Periodicals of Engineering and Natural Sciences ISSN 2303-4521
Vol. 8, No. 3, August 2020, pp.1535-1547

Using TGARCH, TGARCH-M, EGARCH, EGARCH-M, PGARCH and PGARCH-M models Gaussian and non-Gaussian for modeling (EUR/USD) and (GBP/USD) Exchange Rate

Suhail Najm Abdullah

Department of Statistics, University of Baghdad

ABSTRACT:

This paper aims to study characteristics of exchange rate volatility of (EUR/USD) and (GBP/USD) using daily closing prices for two time periods, sub-period form (1 January 2015 until 15 may 2020) and full period from(1 January 2010 until 15 may 2020). This is done by studying (TGARCH, EGARCH, PGARCH, TGARCH-M, EGARCH-M, and PGARCH-M) models. The error term assumed five distributional (Gaussian, Student-t, Student-t with fixed df, (GED) and (GED) with fixed parameter. The results showed that the EGARCH (1,1) has been the best model selected for the return series of (EUR/USD) exchange rate in the both full and sub period series. The best model selected for the return series of (GBP/USD) exchange rate it was EGARCH (1,1) model in the full period and TGARCH(1,1) model in the sub-period. All four models selected are based on the Student's t distribution to provide the leverage parameter of the EGARCH model of (EUR/USD) in the full time period and sub-period and the EGARCH model of (GBP/USD) in the full time.

Keywords: EGARCH Models, volatility, exchange rate

Corresponding Author:

Suhail Najm Abdullah,
Department of Statistics,
University of Baghdad,
Baghdad, Iraq
E-mail: dr.suhail.najm@coadec.uobaghdad.edu.iq

1. Introduction

Things surrounding the exchange rate in modern economic theory are of great interest to the researchers. Foreign exchange rates are known to be the value of a currency as opposed to another currency, which fluctuate in relation to currency demand and supply. Exchange rates are vitally significant in trade and investment levels and their volatility reduces the amount of foreign trade. It is well established that exchange rates are primarily the responsibility of central banks export and import profitability, countries' debt payments. In addition, foreign portfolios of investments are affected by the volatility's on exchange rates. The volatility of the exchange rates has an effect on market flows and capital cycles, deciding terms of trade with other nations. Thus, it guides the economic life of each country's. Exchange-rate volatility can also impact foreign trade country's economics as well as government policy making. Consequently, predicting exchange rate volatility is of considerable significance for decision-makers [4]. There has been extensive debate over the last decades about the volatility of the exchange rate. As a result, several models to analyze this volatility have been created. The models frequently applied in calculating exchange rate volatility are ARCH and GARCH models developed respectively by Engle (1982) and Bollerslev (1986) [3]. Hsieh (1988) first applied the ARCH model for the exchange rate in order to analyze the five exchange rates. He concludes that the indiscriminate ARCH (GARCH) model can describe single part of the nonlinearity of exchange rates [6]. In recent years, many analysts studied forecasting of exchange rate volatility. Specifically, Sandoval (2006) discusses the Asian and Latin American countries exchange rates with respect to the US dollar. Using GJR-GARCH, GARCH and EGARCH models. He showed that from the seven exchange rates, four ones follow asymmetric models that comprised in developing countries. In addition, symmetric model forecasts performed well than those of asymmetric models [13]. In 2011, Vee et al., studied forecasts of US dollar / Mauritian rupee exchange rate volatility. Regular data from 2003 to 2008 and the symmetric model GARCH(1,1) with student t-distribution and (GED) generalized error distribution were used for the estimation. The results of their papers demonstrated that the (GED) provides better results for out-of-sample forecasts of exchange rates [14]. Miletić (2015) studied the hypothesis that the emerging-market exchange rate is more sensitive in negative than positive crises. He used regular exchange rate data for (Romania, Hungary, UK, Serbia, and Japan) and the European Union against the US dollar by using non-symmetrical and symmetrical GARCH models [11]. Dritsaki (2018) examined the characteristics of British pound exchange rate/US dollar volatility using monthly data from August 1953 to January 2017. Both static procedures and dynamic are used for forecasting the ARIMA(0,0,0,1)-EGARCH(1,1) model. The static procedure delivers better results on the forecast than on the dynamic [3]. The euro /US dollar exchange rate volatility characteristics has analyzed using monthly data from August 1953 to January 2017. It was based on ARIMA(0,0,0,1)-EGARCH(1,1) model. The static procedure provides better results on the forecast rather than the dynamic one [4].

This paper attempts to evolve and study the characteristics of exchange rate volatility of (EUR/USD) and (GBP/USD) using daily data for two time period, sub-period form (1 January 2015 until 15 may 2020) and full period from(1 January 2010 until 15 may 2020) by studying (TGARCH,EGARCH, PGARCH,TGARCH-M,EGARCH-M, and PGARCH-M) models. The error term assumed five distributional (Gaussian, Student-t, Student-t with fixed df, Generalized Error Distribution (GED) and Generalized Error with fixed parameter distribution.

2. Materials and methods of analysis

2.1. Data for analysis

The studied data represent the daily closing prices of the GBP / USD index, which shows the value of the British pound against the US dollar, as well as the EUR / USD value index. It shows the value of the euro against the US dollar, by studying the time series of two periods of time for each. The first period extends from 1 January 2015 until 15 may 2020 includes (1402) values, and full period extends from 1 January 2010 until 15 may 2020 including (2705) values after it was transferred. The time series for each of them to the daily return series through the formula:

$$X_t = log(\frac{P_t}{P_{t-1}}) \quad (1)$$

Where $\boldsymbol{X_t}$ is the returns series, $\boldsymbol{P_t}$ is the prices for the current day, and $\boldsymbol{P_{t-1}}$ is the prices for the previous day.

2.2. ADF test: Unit root test

The adopted test to determine whether the series has unit root or it is stationary. Through testing [1], the following hypothesis**:**

$$H_0: \theta = 1 \quad \text{Non-stationary} \quad (2)$$

$$H_1: \theta < 1 \quad \text{Stationary} \quad (3)$$

The ADF test is given by:

$$ADF = \frac{\phi^{\wedge}}{SE(\phi^{\wedge})} \quad (4)$$

Where is $\phi^{\wedge}$ the estimate of ϕ *and* $SE(\phi^{\wedge})$ is stander error of the least squarer estimate of $\phi^{\wedge}$.The null hypothesis is rejected if the $p - value < \alpha$ level

2.3. Jarque-Bera test

It is a goodness-of-fit test [7] that used for checking the sample data if it has coefficient skewness and kurtosis are similar to normal distribution

The test statistic is:

$$JB = T.\left[\frac{S^2}{6} + \frac{(k-3)^2}{24}\right] \quad (5)$$

Where K , S are the sample kurtosis and skewness respectively The hypothesis is:

$$H_0: Normality \quad (6)$$

$$H_1: non - Normality \quad (7)$$

When Accepted the null hypothesis mean that data follow normal distribution.

2.4. Ljung box test

The test used whether there exist autocorrelation r_i in the return series [10]

The hypothesis is:

$$H_0: \rho_1 = \rho_2 = \cdots \rho_k = 0 \qquad \text{k=1,2,…,m} \quad (8)$$

$$H_1: \rho_k \neq 0 \quad for\ some\ values\ of\ k \quad (9)$$

The statistic is:

$$Q(K) = T(T+2)\sum_{i=1}^{k}\frac{r_i^2}{T-i} \quad (10)$$

Where r_i is the residual autocorrelation ,K is the number of the time lags , T is the size of series .when the p-value is greater than $\boldsymbol{\alpha}$ significant indicter that the residuals are no serial correlation .

2.5. Testing for ARCH Effects - (ARCH-LM) test

This test can be used to test the ARCH effect in a series by the i log autocorrelation of the squared by $\rho^{\wedge}{}_i$ [9]. The Ljung-Box is:

$$Q = T(T+2)\sum_{i=1}^{k}\frac{\rho^{\wedge}{}_i^2}{T-i} \sim X^2(m) \quad (11)$$

The ARCH-LM hypothesis is

$$H_0: \alpha_1 = \alpha_2 = \cdots \alpha_k = 0 \quad no\ ARCH\ effect \quad (12)$$

$$H_1: \alpha_1 \neq \alpha_2 \neq \cdots \alpha_i \neq 0 \quad ARCH\ effect \qquad \text{for i=1,2,…,q} \quad (13)$$

The statistic test is:

$$LM = T.R^2 \sim X^2(q) \quad (14)$$

Where T is the total number of observation, q is the number of restrictions and R^2 is based on the regression.

2.6. The threshold GARCH (TGARCH) model

This model commonly used to handle leverage effects by Zakoian and abemananjara 1991. The condition variance equation is given by :

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i + \gamma_i N_{t-i})\alpha_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2 \qquad (15)$$

Where,

$$N_{t-i} = \begin{cases} 1 \ if \ \ a_{t-i} < 0 \ \ \ bad \ news \\ 0 \ if \ \ a_{t-i} \geq 0 \ \ good \ news \end{cases} \qquad (16)$$

αi, γi, and βj are non–negative parameter, the γ is the leverage parameter ,the a positive at−i contributes $\alpha_i \, \alpha_{t-i}^2$ to σ_t^2 whereas a negative at−i has a large impact on $(\alpha_i + \gamma_i)\alpha_{t-i}^2$ with $\gamma i > 0$

2.7. The Exponential GARCH (EGARCH) model

Nelson (1991) developed this model. It is the logarithm of conditional volatility in order to asymmetries effects between negative and positive shocks that the leverage effect is exponential [12]. The conditional variance equation is

$$\log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p}\alpha_i \ |\eta_i| + \sum_{j=1}^{q}\beta_j \log(\sigma_{t-j}^2) + \sum_{k=1}^{r}\gamma_k \, |\eta_{t-k}| \qquad (17)$$

Here, γ is the leverage parameter. Where $|\eta_{t-i}|$ and $\eta_{t-i} = \frac{u_{t-i}}{\sigma_{t-i}}$ .There is an leverage effect when $\gamma < \alpha < -\gamma$ this implies that the negative shock increase volatility.

2.8. The power GARCH (PGARCH) model

This model specification to deal with asymmetry. Ding, Granger and Engle developed it in 1993. The conditional variance [2]:

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{q}\alpha_i \ (|u_{i-1}| - \gamma_i u_{i-1})^{\delta} + \sum_{j=1}^{p}\beta_j(\sigma_{t-j}^{\delta}) \qquad (18)$$

Where $\delta > 0$, $\boldsymbol{\gamma_i \leq 1}$ for i=1,2,…,r and $\boldsymbol{\gamma_i = 0}$ for i > r , and r ≤ p. In the PGARCH model if $\boldsymbol{\gamma_i \neq 0}$**,** this captures asymmetric effects.

2.9. The GARCH-in-Mean (GARCH-M)

These models are used in assessing risk in financial markets by measuring the relationship between risk and return [5] .The mean equation is:

$$X_t = \mu + \lambda X_{t-1} + \varepsilon_t \qquad (20)$$

where μ and λ are constant, X_t is the return, λ is the risk parameter and ε_t are the residuals.

2.10. Distribution assumptions of error term and estimation

In this paper, the volatility estimated is based on (TGARCH,EGARCH,PGARCH,TGARCH-M,EGARCH-M,PGARCH-M). The distributional assumption of the error term assumed five distributional (Gaussian , Student-t ,Student-t with fixed df, Generalized Error Distribution (GED) and Generalized Error with fixed parameter distribution [8]. The GARCH processes are estimated using the maximum likelihood approach by using maximizing the log- likelihood:

$$\mathrm{Log}(\mathrm{L}\theta_t) = -\frac{1}{2}\sum_{t-1}^{T}(\ln 2\pi + \ln h_t + \frac{\varepsilon_t^2}{h_t}) \qquad (21)$$

i) The Normal distribution to the log- likelihood for t observe is:

$$\mathrm{I}_t = \frac{-\frac{1}{2}\log(2\pi) - \frac{1}{2}\log h_t - \frac{1}{2}(y_t - X_t'\theta)^2}{h_t} \qquad (22)$$

ii) The Student –t distribution to the log-likelihood is given by :

$$I_t = \frac{1}{2}log\left[\frac{\pi(\nu-2)\Gamma(^{\nu}/_2)^2}{\Gamma(^{\nu+1}/_2)}\right] - \frac{1}{2}log h_t - \frac{(\nu+1)}{2}log\left[1 + \frac{(y_t - X_t'\theta)^2}{h_t(\nu-2)}\right] \qquad (23)$$

Where the degree of freedom $\nu > 2$ controls the tail behavior.

iii) The GED distribution to the log-likelihood of the form:

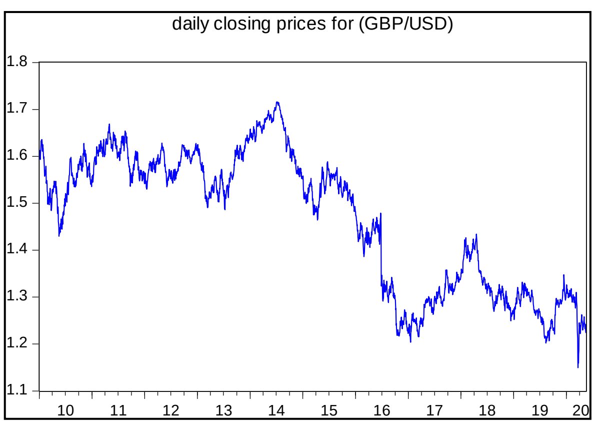
$$I_t = -\frac{1}{2}log\left[\frac{\Gamma(^1/_\nu)^3}{\Gamma(^3/_\nu)(^{\nu}/_2)^2}\right] - \frac{1}{2}log h_t - \left[\frac{\Gamma(^3/_\nu)(y_t - X_t'\theta)^2}{h_t\Gamma(^1/_\nu)}\right]^{\frac{\nu}{2}} \qquad (24)$$

Where the fat-tailed is based on $\nu < 2$.

3. Results and discussion

3.1. The data

Figures 1-4 represent series of daily closing prices and the series of returns for (GBP/USD) and (EUR/USD) from 01/01/2020 to 15/05/2020.



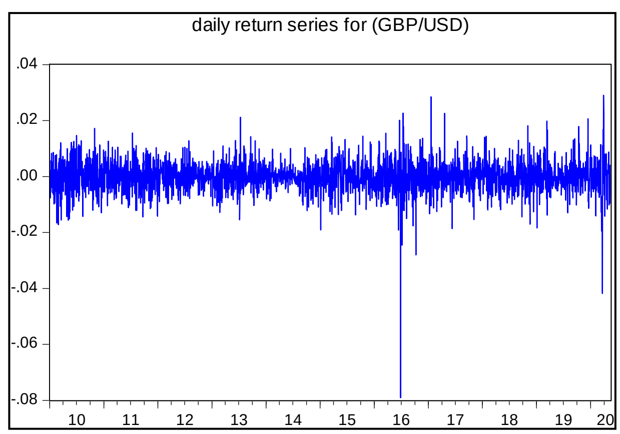


Figure1. Daily closing for (GBP/USD) in full period

Figure 2. Daily return for (GBP/USD) in full period

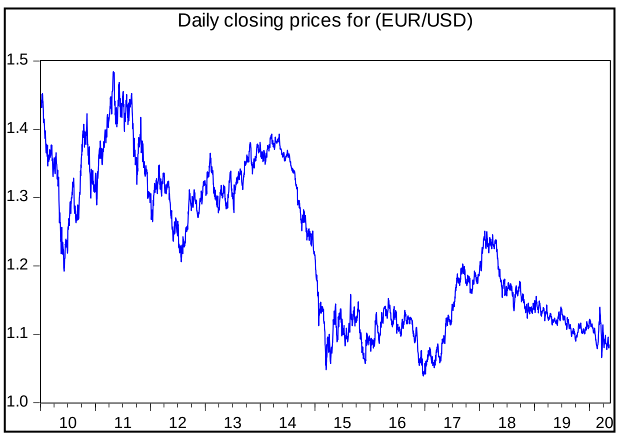


Figure 3. Daily closing for (EUR/USD) in full period

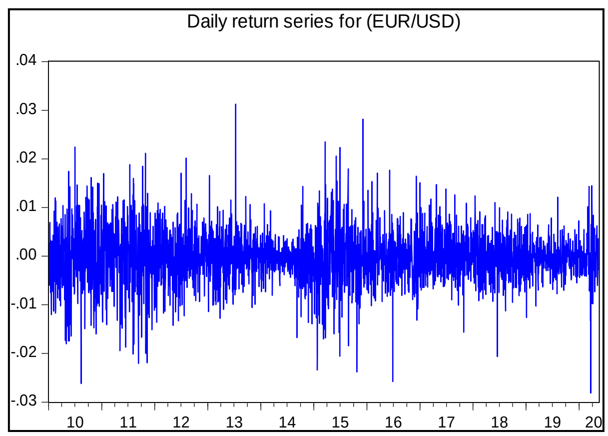


Figure 4. Daily return for (EUR/USD) in full period

3.2. Descriptive statistics

The statistical results in Table 1 indicate that there is convergence at the highest and lowest exchange rates for return series with when comparing the two studied periods for each of GBP/USD and EUR/USD index. In addition, the values of the standard deviation were close to zero indicating low level of dispersion from the average log returns in the exchange market. But, the exchange rates have high kurtosis values, especially in GBP/USD in which big shocks of either signs are more, and the series is clearly leptokurtic. The negative skewness indicates that exchange rates are affected by negative shocks more than positive shocks.

Table 1. Descriptive statistic

	GBP/USD		EUR/USD	
	Full Period	Sub-Period	Full Period	Sub-Period
Mean	-2.19E-19	6.48E-19	-8.50E-19	-8.74E-19
Median	1.89E-05	2.72E-05	0.000106	-7.15E-06
Maximum	0.029165	0.029235	0.031368	0.02823
Minimum	-0.07898	-0.07891	-0.02804	-0.02806

Std. Dev.	0.005615	0.006205	0.005688	0.005287
Skewness	-1.08697	-1.51869	-0.08745	-0.05678
Kurtosis	19.64314	24.00709	5.218491	6.125616

3.3. ADF test (Unit root test)

We use the Augmented Dickey Fuller Test to investigate the unit root and stationary properties of log return series. The result is presented in Table (2) show that the p-values for test statistics less than $\alpha = 0.05$ for the return series of (GBP/USD) and (EUR/USD) exchange rate to period and sub-period series, This is indicated the returns series are stationary means that there is no unit root.

Tables 2. The augmented Dickey Fuller test statistics

		(GBP/USD) Return Series		(EUR/USD) Return Series	
	Time period	Full Period	Sub-Period	Full Period	Sub-Period
	ADF-test statistic	-51.1178*	-35.2369*	-53.8321*	-37.8773*
Test Critical values	1% level	-3.43258	-3.43481	-3.43258	-3.43481
	5% level	-2.86241	-2.8634	-2.86241	-2.8634
	10% level	-2.56728	-2.56781	-2.56728	-2.56781

* Indicates significance at 5% level

3.4. Testing for ARCH Effects - (ARCH-LM) test

Using ARCH-LM test, the results in Table 3 show that the p-value less than 0.05 meaning that the ARCH effect is exists to test the ARCH effect in the return series residuals for (GBP/USD) and (EUR/USD) exchange rate.

Table 3. The ARCH-LM test

Return Series	Time period	F-statistic	p-value	Obs * R^2	p-value
GBP/USD	Full Period	16.12759	0.0001*	16.04376	0.0001*
	Sub-Period	8.090811	0.0045*	8.055789	0.0045*
EUR/USD	Full Period	40.13143	0.0000*	39.57337	0.0000*
	Sub-Period	25.53833	0.0000*	25.11634	0.0000*

* Indicates significance at 5% level

3.5. Estimation results

Tables 4-6 show that results estimation for (TGARCH(1,1), EGARCH(1,1), PGARCH(1,1), TGARCH-M(1,1), EGARCH-M(1,1)) and PGARCH-M(1,1)). The distributional assumption of the error term assumed five distributional (Gaussian, Student-t, Student-t with fixed df=10, GED, and GED with fixed parameter=1.5 distribution for the returns series both of (GBP/USD) and (EUR/USD) exchange rate to period and sub-period series.

Table 4. That results estimation

Time period	Distribution	Coeff	TGARCH		TGARCH-M	
			EUR/UDS	GBP/USD	EUR/UDS	GBP/USD

period	Normal	α_0	1.16E-07*	7.80E-07*	1.21E-07*	7.78E-07*
		α_1	0.021237*	0.053048*	0.021319*	0.052906*
		γ	0.026579*	0.042548*	0.026219*	0.042598*
		β_1	0.96259*	0.903925*	0.962387*	0.904129*
	Student's t	α_0	6.73E-08	2.56E-07*	7.34E-08	2.57E-07*
		α_1	0.023176*	0.024262*	0.023149*	0.024081*
		γ	0.022103*	0.029418*	0.021613*	0.029342*
		β_1	0.96453*	0.953428*	0.964415*	0.953527*
	Generalized error	α_0	8.78E-08*	3.20E-07*	9.50E-08*	3.20E-07*
		α_1	0.022834*	0.031741*	0.022787*	0.031573*
		γ	0.024213*	0.031935*	0.023826*	0.031912*
		β_1	0.962917*	0.943232*	0.96273*	0.943373*
	Student's t with fixed df=10	α_0	6.80E-08	2.88E-07*	7.38E-08	2.90E-07*
		α_1	0.021908*	0.023685*	0.02191*	0.023537*
		γ	0.021615*	0.028079*	0.021154*	0.027974*
		β_1	0.964947*	0.951005*	0.964793*	0.951106*
	GED with fixed Parameter=1.5	α_0	9.01E-08*	3.90E-07*	9.70E-08*	3.88E-07*
		α_1	0.022363*	0.034951*	0.022344*	0.03476*
		γ	0.024217*	0.03281*	0.023823*	0.032736*
		β_1	0.962993*	0.936215*	0.962793*	0.936463*
sub-period	Normal	α_0	1.34E-07*	5.41E-06*	1.34E-07*	5.37E-06*
		α_1	0.033296*	0.112662*	0.033277*	0.11273*
		γ	0.004703	0.084337*	0.004588	0.083462*
		β_1	0.959718*	0.705605*	0.959764*	0.707152*
	Student's t	α_0	1.42E-07	3.62E-06*	1.41E-07	3.72E-06*
		α_1	0.034855*	0.068103*	0.034769*	0.070488
		γ	-0.00507	-0.00648	-0.00503	-0.00899
		β_1	0.962114*	0.827934*	0.962214*	0.823937*
	Generalized error	α_0	1.32E-07	3.96E-06*	1.33E-07	4.02E-06*
		α_1	0.034477*	0.082903*	0.034671*	0.085763*
		γ	-0.00141	0.01172	-0.00156	0.008191
		β_1	0.961146*	0.796024*	0.960992*	0.793468*
	Student's t with fixed df=10	α_0	1.27E-07*	3.54E-06*	1.26E-07*	3.57E-06*
		α_1	0.030843*	0.064854*	0.030729*	0.066094*
		γ	-0.00294	0.000642	-0.00293	-0.00084
		β_1	0.963953*	0.815435*	0.964113*	0.813634*
	GED with fixed parameter=1.5	α_0	1.29E-07*	4.11E-06*	1.30E-07*	4.11E-06*
		α_1	0.03329*	0.088316*	0.033349*	0.089674*
		γ	-0.00031	0.023186	-0.00034	0.021191
		β_1	0.961328*	0.776711*	0.961274*	0.776115*

* significance at 5% level

Table 5. That results estimation

			EGARCH		EGARCH-M	
Time period	Distribution	Coeff	EUR/UDS	GBP/USD	EUR/UDS	GBP/USD
period	Normal	α_0	-0.12251*	-0.41434*	-0.12626*	-0.41667*
		α_1	0.079993*	0.162911*	0.080004*	0.163104*

Time period	Distribution	Coeff	EGARCH EUR/UDS	EGARCH GBP/USD	EGARCH-M EUR/UDS	EGARCH-M GBP/USD
		γ	-0.02746*	-0.02429*	-0.02755*	-0.02422*
		β_1	0.994109*	0.972123*	0.993755*	0.971916*
	Student's t	α_0	-0.1069*	-0.18573*	-0.11159*	-0.18922*
		α_1	0.084395*	0.085667*	0.083837*	0.082402*
		γ	-0.02078*	-0.03175*	-0.02107*	-0.03283*
		β_1	0.995946*	0.988475*	0.995467*	0.98792*
	Generalized error	α_0	-0.11443*	-0.24895*	-0.11961*	-0.24802*
		α_1	0.083281*	0.110729*	0.083041*	0.107906*
		γ	-0.02397*	-0.0285*	-0.02412*	-0.02941*
		β_1	0.995162*	0.984239*	0.994663*	0.98413*
	Student's t with fixed df=10	α_0	-0.10648*	-0.20603*	-0.11113*	-0.21149*
		α_1	0.081504*	0.086921*	0.081079*	0.084591*
		γ	-0.02106*	-0.0307*	-0.02128*	-0.03153*
		β_1	0.995864*	0.986836*	0.995401*	0.986158*
	GED with fixed Parameter=1.5	α_0	-0.11511*	-0.28665*	-0.12049*	-0.28625*
		α_1	0.082394*	0.120672*	0.082169*	0.118717*
		γ	-0.02423*	-0.02715*	-0.0244*	-0.02769*
		β_1	0.995055*	0.981469*	0.994536*	0.981371*
sub-period	Normal	α_0	-0.14533*	-0.9213*	-0.14531*	-0.93036*
		α_1	0.092537*	0.251226*	0.092552*	0.24888*
		γ	-0.0185*	0.003386	-0.0185*	0.001801
		β_1	0.992952*	0.92858*	0.992955*	0.927541*
	Student's t	α_0	-0.14278*	-0.8441*	-0.20184*	-0.88691*
		α_1	0.086139*	0.154288*	0.11837*	0.157351*
		γ	-0.00566	0.011988	0.033986*	0.013726
		β_1	0.992742*	0.929358*	0.98942*	0.925435*
	Generalized error	α_0	-0.14224*	-0.87291*	-0.14133*	-0.91149*
		α_1	0.089562*	0.187267*	0.090003*	0.190184*
		γ	-0.01114	0.008166	-0.01069	0.010388
		β_1	0.993054*	0.929002*	0.993168*	0.92548*
	Student's t with fixed df	α_0	-0.13729*	-0.87899*	-0.13793*	-0.90102*
		α_1	0.081565*	0.157456*	0.081748*	0.158668*
		γ	-0.00787	0.008783	-0.00784	0.009524
		β_1	0.993092*	0.927292*	0.993046*	0.925268*
	GED with fixed parameter=1.5	α_0	-0.1424*	-0.89832*	-0.14145*	-0.92228*
		α_1	0.088838*	0.201456*	0.089038*	0.202722*
		γ	-0.01234	0.006691	-0.01211	0.007297
		β_1	0.993039*	0.927961*	0.993141*	0.925744*

* Indicates significance at 5% level

Table 6. Results estimation

Time period	distribution	Coeff	PGARCH EUR/UDS	PGARCH GBP/USD	PGARCH-M EURO/UDS	PGARCH-M GBP/USD
period	Normal	α_0	1.29E-05	9.31E-05	1.35E-05	9.68E-05

Time period	distribution	Coeff	PGARCH EUR/UDS	PGARCH GBP/USD	PGARCH-M EURO/UDS	PGARCH-M GBP/USD
		α_1	0.040227*	0.088046*	0.040156*	0.088014*
		γ	0.319581*	0.133732*	0.319735*	0.132735*
		β_1	0.963844*	0.908135*	0.963537*	0.908097*
		δ	1.144166*	1.078866*	1.145709*	1.071991*
	Student's t	α_0	4.90E-06	0.000107*	5.19E-06	0.00012*
		α_1	0.042351*	0.043648*	0.041991*	0.042535*
		γ	0.219383	0.393997*	0.220682*	0.410668*
		β_1	0.963545*	0.957862*	0.96336*	0.957931*
		δ	1.24486*	0.860805	1.251483*	0.852626*
	Generalized error	α_0	6.92E-06	9.20E-05	7.30E-06	9.84E-05
		α_1	0.041917*	0.055923*	0.041667*	0.054844*
		γ	0.257406*	0.264862*	0.258129*	0.274871*
		β_1	0.963101*	0.945472*	0.962772*	0.946028*
		δ	1.217133*	0.943045*	1.224111*	0.933414*
	Student's t with fixed df=10	α_0	5.26E-06	0.000132	5.61E-06	0.00015*
		α_1	0.040817*	0.044703*	0.040519*	0.043693*
		γ	0.230453*	0.374407*	0.231468*	0.388896*
		β_1	0.964141*	0.954576*	0.963921*	0.954539*
		δ	1.234131*	0.849448	1.239192*	0.837472*
	GED with fixed parameter=1.5	α_0	7.39E-06	9.99E-05	7.79E-06	0.000106*
		α_1	0.041446*	0.062016*	0.041218*	0.061293*
		γ	0.26393*	0.223371*	0.264472*	0.227748*
		β_1	0.963262*	0.937571*	0.962925*	0.937937*
		δ	1.209084*	0.96446*	1.215604*	0.955884*
sub-period	Normal	α_0	1.22E-05	1.34E-04	1.23E-05	0.000186
		α_1	0.045928*	0.150307*	0.045916*	0.148021*
		γ	0.17566*	0.021326	0.175829	0.015322
		β_1	0.958164*	0.794988*	0.958137*	0.800848*
		δ	1.179547*	1.274507*	1.179503*	1.203368*
	Student's t	α_0	3.69E-06	4.86E-05	3.69E-06	5.36E-05
		α_1	0.04149*	0.078829*	0.041494*	0.080326*
		γ	0.015264	-0.04193	0.015245	-0.05125
		β_1	0.95998*	0.847902*	0.959976*	0.844327*
		δ	1.409105*	1.4574*	1.409044*	1.443975*
	Generalized error	α_0	5.25E-06	9.69E-05	5.21E-06	0.000113
		α_1	0.043723*	0.103622*	0.043935*	0.105113*
		γ	0.074306	-0.01083	0.070283	-0.02303
		β_1	0.959081*	0.830682*	0.958999*	0.82847*
		δ	1.331842*	1.324629*	1.331632	1.298186*
	Student's t with fixed df=10	α_0	4.70E-06	9.16E-05	4.68E-06	9.57E-05
		α_1	0.039307*	0.083577*	0.03926*	0.084101*
		γ	0.046491	-0.0261	0.046803	-0.03049
		β_1	0.961413*	0.841573*	0.961451*	0.840063*
		δ	1.350742*	1.326241*	1.351063*	1.320355*
	GED with	α_0	6.13E-06	0.000122	6.08E-06	0.000136
		α_1	0.043415*	0.113958*	0.043528*	0.114508*

Time period	distribution	Coeff	PGARCH EUR/UDS	PGARCH GBP/USD	PGARCH-M EURO/UDS	PGARCH-M GBP/USD
	fixed parameter=1.5	γ	0.091922	-0.00298	0.089986*	-0.00769
		β_1	0.959176*	0.821135*	0.959163*	0.820444*
		δ	1.302237	1.278625*	1.302027*	1.258927*

* Indicates significance at 5% level

3.6. Criteria for selecting the best model

For comparison of results of the best fit study models, using (Akaike info ,Schwarz, Hannan-Quinn) criteria to the best performance forecast volatility, Table 7 shows that the EGARCH(1,1) is the best model selected for the return series of (EUR/USD) exchange rate in the both full and sub period series. This is because the value of the three criteria are smaller for them, and the log likelihood statistics is a largest after compared with all the specified volatility models in each of the time period. While the best model selected for the return series of (GBP/USD) exchange rate was EGARCH(1,1) model in the full period and TGARCH(1,1) model in the sub-period. All the four models selected are based on the student's t distribution .

Table 7. Best model selected for (EUR/USD) and (GBP/USD) return series

	EUR/USD		GBP/USD	
criterion	Full period	Sup-period	Full period	Sup-period
Akaike info	-7.68932	-7.83001	-7.716369	-7.55494
Schwarz	-7.67623	-7.80756	-7.703276	-7.53249
Hannan-Quinn	-7.68459	-7.82162	-7.711635	-7.54655
Log likelihood	10405.81	5494.838	10442.39	5302.01
Best Model	EGARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)
Distribution	Student's t	Student's t	Student's t	Student's t

3.7. Diagnostic checking

The specified volatility models diagnostic tests are based upon residuals. Table 8 presents the results of normality test for the residuals for the best model selected. All the p-values for Jarque-Bera test statistics less than 0.05 indicate that the residuals are not based on the normal distribution. Table 9 shows the results of Ljung-Box test for lag 1 to lag 14 of autocorrelation. The p-value more than 0.05 means that no serial correlation on the squared residual returns series of (EUR/USD) and (GBP/USD) exchange rate.

Table 8. Results of the Jarque-Bera test for the best models

	EUR/USD		GBP/USD	
Time period	Full period	Sup-period	Full period	Sup-period
Best Model	EGARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)
Jarque-Bera test	448.1659	285.7586	5007.572	4010.745
Prop.	0*	0*	0*	0*

* Indicates significance at 5% level

Table 9. Results of the Ljung-Box test (Q-statistic) for the best models

	EUR/USD				GBP/USD			
Best model	EGARCH(1,1)		EGARCH(1,1)		EGARCH(1,1)		TGARCH(1,1)	
Time period	Full period		sub-period		Full period		sub-period	
lag	Q-Stat	Prob	Q-Stat	Prob	Q-Stat	Prob	Q-Stat	Prob
1	0.1965	0.658	0.1083	0.742	2.9052	0.088	0.5171	0.472
2	0.1973	0.906	0.802	0.67	3.0806	0.214	0.57	0.752
3	0.3697	0.946	0.8053	0.848	3.3461	0.341	1.0121	0.798
4	0.5254	0.971	1.9738	0.741	3.3478	0.501	1.0126	0.908
5	3.8243	0.575	4.7901	0.442	8.6373	0.124	4.7643	0.445
6	3.8874	0.692	5.0686	0.535	8.6544	0.194	4.7644	0.574
7	3.9428	0.786	5.0703	0.651	9.4939	0.219	5.0415	0.655
8	4.7255	0.786	5.2565	0.73	9.552	0.298	5.0616	0.751
9	10.285	0.328	8.975	0.44	9.8097	0.366	5.3374	0.804
10	10.797	0.374	8.9875	0.533	15.75	0.107	9.9089	0.449
11	12.953	0.296	5.7471	0.547	15.961	0.143	10.72	0.514
12	15.579	0.211	7.9649	0.381	16.671	0.162	12.991	0.522
13	15.666	0.268	7.9962	0.455	16.945	0.202	13.185	0.594
14	16.339	0.293	8.0396	0.526	17.245	0.243	13.756	0.632

To test for heteroskedasticity, Table 10 presents the value of ARCH-LM test where the p-value more than 0.05 meaning that no ARCH effect on residual returns series of the (EUR/USD) and (GBP/USD) exchange rate.

Table 10. Results ARCH-LM test for ARCH effect

	Best Model	Time period	F-statistic	Prob.	$n * R^2$	Prob.
EUR/USD	EGARCH(1,1)	Full period	0.196153	0.6579	0.196283	0.6577
	EGARCH(1,1)	Sup-period	0.107856	0.7426	0.108001	0.7424
GBP/ USD	EGARCH(1,1)	Full period	2.902064	0.0886	2.901096	0.0885
	TGARCH(1,1)	Sup-period	0.515132	0.473	0.515678	0.4727

* Indicates significance at 5% level

3.8. Forecasting

For the forecasting of EGARCH(1,1) model in the full and sub period on the returns of (EUR/USD) exchange rate and EGARCH(1,1),TGARCH(1,1) model in period and sub period respectively on the returns of (GBP/USD) exchange rate, we employ the dynamic method. This is to find forecasts using static forecasts (RMS,MAE,MAPE and Theil), where the test values are the lowest for the specific models. Namely, that forecast accuracy details are explained by Table 11.

Table 11. Statics forecast for the models selected

	EURO/USD		GBP/USD	
Time Period	Full Period	Sup-Period	Full Period	Sup-Period
	EGARCH(1,1)	EGARCH-M(1,1)	EGARCH(1,1)	TGARCH(1,1)
RMSE	0.074582	0.065236	0.094341	0.090843
MAE	0.063903	0.052514	0.073895	0.068426
MAPE	5.305801	4.700308	4.923648	5.268033

Theil	0.030206	0.028588	0.032457	0.032823

4. Conclusion

This paper focuses on the modeling for the volatility of exchange rate on (EUR/USD) and (GBP/USD) as the exchange rate is considered as a series of financial times which may present volatility. It is best suited to GARCH family models. The EGARCH(1,1) is the best model selected for the return series of(EUR/USD) exchange rate in the both full and sub period series, while the best model selected for the return series of (GBP/USD) exchange rate was EGARCH(1,1) model in the full period and TGARCH(1,1) model in the sub-period. All the four models selected are based on student's t-distribution. This provides the leverage parameter of the EGARCH model of (EUR/USD) in the full and sub-periods, and the EGARCH model of (GBP/USD) in the full time period is the asymmetric impact of the negative shocks and positive shocks. The leverage impact is existing that is the positive shocks associated with good news. It causes lower volatility in relation to negative shocks associated with bad news. In the TGARCH model of (GBP/USD) in the sub period, the leverage impact is not existing and the impact is asymmetric between the positive and negative shocks. The impact of the negative shocks of the volatility is less than positive shocks. However, the shock does not have the characteristic of continuity for a long period.

References

[1] D. Dickey and W. Fuller, " Divolatility of the estimators for autoregressive time series with a unit root", Journal of American Statistical Association, Vol.74, No.366, pp.427–431, 1979.

[2] Z. Ding, R. Engle, C. Granger, " Long Memory Properties of Stock Market Returns and a New Model", *Journal of Empirical Finance*, pp. 83-106, 1993.

[3]C. Dritsaki, " Modeling and Forecasting of British Pound/US Dollar Exchange Rate: An Empirical Analysis" Springer Nature, 2018

[4]C. Dritsaki, "Modeling the volatility of exchange rate currency using GARCH model", University of Applied Sciences, Kila, Kozani, Greece, 2019.

[5] R. Engle, D. Lilien, and R. Robins, " Estimating time varying risk premia in the term structure, the arch-m model", *Econometrica*, Vol.55, No.2, pp.391–407, 1987.

[6] D. A. Hsieh, “The statistical properties of daily foreign exchange rates”, *Journal of International Economics*, Vol.24, pp.129–145, 1988.

[7] C. Jarque and A. Bera, “A test of non-normal of observations and regression residuals”, *International Statistical Review*, Vol.55, pp.163–172, 1987.

[8] Z. Kovacic, " Forecasting volatility on the Macedonian stock exchange", *International Research Journal of Finance and Economics*, Vol.18, pp.182–212, 2008.

[9] J. Lee and M. King, “A locally most powerful based score test for arch and garch regression distrubances”, *Journal of Business and Economic Statistics*, Vol.7, No.7, pp.259–279, 1993.

[10] G. Ljung and G. Box, "On the measure of lack of fit in time series models”, *Econometrica*, Vol.65, No.2, pp.297–303, 1978.

[11] S. Miletić, “Modeling and Forecasting Exchange Rate Volatility Comparison between EEC and Developed Countries”, *Industrija*, Vol.43, No.1, pp.7-24, 2015.

[12] D. Nelson, “Conditional heteroskedasticity in asset returns: A new approach”, *Econometrica*, Vol.59, No.2, pp. 347–370, 1991.

[13], J. Sandoval , “D Asymmetric Garch models fit better rate volatilities on emerging markets”, *Universidad Extern ado do Colombia,Odeon*. pp. 99-116, 2006.

[14] D.N.C. Vee, P.N. Gonpot and N. Sookia, “Forecasting Volatility of USD/MUR Exchange Rate using a GARCH (1,1) Model with GED and Student’s-t errors”, *University of Mauritius Research Journal*, Vol.17, 2011.