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Studying the change in inclination and semi-major axis of the satellites for low earth orbits

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ABSTRACT

The main point of this paper is to evaluate the change in the inclination (i) and semi-major axis (a) due to tidal perturbation in orbital elements of a low Earth orbiting satellite LEO's. The orbital elements in the sense of Keplerian motion are affect perturbation in a satellite motion and change in the orbital elements must be employed to study the perturbations of the tidal effect on these satellites. These elements remain constant in the absence of perturbation where perturbed equation of motion was numerically integrated using Lagrange's formulas where numerical analysis is the most suitable method to analyze disturbances. The findings demonstrate that the tidal disruption of the orbital elements relies on the satellite's inclination the variation in the ratio $(\Delta \boldsymbol{i}/\boldsymbol{i})$ and $(\Delta \boldsymbol{a}/\boldsymbol{a})$ decreases with increasing the inclination of satellite, while it increases with increasing the time and the difference in inclination reduces as the satellite's inclination rises, and the difference in semi major axis increases as time increases.

Keywords: Orbital elements, tide perturbation, deformation of earth, LEO's, inclination, semi-major axis

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1. Introduction

Perturbations are variations from a natural, idealized, or unperturbed motion; numerical is the most reliable method to analyze disturbances. Due to disruptions induced by other bodies (including the Moon and Sun) and external forces considered in Keplerian motion, the real movement will differ from the theoretical two-body direction (including a nonspherical central body and drag). The influence of disturbances is investigated by three major approaches: basic disturbance techniques (utilizing computational methods), general disturbance methods (utilizing empirical techniques), and semi-analytical techniques. A satellite's orbit is influenced by disturbing factors, including accelerations originating from the central body, drag, pressure of solar radiation, third body, and several other forces, although most are very limited and typically ignored. As machine capabilities grow to be regarded, tides are becoming more significant [1-3]. In low Earth orbit, tidal forces induce measurable shifts in the satellites [4, 5]. The tidal deformation induced by the celestial bodies 'outside changes in the gravitational phenomenon; the force identified as a tidal force. For some point of the celestial body's external gravitational force in or on the surface world, the first equality gravity force influence at the center of mass and the second equality of the remainder could be separated into components. The tidal force tends to bend the electric potential surface; the forms are thereby elongated in the celestial bodies' resulting force (moon and sun). The solar capacity is around 46 percent of the lunar potential based on astronomical observation.

Two major modes of tides were studied in astrodynamics: solid-Earth and oceanic. Though its two kinds are separate, they derive from much of the same origins. Solid-Earth tides are the earth's distortions attributable to troubling powers. Internal effects act from the planet's inner framework, which provides complicated representations of the movements inside the earth with the solid and liquid characteristics of matter. Such powers are now starting to be investigated. Planet tides were simpler to measure and do not include the ocean tide model (as well as called body tides, such as Ray1998). Instead, the celestial tide-generating capacity (for example, the Sun and Moon's direct relevancy on Earth) is measured [6-8].

Different analytic, semi-analytic and numerical techniques are used for solving perturbed equations of motion many researchers over the past few decades were interest to study the motion of the satellite and its lifetime. Asma was developed a simulation for the orbit propagator for LEO satellite including dominant perturbations

for it. This simulation involves modeling the perturbing forces to optimize the propagation. Mishra, S. etal. Were determined orbit of the satellite by using a technique to measure position, velocity and behavior of orbital elements under the effect of the drag. Evaluated the perturbations in CubeSat's orbital elements for LEO satellite using Cowell's method. Mohammed and Abdul-Rahman were studied extensively the behavior of orbital elements by using different A/M, altitudes and eccentricities [19].
Low Earth Orbit LEOs are mainly circular in satellite geodesy. They can usually accommodate missions in the gravity region and are often used by constellations of contact satellites such as Iridium and Global Star. The orbital time at these altitudes ranges between (1.5 and 2) hours [17]. The satellite footprints radius is very limited and ranges between 2000 km and 4000 km (for example, the region on the surface from which the satellite is observable above the horizon) [12, 13]. To study the tidal effects on these satellites, we describe the velocity and position; change in the orbital components must be employed for determining the shape of major orbital semi-axis, $\boldsymbol{a}$, and eccentricity, $\boldsymbol{e}$, the inclination i, longitude of pericenter or argument of pericenter [9-11]
In the lack of perturbation, these elements remain constant. The last element provides the location of the satellite along its orbit can be described here by $M, t, v, e, \Omega,$

2. Theory of tidal perturbation

The word disrupted movement indicates that there is an undisturbed movement. The unperturbed movement in Celestial Mechanics is the orbital movement of two spherically symmetrical bodies, defined by movement formula, whose solution is understood in terms of basic analytical functions[14].
The basic Keplerian formula of movement is

$$\ddot{r} = \frac{GM}{r^3} r + k_s$$

The integration of this formula properly provides the solutions

$$r(t) = r(t\,;\, a_1, \dots, a_6), \qquad \dot{r}(t) = \dot{r}(t\,;\, a_1, \dots, a_6)$$

With $a_1, \dots, a_6$ being free selectable integration constants. Preferably, the Kepler elements a, e, i, ω, M are utilized.
A certain amount of additional powers are currently acting on the near-Earth satellite. They are referred to as destructive forces to differentiate them from the central force (accelerating the central body). Because of these powers, the satellite encounters additional speeding merged into a corresponding disturbing vector k_s. The extended movement formula is written as

$$\ddot{r} = -\frac{GM}{r^3} r + k_s$$

Whereas k_s are forces of perturbing are in specific responsible for:

1. Acceleration since the inhomogeneous and non-spherically mass distributing within the earth (central body), $\ddot{\boldsymbol{r}}_E$.
2. Acceleration since other celestial bodies (planets, Moon and Sun), mostly $\ddot{\boldsymbol{r}}_S, \ddot{\boldsymbol{r}}_M$.
3. Acceleration since earth and oceanic tides, $\ddot{\boldsymbol{r}}_e, \ddot{\boldsymbol{r}}_O$.
4. Acceleration since atmospheric drag, $\ddot{\boldsymbol{r}}_D$.
5. Acceleration since direct and Earth-reflected solar radiation pressure, $\ddot{\boldsymbol{r}}_{SP}, \ddot{\boldsymbol{r}}_A$.

The forces of perturbing lead to cause 1-3 refer to nature gravitational; the residual forces are non-gravitational. The total force is

$$k_s = \ddot{\boldsymbol{r}}_E + \ddot{\boldsymbol{r}}_S + \ddot{\boldsymbol{r}}_M + \ddot{\boldsymbol{r}}_e + \ddot{\boldsymbol{r}}_O + \ddot{\boldsymbol{r}}_D + \ddot{\boldsymbol{r}}_{SP} + \ddot{\boldsymbol{r}}_A.$$

A graphical summary of the perturbing forces and acceleration is given in Figure (1). The resultant overall acceleration is based on the direction r of the satellite, i.e., the quantity that must first be calculated as a length function from the formula's solution (1). A Keplerian movement with time different elements $a(t), e(t), i(t), \omega(t), (t), \vec{M}(t)$ could be viewed as the disrupted satellite movement [18].

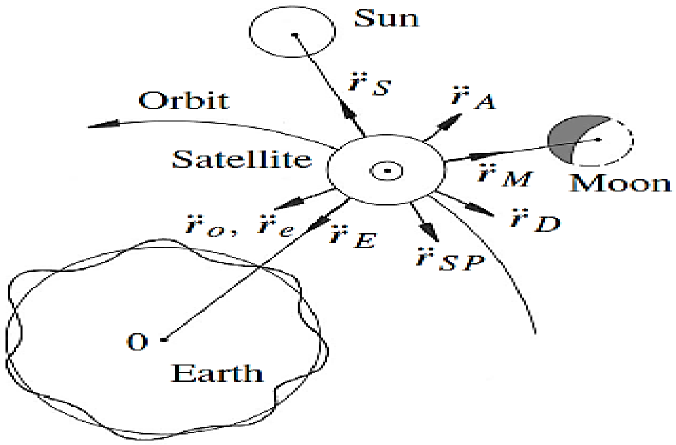

Figure 1. Perturbation forces acting on a satellite [7]

The interaction between the working forces of perturbing and the duration-dependent elements of orbital differences requires the simple formulas expressed by Lagrange. The cumulative energy of satellite movement is defined by

$$E_M = \frac{v^2}{2} - \frac{GM}{r} = -\frac{GM}{2a}$$

The negative terminal of the whole energy, $GM/2a$, is called F, the *functional force*.
With V= capacity as the negative magnitude of the capacity energy and the kinetic energy= T symbol, the next expression of the force function has been found.
$F = V - T$.
In a non-central field force

$$V = \frac{GM}{r} + R$$

$$F = \frac{GM}{r} + R - T = \frac{GM}{2a} + R$$

The R functions have all V components without the central term GM/r and are named the *potential disturbing* or function of disturbing. For the completeness sake an alternative expression of the formula of movement in a non-central field force has been specified [12][19].
$\ddot{r} = \text{grad}\, V = \nabla V$
With perturbation of Lagrange's formulas an association between the potential of disturbing R and the elements of orbital differences is established[15, 16].

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial R}{\partial \vec{M}},$$

$$\frac{de}{dt} = \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial \vec{M}} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega},$$

$$\frac{d\omega}{dt} = -\frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i} + \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e},$$

$$\frac{di}{dt} = -\frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \omega} - \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \Omega},$$

$$\frac{d\Omega}{dt} = -\frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i},$$

$$\frac{d\vec{M}}{dt} = n - \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial e} - \frac{2}{na}\frac{\partial R}{\partial \mathrm{a}}.$$

Where R is the disturbing potential cf. changed to the center of primary mass with the root of the coordinate scheme,

$$R = \frac{GM}{r}(\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n (C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)P_{nm}(\cos\vartheta)$$

the terms with m=0,1 and *n*=1 change to zero, the coefficients are called

when zonal *m=0*,
when tesseral *n=0* and
when sectorial *m=n*.

In the *first stage* the possible disturbing

$$R = \sum_{n=2}^{\infty}\sum_{m=0}^{n} R_{nm}$$

is re-formulated as f the orbital features function:

$$R_{nm} = \frac{GMa_e^n}{a^{n+1}}\sum_{p=0}^{n} F_{nmp}(i)\sum_{q=-\infty}^{+\infty} G_{npq}(e)S_{nmpq}\big(\omega, \vec{M}, \Omega, \Theta\big).$$

Θ is the sidereal time of Greenwich.

3. Result and discussion

The movement formulas utilized in astrodynamics are not easy to solve since they are typically coupled structures of first or second orders, nonlinear, differential formulas that have for the last 300 years defied direct solution. However, advanced machines make it easy for us to utilizing numerical methods. As contrasted with empirical approaches, numerical techniques are characterized by their simplicity and general applicability. With the usage of advanced computing methods, numerical initiative plays only a minor function. This is why computational techniques are still utilized nearly entirely in satellite geodesy orbit computations. The osculating elements of orbital for TDRS's satellite demonstrated in a table (1), figures (2-5) shows the variation in the inclination of the above satellites since the perturbation of central body including tidal of the Solid Earth over a short period 2 day corresponding on about 25 revolutions around the earth and period for 2, 4and 8 days. From these figures, we calculate the variation ($\Delta i/i$), the result is given in table (2) shows that as the inclinations of satellites increase, the ($\Delta i/i$) decrease. Figures (6-9) shows the periodic perturbation on the major semi-axis, and the variation in ($\Delta a/a$) remains constant over by period 1, 2, 4 and 8 days as given in Table (3) the semi major axis has a secular decreasing behavior as well as a simple periodic. Figure (10) show the perturbation effect on another satellite, SCD1 and SCD 2, over a short period (2, 6, and 10 days); the orbital elements are given in Table (4). From Figure, we get the inclination of satellites have a significant function in the effect of perturbation and the result of the variation in ($\Delta i/i$) given in Table (5), the result shows that the variation decrease as the inclination of the satellites increase, while figure (11-12) show the periodic perturbation on the semi-major axis over periods 2, 6 and 10 days. Table (6) gives the variation in ($\Delta i/i$) as the satellites' variation increases around the earth. Also, table (6) shows ($\Delta a/a$) for SCD's satellites throughout the revolution. Not that the time was calculated by Julian time in the paper.

Table 1. osculating orbital elements for (TDRS's) satellites.

TDRS 3	TDRS 8	TDRS 10
$i = 14.0780$	$i = 9.5085$	$i = 7.2189$
$\Omega = 355.8492$	$\Omega = 46.3158$	$\Omega = 52.7840$
$e = 0.0039492$	$e = 0.0016976$	$e = 0.0008299$
$w = 319.6069$	$w = 315.4982$	$w = 270.6012$
$a = 6886$	$a = 6607$	$a = 6845$

Table 2. Change in inclination of (TDRS's) satellites for different times

Satellite	$\boldsymbol{i(degree)}$	$\boldsymbol{(\Delta i/i)}$			
		1 day	**2 days**	**4 days**	**8 days**
TDRS 3	14.0780	0.024	0.025	0.026	0.028
TDRS 8	9.5085	0.025	0.021	0.022	0.024
TDRS 10	7.2189	0.031	0.014	0.015	0.02

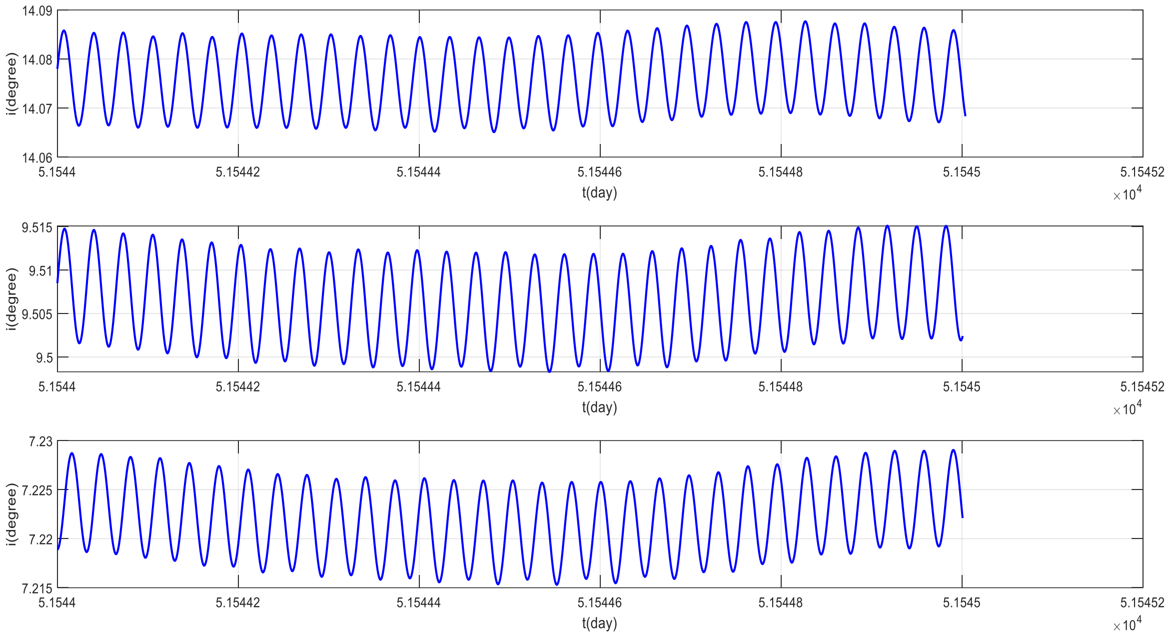

Figure 2. Change in inclination of TDRS 3, TDRS 8and TDRS 10 for 1 day

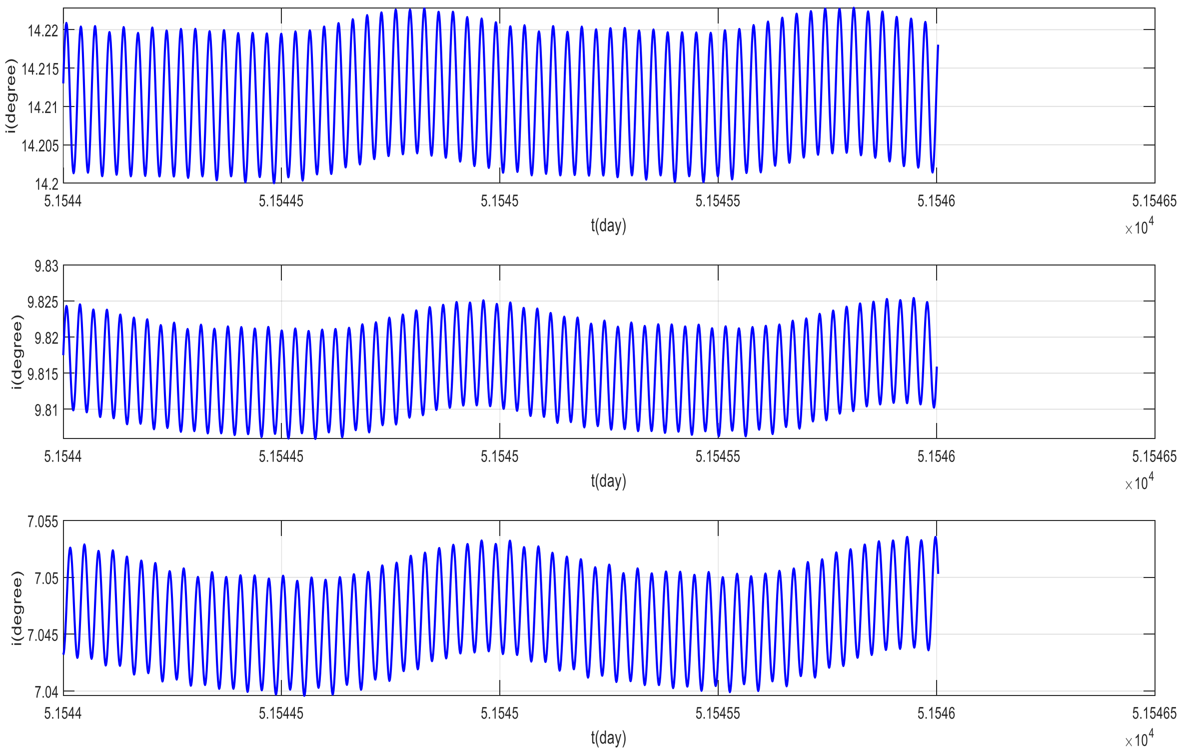

Figure 3. Change in inclination of TDRS 3, TDRS 8and TDRS 10 for 2 days

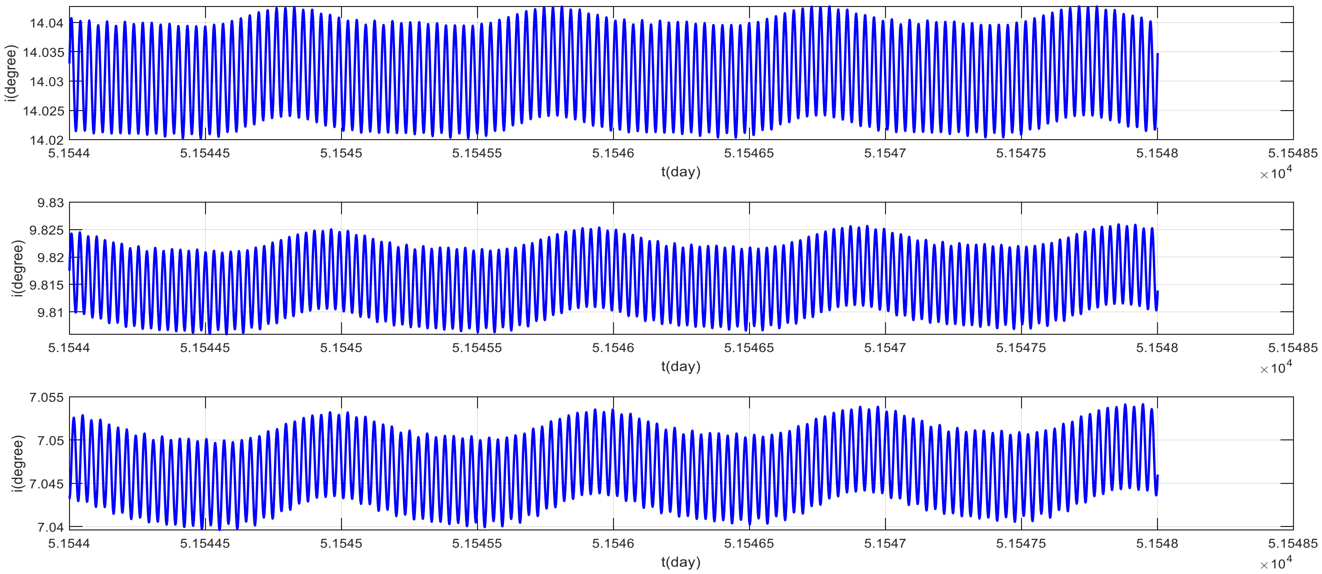

Figure 4. Change in inclination of TDRS 3, TDRS 8and TDRS 10 for 4 days.

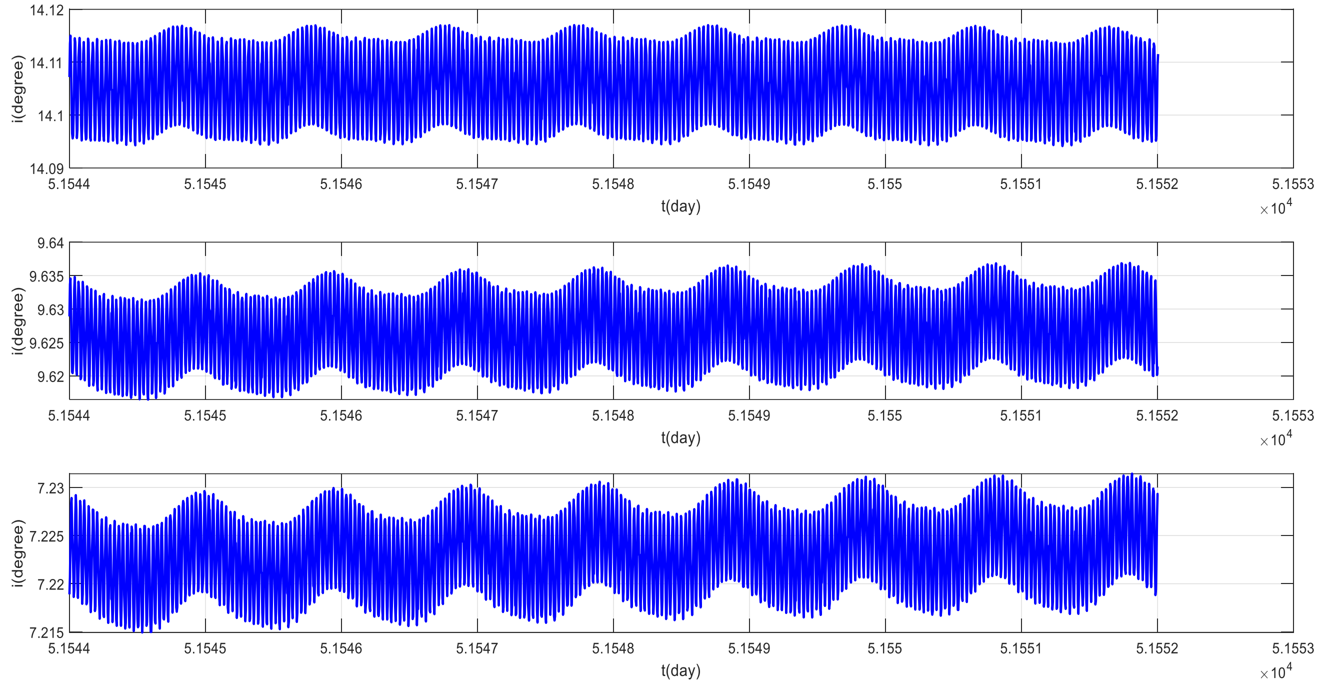

Figure 5. Change in inclination TDRS 3, TDRS 8and TDRS 10 for 8 days.

Table 3. change in a semi-major axis of (TDRS's) satellite for different times

Satellite	$a(Km)$	$(\Delta a/a)$			
		1 day	**2 days**	**4 days**	**8 days**
TDRS 3	6886	0.0015	0.0016	0.0018	0.00185
TDRS 8	6607	0.0008	0.00087	0.0009	0.00092
TDRS 10	6845	0.0006	0.0008	0.00082	0.0009

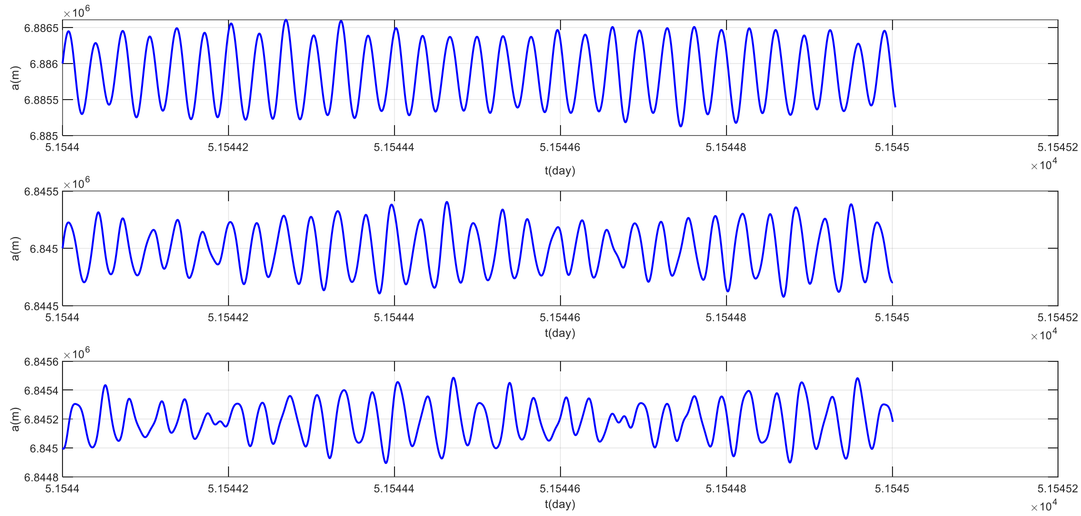

Figure 6. perturbation on a semi-major axis of TDRS 3, TDRS 8and TDRS 10 for 1 day

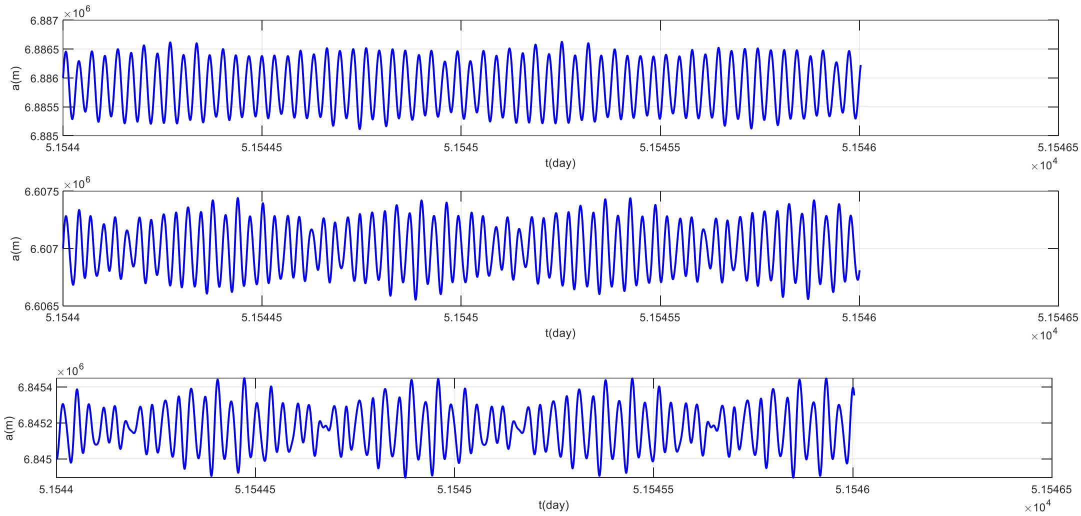

Figure 7. perturbation on a semi-major axis of TDRS 3, TDRS 8and TDRS 10 for a 2 day

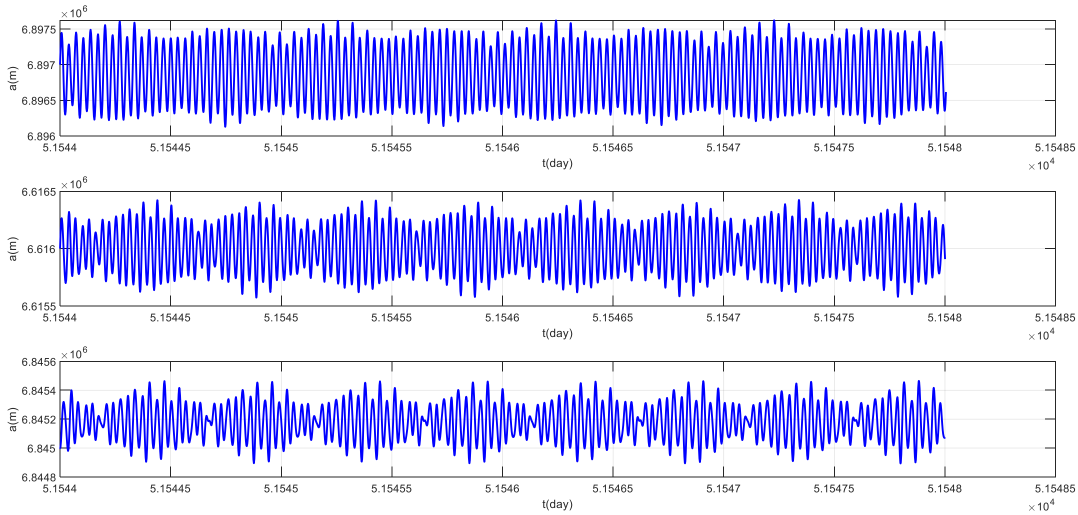

Figure 8. Perturbation on a semi-major axis of TDRS 3, TDRS 8and TDRS 10 for 4 day

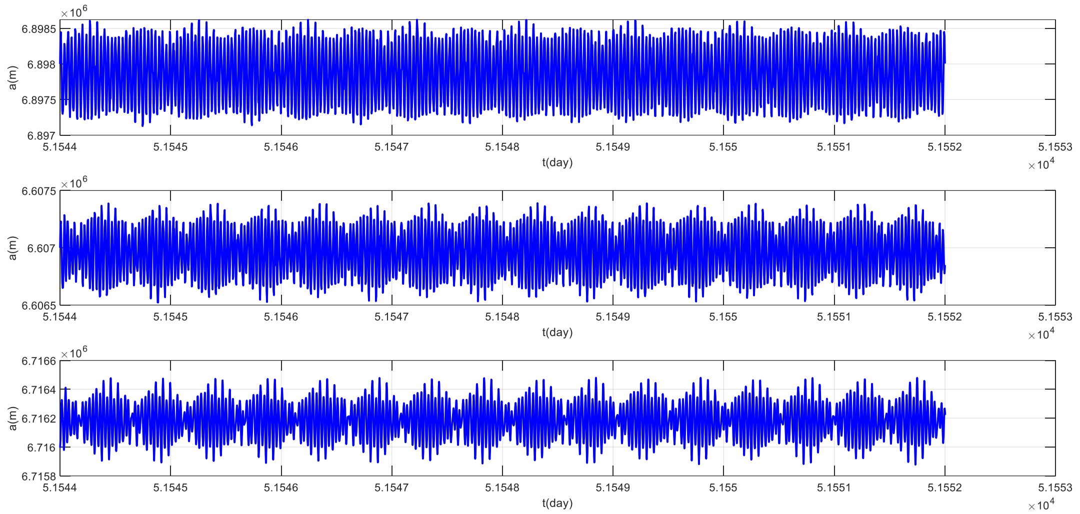

Figure 9. Perturbation on a semi-major axis of TDRS 3, TDRS 8and TDRS 10 for 8 day

Table 4. osculating orbital elements for SCD's satellites

SCD 1	**SCD 2**
$i = 24.9705$	$i = 24.9944$
$RA = 297.6394$	$RA = 148.9115$
$e = 0.0042799$	$e = 0.0017015$
$AP = 140.9110$	$RP = 257.4520$
$a = 6600$	$a = 6670$

Table 5. change in inclination of SCD's satellites for different times

Satellite	$\boldsymbol{i}$ **(*degree*)**	$(\boldsymbol{\Delta i/i})$		
		2 days	**6 days**	**10 days**
SCD 1	24.9705	0.0032	0.0036	0.0039
SCD 2	24.9944	0.0037	0.0038	0.0038

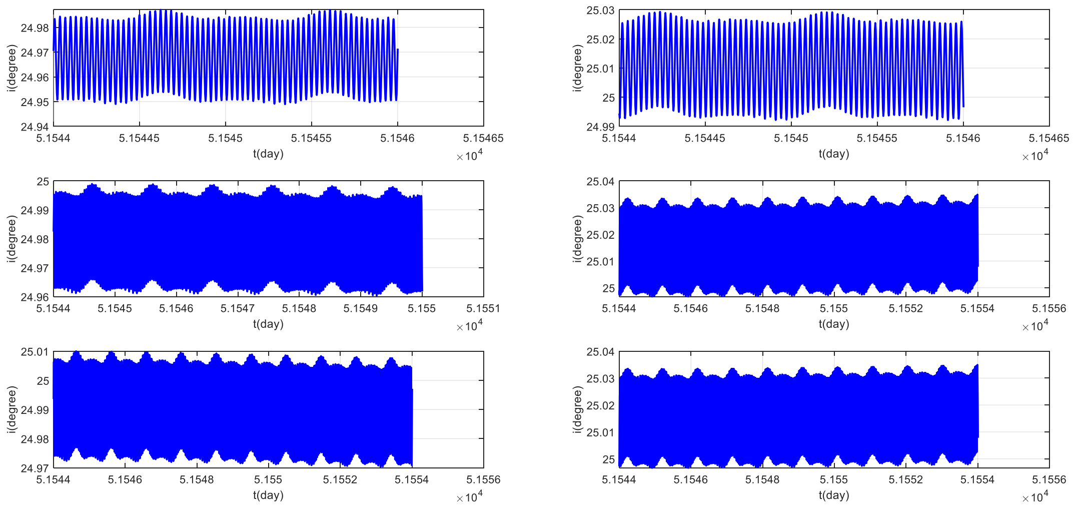

Figure 10. Change in inclination of SCD 1 and SCD 2 for 2, 6, and 10 days

Table 6. perturbation on a semi-major axis of SCD's satellite for a different time

Satellite	$a(Km)$	$(\Delta a/a)$		
		2 days	6 days	10 days
SCD 1	6600	0.0037	0.0039	0.0043
SCD 2	6670	0.0052	0.0061	0.0061

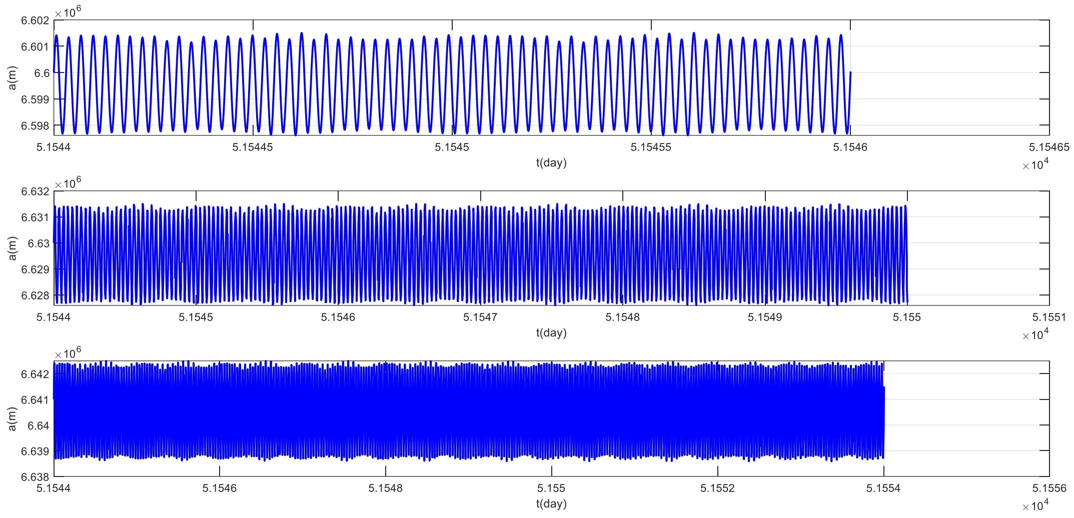

Figure 11. Perturbation on a semi-major axis of SCD 1 for 2, 6, and 10 days

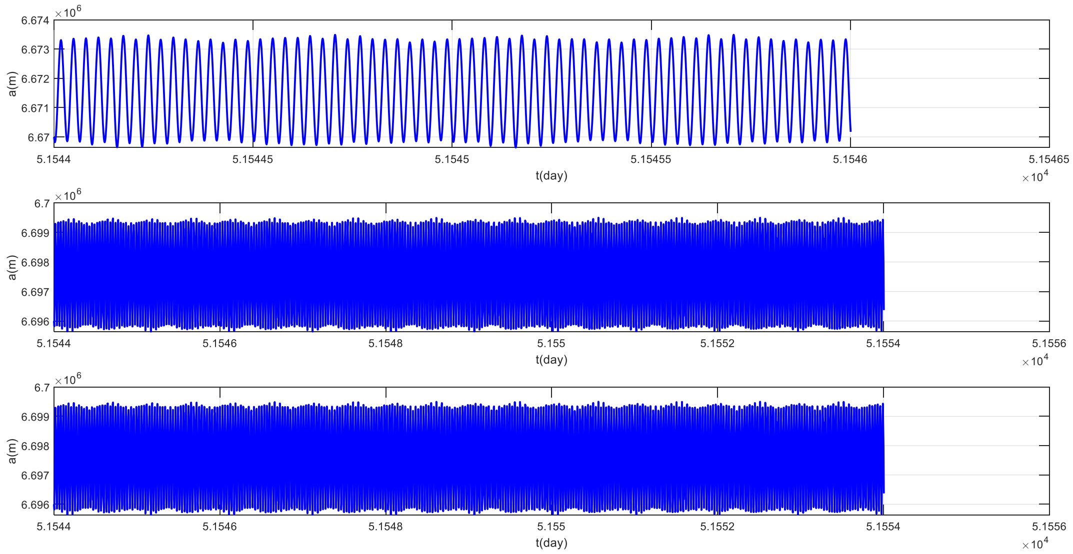

Figure 12. Perturbation on a semi-major axis of SCD 2 for 2, 6, and 10 days

4. Conclusions

The earth tidal deformation cause perturbations in the satellite motion near to earth, in this paper, variation in inclination of TDRS's and SCD's satellites are computed. The results show that the variation in $(\Delta i/i)$ and $(\Delta a/a)$ decreases with increasing the inclination of a satellite, we concluded that the tidal disruption on the elements of orbital relies on the satellite's inclination, and with growing time, we also get the difference in $(\Delta i/i)$ growth. The solid Earth tide changes the inclination of the orbit about at about 3.5× 10−3 equivalent to about 2 seconds in one revolution and it should be mentioned that the solid Earth tide does not much affect the medium height and geostationary satellites. The perturbations were calculated and presented on orbital elements. To analyze disturbances numerical analysis is the most suitable method. The perturbing accelerations acting on a low Earth orbiting satellite were investigated using the second-order vector differential equations of motion of a satellite.

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