x(v+1)^2}{v^{n-1}\,(v+1)^2}}{1-(v+1)^2}$$

$$T(x,v) = \frac{\frac{-e^x}{v^{n-1}}}{1-(v^2+2v+1)}$$

$$T(x,v) = \frac{-e^x}{v^{n-1}\,[-v^2-2v]}$$

$$T(x,v) = \frac{e^x}{v^n(v+2)}$$

Take inverse of SEE transform we get

$$u(x,t) = e^x.e^{-2t} = e^{x-2t} \ .$$

Example (6): Consider 2nd order linear homogenous (Klein-Gordan) equation

$$u_{tt} = u_{xx} + u_x + 2u \quad -\infty < x < \infty \quad t > 0$$

Subject to the initial conditions:

$$u(x,0) = e^x \quad , \quad u_t(x,0) = 0$$

Using the SEE integral transform, we have

$$S[u_{tt}] - S[u_{xx}] - S[u_x] - 2S[u\ ] = 0$$

$$v^2T(x,v) - \frac{e^x}{v^{n-1}} - T''(x,v) - T'(x,v) - 2T(x,v) = 0$$

$$-T''(x,v) - T'(x,v) + (v^2 - 2)T(x,v) = \frac{e^x}{v^{n-1}}$$

$$\frac{1}{(v^2-2)}T''(x,v)+\frac{1}{(v^2-2)}T'(x,v)-T(x,v)=\frac{-e^x}{v^{n-1}(v^2-2)}$$

$$T(x,v)=\frac{\frac{-e^x}{v^{n-1}(v^2-2)}}{\frac{1}{(v^2-2)}D^2+\frac{1}{(v^2-2)}D-1}=\frac{-e^x}{v^{n-1}[1+1-(v^2-2)]}$$

$$T(x,v)=\frac{-e^x}{v^{n-1}(2-v^2+2)}=\frac{-e^x}{v^{n-1}(-v^2+4)}=\frac{-e^x}{-v^{n-1}(v^2-4)}$$

$$T(x,v)=\frac{e^x}{v^{n-1}(v^2-4)}$$

Take inverse we have

$$u(x,t)=e^x\cosh(2t)\ .$$

4. Conclusion

In the present paper, a new integral transform namely SEE transform was applied to solve linear ordinary and partial differential equations with constant coefficients. Also, its applicability demonstrated using different partial differential equations (wave, heat, Laplace), we find the particular solutions of these equations.

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