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Estimation of the reliability system in model of stress- strength according to distribution of inverse Rayleigh

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ABSTRACT

In many real-life scenarios, stress-strength model is a significant level that notices efficincy in reliability system. Thus, this paper considers the stress–strength model with reliability estimation R=P(X<Y) based on the Distribution of Inverse Rayleigh (IRD). Few classical methods of estimation such as; Likelihood as Maximum (MLE), Uniformly Unbiased Minimum Variance estimator (UMVUE), and Moment method (MOM), and three types of shrinkage weight factors estimation methods were compared. Also, a simulation of Monte Carlo is utilized for comparing among proposed methods based on Mean Square Error (MSE).

Keywords: Reliability of stress–strength, Method of MLE, method of Moment, Uniformly Unbiased minimum variance method, Method of shrinkage

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1. Introduction

Numerous researches have been conducted in previous years for estimation and providing bounds for confidence for component reliability utilizing possibile arguments of a definite failure physical model. However, the model of stress-strength (S-S) define the component life as subjected to stress as Y with X strength. The component fails when the stress applied to it which surpasses the strength, while the component works whenever Y less than X. The reliable probability of the stress-strength is denoted via:

$$R = p(stress < strength), \quad \text{or}$$

$$R = p(Y < X) \tag{1}$$

Many applications in different areas of science such as statistics, engineering, structural and aircraft take benefits by applying the S-S model. In 1956, Birnbaum was the first who regarded stress-strength model [23]. After that, Church and Harris (1970) introduced R estimation if X and Y are distributed normally [7]. Generally, supposed X and Y are 2 variables being randomly independent; where X represents the strength while the stress is represented by the random variable Y [4], [6], [8], [9], [13], [14].

Instead, IRD can be regarded as an approximate of several experimental units ~~s~~ type's distributions of lifetimes. This distribution was first suggested by Trayer [17]. In 1972, Voda presented for IRD, some MLE estimator properties [22]. Gharraph offered five location measures for IRD; these measures are mean of harmonic, mode, mean, median, and geometric mean. Also, the unknown parameter was estimated utilizing various estimation methods [12]. In 1996, Mukarjee and Maitim took the percentile Inverse Rayleigh estimator as Parameter [20]. Abdel-Monem suggested few results prediction of and estimation for the IRD [1]. [2] proposed fourloss functions to develope Bayesian estimators of the parameter based on IRD. Ref. [5] proposed a model for lower record value based on the IR distribution.

Among these models, IRD offers flexibility as larger in data of modeling complex where the obtained results appear quite genuine and sound. Therefore, this paper estimates the stress and strength reliability if the strength and stress follow one parameter IRD via various estimation methods.
The probability function of density (PDF) of A R.V. X follows one parameter IRD ($X\sim IRD(\alpha_1)$) is given by [16];

$$f(x;\alpha)=\frac{2\alpha}{x^3}\exp\left(\frac{-\alpha}{x^2}\right) \quad ; \ x>0 \ \ , \ \alpha 1>0 \tag{2}$$

Since α is the scale IRD parameter, the cumulative corresponding distribution function (CDF) is:

$$F(x;\alpha)=\exp\left(-\frac{\alpha}{x^2}\right) \quad ; x>0 \ \ , \ \alpha 1>0 \tag{3}$$

Consequently, when $Y\sim IRD(\alpha_2)$, after that the PDF of Y turns as

$$g(y;\ \alpha_2)=\frac{2}{x^3}\exp\left(\frac{-\alpha_2}{y^2}\right) \quad ; \ y>0 \ \ , \alpha_2>0 \tag{4}$$

Thus, $R=P(Y<X)$

$$=\int_0^\infty\int_0^x f(x)g(y)dxdy$$

$$=\int_0^\infty F_y(x)f(x)dx$$

$$=\int_0^\infty \exp\left(-\frac{\alpha_2}{y^2}\right)\ \frac{2\alpha_1}{x^3}\exp\left(\frac{-\alpha_1}{x^2}\right)dx$$

$$=\int_0^\infty \frac{2\alpha_1}{x^3}\exp\left(\frac{-(\alpha_1+\alpha_2)}{x^2}\right)dx$$

Then, $R=\frac{\alpha_1}{\alpha_1+\alpha_2}$ (5)

2. Classical estimation methods of R

2.1. MLE estimation:

The MLE estimation is most significant and diffuse methods of parameter estimation which was first introduced by [18]. Most statisticians prefer this estimation when the sample size is large [10]. The principle behind this method is that for the X random variable if (x_1, x_2, x_3,…,x_n) are the n observation or sample values and for the random variable Y if (y_1,y_2,y_3,…,y_m) are the m observation or sample values then the estimated value of the parameters is the value most likely to produce the observed values.

$$L(x,y,\alpha_1,\alpha_2)=\prod_{i=1}^n f(x_i).\prod_{j=1}^m g(y_j) \tag{6}$$

$$=\prod_{i=1}^n\frac{2\alpha_1}{xi^3}\exp\left(\frac{-\alpha_1}{xi^2}\right) \ .\prod_{j=1}^m\frac{2\alpha_2}{yj^3}\exp\left(\frac{-\alpha_2}{yj^2}\right)$$

$$=(2\alpha_1)^n\prod_{i=1}^n\frac{1}{x^3}\exp\left(-\alpha_1\sum_{i=1}^n\frac{1}{x^2}\right).(2\alpha_2)^m\prod_{i=1}^m\frac{1}{y^3}\exp\left(-\alpha_1\sum_{i=1}^m\frac{1}{y^2}\right) \tag{7}$$

Considering log. of (7) and then differentiating partially the result with regard to $\alpha_1,\alpha_{2,}$respectively to get below:

$$\frac{\partial lnL}{\alpha_1}=\frac{n}{\alpha_1}-\sum_{i=1}^n\frac{1}{x_i^2} \tag{8}$$

And

$$\frac{\partial lnL}{\alpha_2}=\frac{m}{\alpha_2}-\sum_{j=1}^m\frac{1}{y_j^2} \tag{9}$$

Equalized (8) and (9) to zero, we obtained the following:

$$\hat{\alpha}_{1MLE}=\frac{n}{T_1} \quad , \text{where } T_1=\sum_{i=1}^n\frac{1}{x_i^2} \tag{10}$$

And

$$\hat{\alpha}_{2MLE}=\frac{m}{T_2} \quad , \text{where } T_2=\sum_{j=1}^m\frac{1}{y_j^2} \tag{11}$$

Substituted (10), (11) in (5) for obtaining MLE estimator for reliability of stress-strength (R) for IRD as following:

$$\hat{R}_{MLE}=\frac{\hat{\alpha}_{1MLE}}{\hat{\alpha}_{1MLE}+\hat{\alpha}_{2MLE}} \tag{12}$$

2.2. Method of moments

The idea of the moment's method estimator is to use the sample moments as estimators for the distribution parameters [19]. In this subsection, there are two populations of IRD X and Y with unknown scale parameters $\alpha_1,\alpha_{2,}$ respectively, the k^{th} moments of the IRD population is giving as:

$E(x^k) = \Gamma\left(1-\frac{k}{2}\right)\alpha 1^k \qquad k = 1,2, \dots\dots$

$E(y^k) = \Gamma\left(1-\frac{k}{2}\right)\alpha 2^k \qquad k = 1,2, \dots\dots$

(13)

Thus, the first moment (mean) of X and Y are, respectively

$E(x) = \Gamma\left(\frac{1}{2}\right)\alpha_1 \qquad \alpha_1 > 0$

$E(y) = \Gamma\left(\frac{1}{2}\right)\alpha_2 \qquad \alpha_2 > 0$

While the sample mean of X and Y are respectively as follow:

$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$\bar{Y} = \frac{1}{m}\sum_{i=1}^{m} y_i$

Equalize the populations mean with the sample mean, the estimates of α_1, α_2 become

$\hat{\alpha}_{1MOM} = (\frac{1}{n}\sum_{i=1}^{n} x_i)/\Gamma\left(\frac{1}{2}\right)$ (14)

$\hat{\alpha}_{2MOM} = (\frac{1}{m}\sum_{i=1}^{m} y_i)/\Gamma\left(\frac{1}{2}\right)$ (15)

Where $\Gamma\left(\frac{1}{2}\right) = \sqrt[2]{\pi}$

Substituted (14), (15) in (5) to obtainmoment estimator for stress-strength reliability (R) for IRD as following:

$\hat{R}_{MOM} = \frac{\hat{\alpha}_{1MOM}}{\hat{\alpha}_{1MOM}+\hat{\alpha}_{2MOM}}$ (16)

2.3. UNIFORMLY UNBIASED MINIMUM VARIANCE ESTIMATORS (UMVUE)

The UMVUE has a vital role in the theory of point estimation. Such method depends on minimizing the mean square error among estimators being unbiased where estimator $\hat{\alpha}$ as unbiased of α is named (UMVUE) when and only when Var ($\hat{\alpha}$) ≤ Var ($\hat{\alpha}_{ub}$) for whatever $x \in X$ and whatever other estimator as unbiased of α [11]. To find the UMVU of the scale parameter α_1, α_2 of X and Y as random variables respectively of IRD which fits the class of exponential or family of exponential that densities-containing at the form

$f(x;\alpha) = a(\alpha)b(x)\exp\left(\sum_{j=1}^{m}\rho_j(\alpha)k_j(x)\right)$, since $a(\alpha), b(x) > o, \alpha < x < \beta$ and $\alpha = \alpha_1\alpha_2, \dots, \alpha_k$ with $\gamma_j < \alpha_j < \delta_j$ and each of $\alpha, \beta, \gamma_j and \delta_j$ are constant

Put $a(\alpha) = 2\alpha$; $b(x) = 1/x^3$; $\rho_j(\alpha) = -\alpha$; $k_j(x) = \frac{1}{x^2}$

Thus, T_i is a sufficient complete statistic for (α_i) for i=1, 2.

For distribution finding of $T_1 = \sum_{i=1}^{n}\frac{1}{x_i^2}$ and $T_2 = \sum_{j=1}^{m}\frac{1}{y_j^2}$, suppose $z_1 = \frac{1}{X^2}$, $z_2 = \frac{1}{y^2}$ consequently, $X = \frac{1}{\sqrt[2]{z1}}$ and $y = \frac{1}{\sqrt[2]{z2}}$

$\xi(z_1) = f(X = \frac{1}{\sqrt[2]{z1}})\frac{dx}{dz_1}$ (17)

$\xi(z_2) = g(Y = \frac{1}{\sqrt[2]{z2}})\frac{dy}{dz_1}$ (18)

Substitute (2) in (17) and (4) in (18) to get

$\xi(z_1) = \alpha_1\exp(-\alpha_1 z_1)$ and $\xi(z_2) = \alpha_2\exp(-\alpha_2 z_2)$ (19)

Clear that $Z_1 \sim Exp(\alpha_1)$ and $Z_2 \sim Exp(\alpha_2)$, hence $T_1 \sim \Gamma(n, \alpha_1)$ and $T_2 \sim \Gamma(n, \alpha_2)$ with the following density functions

$U(t_1) = \frac{\alpha_1^n}{\Gamma(n)}t^{n-1}.\exp(-\alpha_1 t_1) \quad ; \quad t_1 > 0, \alpha_1 > 0, \ n > 0$ (20)

$U(t_2) = \frac{\alpha_2^m}{\Gamma(m)}t^{m-1}.\exp(-\alpha_2 t_2) \quad ; \quad t_2 > 0, \alpha_2 > 0, \ m > 0$ (21)

Thus $E\left(\frac{1}{T_1}\right) = \frac{\alpha_1}{n-1}$

So, the unbiased estimator of (α_1) is $(\frac{n-1}{T_1})$, therefore according to theorem of Lehmann-Scheffe (UMVUE) of (α_1) is

$$\hat{\alpha}_{1(UMVU)} = \frac{n-1}{T_1} \quad (23)$$

By the same way we can obtain (UMVUE) of (α_2) as below

$$\hat{\alpha}_{1(UMVU)} = \frac{m-1}{T_2} \quad (24)$$

Substituted (23), (24) in (5) to obtain UMVU estimator for stress-strength reliability in IRD as following:

$$\hat{R}_{UMVU} = \frac{\hat{\alpha}_{1(UMVU)}}{\hat{\alpha}_{1(UMVU)}+\hat{\alpha}_{1(UMVU)}} \quad (25)$$

3. Shrinkage estimation method (Sh)

Shrinkage technique was described for the first time by Thompson in 1968 for the univariate population mean (α) depending on prior knowledge of unknown parameters where there are two extreme mean values that can be combined to make one more centralized mean value by using shrinkage weight factor $K(\hat{\alpha})$; $0 \leq K(\hat{\alpha}) \leq 1$ via the formula

$$\hat{\alpha}_{sh} = K(\hat{\alpha})\alpha_{ub} + (1 - K(\hat{\alpha}))\alpha_0 \quad (26)$$

Where, α_{ub} is estimator as unbiased of α which is distinct in subsection (2.2) above and α_0 is an initial estimate as the closed value of α that will be considered as prior information, while the weight factor $K(\hat{\alpha})$ can be considered as a function of unbiased estimator$\hat{\alpha}_{ub}$, or as a constant or it might be detected via minimizing MSE of $\hat{\alpha}_{sh}$. Furthermore, $K(\hat{\alpha})$ mentions the belief inα_{ub}, and $(1 - K(\hat{\alpha}))$ symbolizes to approve of α_0 notice [2], [18] and [22].

Observe, $E\left(\hat{\alpha}_{iub} = \frac{\omega-1}{T_i}\right) = \alpha$ and $var\left(\hat{\alpha}_{iub} = \frac{\omega-1}{T_i}\right) = \frac{\alpha_i^2}{\omega-2}$ (27)

Since, i=1, 2 and ω mention n or m, respectively based on i.

3.1. Constant shrinkage weight factor (CSh_{wf})

At this part, the constant shrinkage assumption of the weight factor is as following:

$K(\hat{\alpha}_1) = c_1 = 0.2$ and $K(\hat{\alpha}_2) = c_2 = 0.2$

Then substitute in formula (26) to get the following shrinkage estimators

$$\hat{\alpha}_{1_{sh1}} = c_1\hat{\alpha}_{1ub} + (1 - c_1)\alpha_{1_0} \quad (28)$$

$$\hat{\alpha}_{2_{sh1}} = c_2\hat{\alpha}_{2ub} + (1 - c_2)\alpha_{2_0} \quad (29)$$

Where, αi_0 (i=1, 2) are prior information of $\alpha_i$as we mentioned above.

Substitute (28) and (29) in formula (5) to get the S-S reliability estimation (R) utilizing estimator shrinkage of $\hat{R}_{sh1}$ as

$$\hat{R}_{sh1} = \frac{\hat{\alpha}_{1_{sh1}}}{\hat{\alpha}_{1_{sh1}}+\hat{\alpha}_{2_{sh1}}} \quad (30)$$

3.2. Shrinkage weight function (Sh_{wf})

Sh_{wf} can be discussed as n and m functions, respectively in formula (26) as bellow:

$K(\hat{\alpha}_1) = \frac{\sin n}{n} = c_3$ and $K(\hat{\alpha}_2) = \frac{\sin n}{n} = c_4$

So we get

$$\hat{\alpha}_{1sh_2} = c_3\alpha_{1ub} + (1-c_3)\alpha_{1_0} \quad (31)$$

$$\hat{\alpha}_{2sh_2} = c_4\alpha_{2ub} + (1-c_4)\alpha_{2_0} \quad (32)$$

Substitute (31) and (32) in formula (5) to get the S-S reliability estimation (R) utilizing estimator shrinkage of $\hat{R}_{sh2}$ as

Thus, $\hat{R}_{sh2} = \frac{\hat{\alpha}_{1_{sh2}}}{\hat{\alpha}_{1_{sh2}}+\hat{\alpha}_{2_{sh2}}}$ (33)

3.3. Thompson modified type shrinkage weight function (MTShwf)

At this part, we propose the modification as follow to the weight shrinkage factor of Thompson estimator type

$$\Theta(\hat{\alpha}_{iub}) = \frac{(\hat{\alpha}_{iub}-\hat{\alpha}_{i_0})^2}{(\hat{\alpha}_{iub}-\hat{\alpha}_{i_0})+var(\hat{\alpha}_{iub})}(0.01) \quad for\ i = 1,2$$

Thus, the Thompson modified type as an estimator of shrinkage will be

$$\hat{\alpha}_{iTH} = \Theta(\hat{\alpha}_i)\hat{\alpha}_{iub} + (1-\Theta(\hat{\alpha}_i))\alpha_{i_0} for\ i = 1,2 \quad (34)$$

Now, to get the Thompson modified type shrinkage estimation of the (S-S) reliability substitute formula (34) in the formula (5) as below

$$\hat{R}_{TH} = \frac{\hat{\alpha}_{1_{TH}}}{\hat{\alpha}_{1_{TH}}+\hat{\alpha}_{2_{TH}}} \quad (35)$$

4. Computational study and numerical results

4.1. Mote Carlo Simulation (MCS)

MCS was used to investigate the concert comparison between the different reliability estimators which is called; MLE, MOM, UMVUE, CSh_{wf}, CSh_{wf}, and $MTSh_{wf}$ in this subsection. Different samples were utilized sizes = 10, 25, 50 and 75, based on MSE criteria with 1000 trials. The steps of MCS for this purpose as follows;

Step1: Generate random samples as$u_1, u_2, \dots, u_n$ which follow the continuous distribution uniform which well-defined on the interval (0, 1).

Step2: Initialize random samples follow the uniform continuous distribution over the interval (0, 1) as $w_1, w_2, \dots, w_m$

Step3: Transforming the mentioned random uniform samples to samples as random following IRD utilizing the (CDF) as following;

$$F(x) = e^{\frac{-\alpha}{x^2}}$$

$$Ui = e^{\frac{-\alpha}{x^2}}$$

$$x_i = [\alpha_1/-\ln(Ui)]^{\frac{1}{2}}$$

And, by the same method, we get

$$y_i = [\alpha_2/-\ln(Ui)]^{\frac{1}{2}}$$

Step4: Recalling R from formula (5).

Step5: finding R of the MLE, MOM, UMVUE using formulas (12), (16), and (25), respectively.

Step 6: Compute CSh_{wf}, Shwf, and $MTSh_{wf}$ estimators of R using formulas (30), (33) and (35), respectively.

Step 7: According to replication of (L=1000), compute MSE as following:

$$\text{MSE} = \frac{1}{L}\sum_{i=1}^{L}(\hat{R}_i - R)^2$$

4.2. Results analysis

In this section, numerical reliability system results in a model of Stress- strength based on IRD for several estimators' values will be illustrated. Four sample problem sizes 10, 25, 50, 75 have been implemented in 1000 repetitions, based on two parameters values α_1 and α_2. The summary results are given in Tables (1-8) below. The Mote Carlo simulation was coded using Matlab b 2016. Tables [1, 3, 5, and 7] show the reliability of all the different methods. For performance verifying, the estimation methods proposed have reasonabe MSE (Tables 2, 4, 6, and 8).

Tables 2, 4, 6, and 8 illustrated that the Modified Thompson type shrinkage estimator had minimum mean square error for the estimator of S-S reliability of the Invers Rayleigh Distribution since shrinkage weight factor of the 2nd rank and then followed by CSh_{Wf}, MOM, UMVUE, and MLE, respectively. At most when n fixed and m change, MSE decreases. Tables [1-8] present the simulation results as follows;

Table 1. Estimation value of R, $\beta_1 = 2$ and $\beta_2 = 1.5$

n	m	R	$\hat{R}_{MLE}$	$\hat{R}_{UB}$	$\hat{R}_{MOM}$	$\hat{R}_{CShwf}$	$\hat{R}_{Shwf}$	$\hat{R}_{MTShwf}$
10	10	0.571	0.4743	0.4743	0.5894	0.5138	0.5738	0.5697
	25	0.571	0.4761	0.4632	0.5874	0.5119	0.5758	0.5705
	50	0.571	0.4766	0.4596	0.588	0.5085	0.5746	0.5705
	75	0.571	0.4853	0.4667	0.5896	0.5128	0.576	0.5707
25	10	0.571	0.4552	0.4678	0.5856	0.5075	0.5885	0.5703
	25	0.571	0.465	0.465	0.5947	0.5081	0.5726	0.5705
	50	0.571	0.4577	0.4537	0.5899	0.5015	0.5752	0.5698
	75	0.571	0.4624	0.457	0.589	0.503	0.5824	0.5698
50	10	0.571	0.4668	0.4836	0.5993	0.515	0.5718	0.5701
	25	0.571	0.457	0.4611	0.5939	0.5055	0.578	0.5701
	50	0.571	0.4536	0.4536	0.5897	0.5012	0.5781	0.5702
	75	0.571	0.4526	0.4513	0.589	0.5002	0.5776	0.569
75	10	0.571	0.4728	0.4911	0.6044	0.5191	0.5829	0.5706
	25	0.571	0.4713	0.4768	0.6036	0.5128	0.5752	0.5705
	50	0.571	0.462	0.4633	0.5964	0.5044	0.5823	0.5698
	75	0.571	0.4582	0.4582	0.5981	0.5077	0.5749	0.57

Table 2. MSE value of R= 0.571 when $\beta_1 = 2$ and $\beta_2 = 1.5$

n	m	$\hat{R}_{MLE}$	$\hat{R}_{UB}$	$\hat{R}_{MOM}$	$\hat{R}_{CShwf}$	$\hat{R}_{Shwf}$	$\hat{R}_{MTShwf}$
10	10	0.0596	0.0596	0.0206	0.0162	0.5075	0.0004
	25	0.0578	0.0597	0.019	0.0157	0.0021	0.0009
	50	0.0594	0.0619	0.0199	0.017	0.0037	0.00001
	75	0.0554	0.0579	0.0186	0.0153	0.0006	0.00007
25	10	0.0666	0.0647	0.0215	0.0181	0.9361	0.0001
	25	0.0614	0.0614	0.0214	0.0178	0.029	0.008
	50	0.0659	0.0666	0.0211	0.0199	0.0046	0.0002
	75	0.0642	0.0651	0.0225	0.0201	0.024	0.0002
50	10	0.0625	0.0603	0.0206	0.0162	0.7429	0.0003
	25	0.0646	0.064	0.021	0.0183	0.0048	0.0002
	50	0.0681	0.0681	0.0217	0.0199	0.0032	0.000008
	75	0.0666	0.0669	0.0222	0.0208	0.0044	0.0007
	10	0.0602	0.0579	0.0202	0.0146	0.1914	0.0009

n	m	$\hat{R}_{MLE}$	$\hat{R}_{UB}$	$\hat{R}_{MOM}$	$\hat{R}_{CShwf}$	$\hat{R}_{Shwf}$	$\hat{R}_{MTShwf}$
75	25	0.0618	0.0611	0.02	0.0162	0.0012	0.0001
	50	0.0654	0.0652	0.0221	0.0199	0.0106	0.0003
	75	0.0618	0.0618	0.0199	0.0171	0.0075	0.0002

Table 3. Estimation value of R, when $\beta_1 = 2.5$ and $\beta_2 = 1.5$

n	m	R	$\hat{R}_{MLE}$	$\hat{R}_{UB}$	$\hat{R}_{MOM}$	$\hat{R}_{CShwf}$	$\hat{R}_{Shwf}$	$\hat{R}_{MTShwf}$
10	10	0.625	0.5125	0.5125	0.6079	0.5652	0.7111	0.6236
	25	0.625	0.5248	0.5121	0.6114	0.5618	0.627	0.6233
	50	0.625	0.5234	0.5066	0.6129	0.5608	0.6317	0.6238
	75	0.625	0.5218	0.5036	0.6155	0.5604	0.6289	0.624
25	10	0.625	0.5148	0.5276	0.6217	0.5692	0.6598	0.6179
	25	0.625	0.5036	0.5036	0.6126	0.5584	0.6325	0.6166
	50	0.625	0.4959	0.4919	0.6082	0.5537	0.6302	0.6162
	75	0.625	0.5038	0.4985	0.6123	0.5541	0.6305	0.6152
50	10	0.625	0.5027	0.5193	0.6179	0.5629	0.6928	0.6169
	25	0.625	0.5081	0.512	0.6167	0.5548	0.6325	0.6147
	50	0.625	0.5092	0.5092	0.6216	0.5612	0.6289	0.6166
	75	0.625	0.5048	0.5035	0.6193	0.5565	0.6345	0.6159
75	10	0.625	0.509	0.5267	0.6233	0.5652	0.6884	0.6178
	25	0.625	0.5002	0.5055	0.6153	0.5536	0.6312	0.6147
	50	0.625	0.4986	0.4999	0.6146	0.5524	0.6352	0.615
	75	0.625	0.5094	0.5094	0.6227	0.5586	0.633	0.6151

Table 4. MSE value of $R = 0.625$, $\beta_1 = 2.5$ and $\beta_2 = 1.5$

n	m	$\hat{R}_{MLE}$	$\hat{R}_{UB}$	$\hat{R}_{MOM}$	$\hat{R}_{CShwf}$	$\hat{R}_{Shwf}$	$\hat{R}_{MTShwf}$
10	10	0.0639	0.0639	0.0194	0.0168	0.9923	0.0004
	25	0.063	0.0652	0.02	0.0195	0.0058	0.0004
	50	0.0627	0.0657	0.02	0.0195	0.0041	0.0001
	75	0.0633	0.0666	0.02	0.0185	0.001	0.0001
25	10	0.0629	0.0607	0.0189	0.0163	0.255	0.0009
	25	0.0677	0.0677	0.0202	0.0196	0.0121	0.001
	50	0.0707	0.0716	0.0211	0.0208	0.0013	0.001
	75	0.0715	0.0726	0.0214	0.0213	0.0045	0.0015
50	10	0.0694	0.0663	0.0203	0.0187	0.9726	0.0012
	25	0.0731	0.0724	0.0229	0.0231	0.0063	0.0016
	50	0.0661	0.0661	0.02	0.0188	0.0022	0.0013
	75	0.0706	0.0709	0.0209	0.0205	0.0115	0.0013
75	10	0.0698	0.0666	0.0205	0.0179	0.5205	0.0007
	25	0.0739	0.0729	0.0224	0.0226	0.0144	0.0016
	50	0.073	0.0727	0.0213	0.0227	0.0146	0.0013
	75	0.0678	0.0678	0.0209	0.0206	0.0086	0.0017

Table 5. Estimation value of R, $\beta_1 = 2$ and $\beta_2 = 2$

n	m	R	$\hat{R}_{MLE}$	$\hat{R}_{UB}$	$\hat{R}_{MOM}$	$\hat{R}_{CShwf}$	$\hat{R}_{Shwf}$	$\hat{R}_{MTShwf}$
10	10	0.5	0.4263	0.4263	0.5582	0.4499	0.5393	0.4924
	25	0.5	0.3989	0.3864	0.5419	0.4334	0.5065	0.4911
	50	0.5	0.4139	0.3971	0.5476	0.4382	0.5059	0.4914
	75	0.5	0.4149	0.3969	0.5539	0.4368	0.5046	0.4912
25	10	0.5	0.4056	0.418	0.5568	0.4396	0.4998	0.4897
	25	0.5	0.4145	0.4145	0.5658	0.4422	0.5071	0.4921
	50	0.5	0.4115	0.4075	0.5621	0.4433	0.5054	0.4932
	75	0.5	0.4063	0.401	0.5596	0.4375	0.5083	0.4913
50	10	0.5	0.4012	0.4177	0.5618	0.4423	0.497	0.4923
	25	0.5	0.4106	0.4146	0.5638	0.4381	0.5025	0.4898
	50	0.5	0.405	0.405	0.5601	0.4365	0.5072	0.4913
	75	0.5	0.4077	0.4064	0.5672	0.4401	0.5042	0.4923
75	10	0.5	0.4083	0.4262	0.567	0.4446	0.4689	0.4911
	25	0.5	0.4065	0.4117	0.5626	0.4346	0.5056	0.4888
	50	0.5	0.4118	0.4131	0.5658	0.4385	0.507	0.4908
	75	0.5	0.4063	0.4063	0.5673	0.4385	0.508	0.4911

Table 6. MSE value of R= 0.5 when $\beta_1 = 2$ and $\beta_2 = 2$

n	m	$\hat{R}_{MLE}$	$\hat{R}_{UB}$	$\hat{R}_{MOM}$	$\hat{R}_{CShwf}$	$\hat{R}_{Shwf}$	$\hat{R}_{MTShwf}$
10	10	0.0491	0.0491	0.0224	0.0131	0.8567	0.001
	25	0.0561	0.0574	0.0222	0.0165	0.0041	0.0011
	50	0.0514	0.053	0.0223	0.0158	0.0069	0.0009
	75	0.0528	0.0545	0.0236	0.0164	0.003	0.0009
25	10	0.0587	0.0577	0.0259	0.0172	0.6037	0.0015
	25	0.0536	0.0536	0.025	0.0152	0.002	0.0007
	50	0.0531	0.0535	0.0229	0.0137	0.0009	0.0005
	75	0.0565	0.057	0.0247	0.0157	0.0108	0.0011
50	10	0.0578	0.0563	0.0249	0.0149	0.2085	0.0007
	25	0.0573	0.057	0.0273	0.0177	0.0074	0.0014
	50	0.0589	0.0589	0.0252	0.0166	0.0052	0.0009
	75	0.0543	0.0544	0.0241	0.0152	0.0009	0.0006
75	10	0.0562	0.0548	0.0258	0.0154	0.7414	0.0012
	25	0.0604	0.06	0.0276	0.0187	0.0098	0.0017
	50	0.0567	0.0566	0.0262	0.0171	0.0194	0.001
	75	0.0557	0.0557	0.0253	0.0162	0.0118	0.0011

Table 7. Estimation value of R, , $\beta_1 = 1.5$ and $\beta_2 = 2$

n	m	R	$\hat{R}_{MLE}$	$\hat{R}_{UB}$	$\hat{R}_{MOM}$	$\hat{R}_{CShwf}$	$\hat{R}_{Shwf}$	$\hat{R}_{MTShwf}$
10	10	0.428571	0.3581	0.3581	0.5178	0.3751	0.4898	0.4198
	25	0.428571	0.3539	0.3418	0.5143	0.3714	0.4279	0.4198
	50	0.428571	0.356	0.34	0.5145	0.3702	0.4345	0.4191
	75	0.428571	0.3613	0.344	0.5193	0.3718	0.4308	0.4193
25	10	0.428571	0.3597	0.3718	0.5288	0.3772	0.4404	0.4205
	25	0.428571	0.3562	0.3562	0.528	0.3756	0.4334	0.4219

n	m	R	$\hat{R}_{MLE}$	$\hat{R}_{UB}$	$\hat{R}_{MOM}$	$\hat{R}_{CShwf}$	$\hat{R}_{Shwf}$	$\hat{R}_{MTShwf}$
	50	0.428571	0.3564	0.3526	0.5307	0.3732	0.4325	0.4208
	75	0.428571	0.3534	0.3483	0.522	0.3688	0.4353	0.4193
	10	0.428571	0.3393	0.3549	0.5208	0.3694	0.4525	0.4193
	25	0.428571	0.3532	0.3571	0.533	0.3738	0.4344	0.421
50	50	0.428571	0.0474	0.0473	0.0321	0.013	0.0006	0.0005
	75	0.428571	0.3496	0.3483	0.5306	0.3718	0.4326	0.42
	10	0.428571	0.3526	0.3698	0.5342	0.377	0.4698	0.4194
	25	0.428571	0.3444	0.3495	0.5298	0.3713	0.439	0.4194
75	50	0.428571	0.3536	0.3549	0.5331	0.3731	0.4346	0.4205
	75	0.428571	0.348	0.348	0.528	0.3676	0.4324	0.4183

Table 8. MSE value of R= 0.428571 $\beta_1 = 1.5$ and $\beta_2 = 2$

n	m	$\hat{R}_{MLE}$	$\hat{R}_{UB}$	$\hat{R}_{MOM}$	$\hat{R}_{CShwf}$	$\hat{R}_{Shwf}$	$\hat{R}_{MTShwf}$
	10	0.0462	0.0462	0.0299	0.013	0.8415	0.001
	25	0.0457	0.0462	0.0284	0.0128	0.0124	0.0011
10	50	0.045	0.0456	0.0288	0.0136	0.0247	0.0012
	75	0.0454	0.046	0.0301	0.0133	0.0073	0.0012
	10	0.0495	0.0495	0.0325	0.0128	0.1225	0.0008
	25	0.0463	0.0463	0.03	0.0119	0.0005	0.0004
25	50	0.0469	0.0471	0.031	0.0126	0.007	0.0007
	75	0.0491	0.0492	0.0304	0.0144	0.0075	0.0009
	10	0.0508	0.0504	0.0314	0.0147	0.7594	0.0008
	25	0.0474	0.0473	0.0321	0.013	0.0006	0.0005
50	50	0.049	0.049	0.0306	0.0145	0.0026	0.001
	75	0.047	0.047	0.0318	0.013	0.0032	0.0009
	10	0.0481	0.0478	0.0335	0.0129	0.3801	0.0012
	25	0.0469	0.0467	0.0312	0.0136	0.006	0.001
75	50	0.0471	0.0471	0.0314	0.0132	0.0015	0.0007
	75	0.0502	0.0502	0.0326	0.0151	0.0073	0.0013

5. Conclusion

IRD was an important role in the life test and reliability domain. This paper evaluated the stress–strength model by estimating the reliability R=P(X<Y) based on IRD. Different estimation methods as; MLE, Moment method, Uniformly Minimum Variance Unbiased estimator, constant shrinkage weight factors, Shrinkage weight function, and Modified Thompson type shrinkage weight factor were compared. Then, Monte Carlo simulation was used to compare among all the suggested methods depending on the statistical indicator Mean Squared Error (MSE). The results indicated the Modified Thompson type shrinkage weight factor was more precise than the others in the sense of MSE.

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