Estimating the reliability function of the asymmetrical hybrid parallel-series system: Applied study at the state company for vegetable oils industry

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ABSTRACT

The research studied and analyzed the hybrid parallel-series systems of asymmetrical components by applying different experiments of simulations used to estimate the reliability function of those systems through the use of the maximum likelihood method as well as the Bayes standard method via both symmetrical and asymmetrical loss functions following Rayleigh distribution and Informative Prior distribution.

The simulation experiments included different sizes of samples and default parameters which were then compared with one another depending on Square Error averages. Following that was the application of Bayes standard method by the Entropy Loss function that proved successful throughout the experimental side in finding the reliability function for the soap manufacturing machines of the State Company for Vegetable Oils Industries whose behavior was based on the hybrid parallel-series system of a symmetrical component that followed both the exponential distribution and the Rayleigh distribution.

Keywords: Asymmetrical Hybrid Parallel-Series System, Exponential Distribution, Rayleigh distribution, Reliability Function, Maximum Likelihood Method, Standard Bayes Method, Square Error Loss Function, Entropy Loss Function

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1. Introduction

The widespread of industry and the advancements in mechanical, electrical and electronic fields of knowledge invested in machines and equipment throughout the past century, brought with it an increased interest in the concept of reliability to be applied in modern researches and studies, an interest that seemed logical in the light of the rapidly advancing technologies that paved the road for the use of complex systems in different walks of life. Those developments made it a persistent need finding out the reasons behind the stoppages and failures of all kind of machines and equipment, since a failure in the work of a machine or equipment would lead to material losses, not to mention the reduction in production of course.

The majority of the previous studies in this field focused on studying the reliability of simple systems consisted of symmetrical or asymmetric components with series-parallel reliability systems, whereas many of the production sources are being made of working systems that are complex, nested in structure and asymmetrical in their behavior. Therefore, in this research, the hybrid parallel-series systems of asymmetrical components were studied and analyzed by applying different experiments of simulations used to estimate the reliability function of those systems

One of the old researches in this field was done by Raheem (2014) dedicated to study the estimators of the reliability function of the hybrid parallel-series systems and improved systems by applying the Bayes method [1]. On the other hand, Al-Saaidi (2016) succeeded in setting a model and estimating the reliability function for the A-out-of-H system and series system for cases where its components are not identical [2]. Whereas the Weibull Rayleigh distribution was applied by Merovci and Elbatal (2015), where the three-parameter hypothetical span life model was defined and studied. Some structural features of the new distribution were considered by applying the maximum likelihood method as well as the least squares method to estimate the parameters as well as to work out the information matrix of Fischer [3]. The researchers were able to explain the usefulness of the suggested model by applying it on real data. Uwaid (2012) used balanced and unbalanced loss functions to compare the Bayes estimators for the reliability function parameter, and failure rate of the Rayleigh distribution [4]. Dey (2012) considered the Bayes estimators of the parameter and reliability function of the Invers Rayleigh distribution using both symmetric and asymmetric loss functions and compared its performance based on a Monte Carlo simulation study [5].

On the other hand, Sha, et al. in 2015, took into consideration the statistical interference of two main hybrid systems of series-parallel and parallel-series systems based on masked data for the lifespan of the components. They had been designed by the bivariate exponential distribution of the variables involved [6]. Through his study, Luguterah (2016), suggested a new generalization for Rayleigh distribution called the odd generalized exponential Rayleigh distribution, where mathematical features were derived and where the application of the new model proved it to be appropriate for obtaining a better suitability compared to some currently used distributions [7].

Likewise, Monte Carlo simulation technique was used in [8] to compare the performance of MLE and the standard Bayes estimators of the reliability function of the one parameter exponential distribution. Two types of loss functions are adopted, namely, squared error loss function and modified square error loss function with informative and non-informative prior.

This study was organized in a way that the second part would deal with both the exponential distribution and the Rayleigh distribution along with the reliability function for each of them. The third part involves estimating the parameters and the reliability function for both the exponential and Rayleigh distributions through the application of maximum likelihood method and the standard Bayes method by means of identical and non-identical loss functions. The fourth part has dealt with the types of the reliability systems, focusing afterwards on the asymmetrical hybrid parallel-series system and how to find out its reliability function parameters by means of the estimation methods referred to in the third part. The fifth part covered the simulation experiments by applying the different simulation experiments applied in estimating the reliability function of the asymmetrical hybrid parallel-series system and that by means of the Maximum Likelihood method and the standard Bayes method with different loss functions, with the parameters estimated through those methods being compared with each other depending on the mean square error as a criterion. The sixth part was devoted to the practical application side using real data through the estimation of the reliability function for the soap manufacturing system of the state company for vegetable oils industry, since the industry included partial systems following the asymmetrical hybrid parallel-series system behavior. The seventh and last part was dedicated for the most important conclusions reached through the fifth and sixth parts.

2. The exponential and Rayleigh distribution

The probability density function for the random variable Y that follows the exponential distribution of the parameter α could be defined according to the following formula:

$$f(y;\alpha)=\frac{1}{\alpha}\,e^{-\frac{y}{\alpha}} \qquad ;\ y,\alpha>0 \tag{1}$$

While the formula of the cumulative density function is:

$$F(y;\alpha)=\Pr(Y\le y)=\int_0^y f(y)dy=1-\ e^{-\frac{y}{\alpha}} \tag{2}$$

Also, the reliability function could be defined as follows:

$$R(y;\alpha)=\Pr(Y>y)=1-F(y;\alpha)=\ e^{-\frac{y}{\alpha}} \tag{3}$$

The random variable v that follows the Rayleigh distribution with one parameter has a probability density function that could be defined by the following formula:

$$f(v\mid\omega)=\frac{v}{\omega^2}\,e^{-\left(\frac{v^2}{2\omega^2}\right)} \qquad ;\ v\ge 0\ ,\omega>0 \tag{4}$$

The cumulative density function, reliability function and risk function for the Rayleigh distribution are as follows:

$$F(v) = 1 - e^{-\left(\frac{v^2}{2\omega^2}\right)} \tag{5}$$

$$R(v) = e^{-\left(\frac{v^2}{2\omega^2}\right)} \tag{6}$$

$$h(v) = \frac{v}{\omega^2} \tag{7}$$

3. Methods of estimating the parameter and the reliability function

3.1. Maximum likelihood method

If $y_1, y_2, \dots, y_n$ represents a random independent sample of the observations of a variable following the exponential distribution, then likelihood function for the random variable y is:

$$L(y_i|\alpha) = \prod_{i=1}^{n} f(y_i|\alpha) = \prod_{i=1}^{n} \alpha^{-1} e^{-\frac{y_i}{\alpha}} = \alpha^{-n} e^{-\frac{\sum_{i=1}^{n} y_i}{\alpha}} \tag{8}$$

Thus we can get the estimator for the maximum likelihood method for the parameter α by calculating the natural logarithm of the formula (8), working out the partial derivative for the α parameter, and then equalizing the derivative to zero.

$$\hat{\alpha}_{ML} = \frac{\sum_{i=1}^{n} y_i}{n} \tag{9}$$

The reliability function of the exponential distribution could be estimated depending on the maximum likelihood method's state of stability and that by substituting equation (9) into equation (3) according to the following formula, as introduced in [9]:

$$\hat{R}(y)_{ML} = e^{-\left(\frac{\sum_{i=1}^{n} y_i}{n}\right) y} \tag{10}$$

Concerning the random variable that follows Rayleigh distribution, it is possible to get the estimator for the maximum likelihood for parameter ω by taking the natural logarithm of the likelihood function and then working out the partial derivative for the ω parameter and then equalizing the derivative to zero, as follows:

$$L(v_i|\omega) = \prod_{i=1}^{n} f(v_i|\omega) = \prod_{i=1}^{n} \frac{v}{\omega^2} e^{-\left(\frac{v^2}{2\omega^2}\right)} = \left(\frac{v}{\omega^2}\right)^n e^{-\frac{\sum_{i=1}^{n} v_i^2}{2\omega^2}} \tag{11}$$

$$LnL(v_i|\omega) = \sum_{i=1}^{n} Ln(v_i) - 2n\ Ln(\omega) - \frac{\sum_{i=1}^{n} v_i^2}{2\omega^2}$$

$$\hat{\omega}_{ML} = \left(\frac{\sum_{i=1}^{n} v_i^2}{2n}\right)^{\frac{1}{2}} \tag{12}$$

By substituting equation (12) into equation (6), Uwaid (2012) in [4], got the maximum likelihood estimator for the reliability function of the Rayleigh distribution, as follows:

$$\hat{R}(v)_{ML} = e^{-\left(\frac{v^2}{2\hat{\omega}^2}\right)} = e^{-\left(\frac{n\,v^2}{\sum_{i=1}^{n} v_i^2}\right)} \tag{13}$$

3.2. Standard Bayes method

The Bayes standard method supposes that the parameters needed to be estimated are random variables about which prior information must be available and must be represented as a Prior Distribution so that by using the Bayes Inversion Formula which combines the Prior Distribution and the Likelihood Function it becomes possible to get a probable density function that is known as the Posterior Distribution as per the following formula , as introduced in [10]:

$$P(\varphi|x) = \frac{L(x|\varphi)\, P(\varphi)}{\int_{\forall\varphi} L(x|\varphi)\, P(\varphi) d\varphi} \tag{14}$$

Where:

$L(x|\varphi)$ represents the likelihood function for an n size sample of the random variable X observations.

$P(\varphi)$ represents the primary probability density function for parameter φ

$P(\varphi|X)$ represents the posterior probability density function for parameter φ

3.2.1. Prior distribution

There are a number of prior probable distributions, some of which are those distributions where there is not enough information about the parameter needed to be estimated, or where no parameter information is available at all. In such cases, the prior distribution is obtained depending on the domain of the parameter needed to be estimated. This kind of distribution is called the Non-Informative Prior Distribution. However, when prior information about the parameter needed to be estimated is available from old experiments, then the Conjugate Prior density function, which is also known as the Informative Prior distribution, is applied. These are parameter-known probability functions that are selected depending on the likelihood function of the observations of the random variable for the parameters needed to be estimated. Accordingly, those functions must be specific and should be appropriate for the probability distribution. Bergor in [11], considered the conjugate prior function for both the exponential distribution and Rayleigh distribution follow the Inverted Gamma Distribution for (μ, σ)parameters and their formula , as follows:

$$P(\alpha) \propto \alpha^{-(\mu+1)}\, e^{-\frac{\sigma}{\alpha}} \qquad ; \ \alpha, \mu, \sigma, > 0 \tag{15}$$

$$P(\omega) \propto \omega^{-(\mu+1)}\, e^{-\frac{\sigma}{2\omega^2}} \qquad ; \ \omega, \mu, \sigma, > 0 \tag{16}$$

3.2.2. Posterior distribution

The Posterior distribution for the parameter of the random exponential distribution is found by substituting equations (8) and (15) into equation (14) as follows:

$$P(\alpha|y) = \frac{\alpha^{-n} \ \alpha^{-\mu-1}\ e^{-\frac{\sigma}{\alpha}}\, e^{-\frac{\sum_{i=1}^{n} y_i}{\alpha}}}{\int_0^\infty \alpha^{-n}\, \alpha^{\mu-1}\, e^{-\frac{\sigma}{\alpha}}\ e^{-\frac{\sum_{i=1}^{n} y_i}{\alpha}}\, d\alpha} = \frac{\alpha^{-\{(n+\mu)+1\}}\ e^{-\frac{\sigma+\sum_{i=1}^{n} y_i}{\alpha}}}{\int_0^\infty \alpha^{-\{(n+\mu)+1\}}\, e^{-\frac{\sigma+\sum_{i=1}^{n} y_i}{\alpha}} d\alpha}$$

Thus the formula for the posterior distribution of the exponential distribution is:

$$P(\alpha|y) = \frac{(\sigma + \sum_{i=1}^{n} y_i)^{(n+\mu)}}{\Gamma(n+\mu)}\ \alpha^{-\{(n+\mu)+1\}}\, e^{-\frac{(\sigma+\sum_{i=1}^{n} y_i)}{\alpha}} \tag{17}$$

Also, it is possible to find the posterior distribution for the random Rayleigh distribution parameter by substituting equation (11) and equation (16) into equation (14) as follows:

$$P(\omega|v) = \frac{\omega^{-(\mu+1)}\ \left(\frac{v}{\omega^2}\right)^n e^{-\frac{\sigma}{2\omega^2}}\ e^{-\frac{\sum_{i=1}^{n} v_i^2}{2\omega^2}}}{\int_0^\infty \omega^{-(\mu+1)}\ \left(\frac{v}{\omega^2}\right)^n e^{-\frac{\sigma}{2\omega^2}}\ e^{-\frac{\sum_{i=1}^{n} v_i^2}{2\omega^2}}\ d\omega}$$

$$P(\omega|v) = \frac{\omega^{-(2n+\mu+1)}\ e^{-\frac{\sigma+\sum_{i=1}^{n} v_i^2}{2\omega^2}}}{\int_0^\infty \omega^{-(2n+\mu+1)}\ e^{-\frac{\sigma+\sum_{i=1}^{n} v_i^2}{2\omega^2}}\ d\omega} \tag{18}$$

Moreover, it is possible to find the integrating denominator through the following conversion:

$$I = \int_0^\infty \omega^{-(2n+\mu+1)}\ e^{-\frac{\sigma+\sum_{i=1}^{n} v_i^2}{2\omega^2}}\ d\omega \tag{19}$$

$$x = \frac{\sigma + \sum_{i=1}^{n} v_i^2}{2\omega^2} \tag{20}$$

$$\omega = \left(\frac{\sigma + \sum_{i=1}^{n} v_i^2}{2x}\right)^{\frac{1}{2}} \tag{21}$$

$$|J\,| = \left|\frac{d\omega}{dx}\right| = \left|-\frac{\left(\sigma + \sum_{i=1}^{n} v_i^2\,\right)^{\frac{1}{2}}}{(2x)^{\frac{3}{2}}}\right| \Rightarrow d\omega = \frac{\left(\sigma + \sum_{i=1}^{n} v_i^2\,\right)^{\frac{1}{2}}}{(2x)^{\frac{3}{2}}}\ dx \tag{22}$$

By substituting equations (20), (21), and (22) into equation (19), we can obtain:

$$I = \frac{1}{2}\ \left(\frac{2}{\sum_{i=1}^{n} v_i^2 + \sigma}\right)^{\frac{2n+\mu}{2}} \int_0^\infty x^{\frac{2n+\mu}{2}-1}\ e^{-x}\ \ dx$$

And through the characteristics of Gamma function, we will observe that:

$$\int_0^\infty x^{\frac{2n+\mu}{2}-1}\ e^{-x}\ dx = \left\{\Gamma\left(\frac{2n+\mu}{2}\right)\right\}$$

This in turn leads to:

$$I = \frac{1}{2}\ \left(\frac{2}{\sum_{i=1}^{n} v_i^2+\sigma}\right)^{\frac{2n+\mu}{2}}\ \Gamma\left(\frac{2n+\mu}{2}\right) \qquad (23)$$

By substituting the result of the integration that was obtained by equation (23) into equation (18), the posterior distribution function for the Rayleigh distribution parameter will be:

$$P(\omega|v) = \frac{2\ \left(\frac{\sum_{i=1}^{n} v_i^2+\sigma}{2}\right)^{\frac{2n+\mu}{2}}\ e^{-\frac{\sigma+\sum_{i=1}^{n} v_i^2}{2\omega^2}}}{\Gamma\left(\frac{2n+\mu}{2}\right)\ \omega^{(2n+\mu+1)}} \qquad (24)$$

3.2.3. Loss functions

The loss function, according to Bayes method, is the difference between the estimated value of the parameter and its real value, that is $\widehat{\varphi}-\varphi$. The Loss function is usually denoted as $L(\hat{\varphi},\varphi)$, where the following conditions are met:

$i.\ \ L(\hat{\varphi},\varphi) = 0 \qquad ;\ \forall\ \hat{\varphi} = \varphi$

$ii.\ \ L(\hat{\varphi},\varphi) > 0 \qquad ;\ \forall\ \hat{\varphi} > \varphi$

By means of the Risk function which is also known as being the mathematical prediction of the loss function, we know that the amount that puts the loss function to minimum is the standard Bayes estimator for the parameter φ as is shown in the following formula:

$$\varphi_B = Risk(\hat{\varphi},\varphi) = E\{L(\hat{\varphi},\varphi)\} = \int_{\forall\alpha} L(\hat{\varphi},\varphi)\ P(\varphi|x)d\varphi \qquad (25)$$

Likewise, if $g(\varphi)$ is a function by the parameter φ, then the standard Bayes method estimator for $g(\varphi)$ function is:

$$\hat{g}(\varphi)_B = \int_{\forall\alpha} g(\varphi)\ L(\hat{\varphi},\varphi)\ P(\varphi|x)d\varphi \qquad (26)$$

Based on symmetry, researchers classify loss functions into two categories: the first one, the Symmetric Loss Functions, supposes the amount of loss obtained through estimation to be equal on both positive and negative sides, that is $L(\hat{\varphi},\varphi) = |\hat{\varphi}-\varphi|$, with the most important and most common symmetric loss function being the Squared Error Function, which could be mathematically defined as follows:

$$L(\widehat{\varphi},\varphi) = (\widehat{\varphi}-\varphi)^2 \qquad (27)$$

Generally speaking, it is possible to find the Bayes method estimator for the (φ) parameter by means of square error function as in formula (25):

$$\widehat{\varphi}_{BS} = R(\hat{\varphi},\varphi) = E(\widehat{\varphi}-\varphi)^2 = \int_{\forall\alpha} (\widehat{\varphi}-\varphi)^2\ P(\varphi|x)d\varphi$$

$$\widehat{\varphi}_{BS} = E(\varphi|x) = \int_{\forall\varphi} \varphi P(\varphi|x)d\varphi$$

The standard Bayes method estimator for the reliability function by means of the square error loss function as by formula (26) would be:

$$\hat{R}(x,\varphi)_{BS} = \int_{\forall\varphi} R(x,\varphi)\ P(\varphi|x)d\varphi \qquad (28)$$

The second category of the loss functions is the asymmetric loss functions that supposes that the amount of loss obtained through the estimation process is not being equal on both positive and negative sides. There are a number of asymmetric loss functions but the one used in this study is the entropy loss function that has the following formula:

$$L(\hat{\varphi},\varphi) = \left(\frac{\widehat{\varphi}}{\varphi}\right) - Ln\left(\frac{\widehat{\varphi}}{\varphi}\right) - 1 \qquad (29)$$

The Bayes method estimator obtained through the Entropy Loss function depending on the Bayes risk function formula is:

$$\widehat{\varphi}_{BE} = R(\hat{\varphi},\varphi) = E\left\{\left(\frac{\widehat{\varphi}}{\varphi}\right) - Ln\left(\frac{\widehat{\varphi}}{\varphi}\right) - 1\right\} = \int_{\forall\varphi}\left\{\left(\frac{\widehat{\varphi}}{\varphi}\right) - Ln\left(\frac{\widehat{\varphi}}{\varphi}\right) - 1\right\}P(\varphi|x)d\varphi$$

$$\widehat{\varphi}_{BE} = [E(\varphi^{-1} \mid x)]^{-1} = \left\{\int_{\forall\varphi}\varphi^{-1}P(\varphi|x)d\varphi\right\}^{-1}$$

Likewise, the Bayes method estimator for the reliability function obtained using the entropy loss function and through the formula (26) would be:

$$\hat{R}(x)_{BE} = [E\{R(x,\varphi)^{-1} \mid x\}]^{-1} = \left\{\int_{\forall\varphi}\{R(\varphi)\}^{-1}P(\varphi|x)d\varphi\right\}^{-1} \quad (30)$$

3.2.4 The standard Bayes method estimators for the reliability function

The standard Bayes method estimator for the exponential-distribution reliability function obtained by applying the square loss function through the equations (3), (17) and (28) could be as follows:

$$\hat{R}(y)_{BS} = E\{R(y,\alpha)|y\} = \int_{\forall\alpha}R(y)\,P(\alpha|y)d\alpha$$

$$\hat{R}(y)_{BS} = \int_0^\infty\left(e^{-\frac{y}{\alpha}}\right)\frac{(\sigma+\sum_{i=1}^n y_i)^{n+\mu}}{\Gamma(n+\mu)}\quad\alpha^{-(n+\mu+1)}\,e^{-\frac{(\sigma+\sum_{i=1}^n y_i)}{\alpha}}\,d\alpha$$

$$\hat{R}(y)_{BS} = \frac{(\sigma+\sum_{i=1}^n y_i)^{n+\mu}}{\Gamma(n+\mu)}\int_0^\infty\alpha^{-(n+\mu+1)}\,e^{-\frac{(\sigma+y+\sum_{i=1}^n y_i)}{\alpha}}\,d\alpha$$

So:

$$\hat{R}(y)_{BS} = \left\{\frac{\sigma+\sum_{i=1}^n y_i}{(\sigma+y+\sum_{i=1}^n y_i)}\right\}^{(n+\mu)} \quad (31)$$

Similarly, it is possible to find the standard Bayes method estimator for the exponential distribution reliability function by using the entropy loss function through the equations (3), (17) and (30) as follows:

$$\hat{R}(y)_{BE} = [E\{R(y,\alpha)^{-1} \mid y\}]^{-1} = \left\{\int_{\forall\alpha}\{R(y,\alpha)\}^{-1}P(\alpha|y)d\alpha\right\}^{-1}$$

$$\hat{R}(y)_{BE} = \left\{\int_0^\infty\left(e^{\frac{y}{\alpha}}\right)\frac{(\sigma+\sum_{i=1}^n y_i)^{n+\mu}}{\Gamma(n+\mu)}\quad\alpha^{-(n+\mu+1)}\,e^{-\frac{(\sigma-y+\sum_{i=1}^n y_i)}{\alpha}}\,d\alpha\right\}^{-1}$$

$$\hat{R}(y)_{BE} = \left\{\frac{(\sigma+\sum_{i=1}^n y_i)^{n+\mu}}{\Gamma(n+\mu)}\int_0^\infty\alpha^{-(n+\mu+1)}\,e^{-\frac{(\sigma-y+\sum_{i=1}^n y_i)}{\alpha}}\,d\alpha\right\}^{-1}$$

$$\hat{R}(y)_{BE} = \left\{\frac{\sigma+\sum_{i=1}^n y_i}{(\sigma-y+\sum_{i=1}^n y_i)}\right\}^{-(n+\mu)} \quad (32)$$

The standard Bayes method estimator for the Rayleigh distribution reliability function through the use of the square loss function could also be found through the equations (6), (24) and (28) as is shown below:

$$\hat{R}2(v)_{BS} = E\{R2(v,\omega)|V\} = \int_{\forall\omega}R2(v)\,P(\omega|v)d\omega$$

$$\hat{R}2(v)_{BS} = \int_0^\infty e^{-\left(\frac{v^2}{2\omega^2}\right)}\,\frac{2\left(\frac{\sum_{i=1}^n v_i^2+\sigma}{2}\right)^{\frac{2n+\mu}{2}}\quad e^{-\frac{\sigma+\sum_{i=1}^n v_i^2}{2\omega^2}}}{\Gamma\left(\frac{2n+\mu}{2}\right)\,\omega^{(2n+\mu+1)}}\,d\omega$$

$$\hat{R}2(v)_{BS} = \frac{2\left(\frac{\sum_{i=1}^{n} v_i^2 + \sigma}{2}\right)^{\frac{2n+\mu}{2}}}{\Gamma\left(\frac{2n+\mu}{2}\right)} \int_0^{\infty} \omega^{-(2n+\mu+1)}\ e^{-\frac{v^2+\sigma+\sum_{i=1}^{n} v_i^2}{2\omega^2}}\ d\omega$$

To solve the integration above, some conversions can be applied based on [4], so we obtain:

$$\hat{R}2(v)_{BS} = \frac{\left(\frac{\sigma + \sum_{i=1}^{n} v_i^2}{v^2}\right)^{\frac{2n+\mu}{2}}}{\left(\frac{\sigma + \sum_{i=1}^{n} v_i^2}{v^2} + 1\right)^{\frac{2n+\mu}{2}}}$$

$$\hat{R}2(v)_{BS} = \left(1 + \frac{v^2}{\sigma + \sum_{i=1}^{n} v_i^2}\right)^{-\frac{2n+\mu}{2}} \qquad (33)$$

Through equations (6), (24), and (30) it is possible to find the standard Bayes method estimator for the Rayleigh distribution of the reliability function by applying the Entropy loss function as follows:

$$\hat{R}2(v)_{BE} = [E\{ R2(v,\omega)^{-1} \mid V\}]^{-1} = \left\{ \int_{\forall\omega} \{R2(v,\omega)\}^{-1} P(\omega|v)\, d\omega \right\}^{-1}$$

$$\hat{R}2(v)_{BE} = \left\{ \int_0^{\infty} e^{-\left(\frac{v^2}{2\omega^2}\right)}\ \frac{2\left(\frac{\sum_{i=1}^{n} v_i^2 + \sigma}{2}\right)^{\frac{2n+\mu}{2}}\ e^{-\frac{\sigma+\sum_{i=1}^{n} v_i^2}{2\omega^2}}}{\Gamma\left(\frac{2n+\mu}{2}\right)\ \omega^{(2n+\mu+1)}}\ d\omega \right\}^{-1}$$

The above integration is solved by carrying out conversions so we get:

$$\hat{R}2(v)_{BE} = \left(1 - \frac{v^2}{\sigma + \sum_{i=1}^{n} v_i^2}\right)^{\frac{2n+\mu}{2}} \qquad (34)$$

4. Reliability systems

The reliability function is defined as the likelihood of having the system working properly after z period of time. It could also be defined as the possibility of the system unfailing throughout the period of $[0, z]$. The system is composed of a number of components and its importance lies in the nature of the relationships connecting those components, and it is upon the nature of those connections that the system is identified. Consequently, it is necessary to know the behavior of those components and the way they influence the overall behavior of the system eventually. The studies in [2, 12], have considered the reliability systems which divided into the following:

1. The Series System: The components making up this system are serial-connected to each other so the failure of any component leads to the failure and hence the stoppage of the entire system. In other words, if a system is made up of n of components (machines) then its reliability is mathematically expressed as follows:

$$R_s(z) = R_1(z).R_2(z) \ldots R_n(z) \prod_{i=1}^{n} R_i(z) \qquad , \qquad i = 1,2,\ldots,m \qquad (35)$$

2. The Parallel System: It is a system where its components are connected parallel to each other so if any of those components gets faulty, the system would still work and would not fail. The parallel system could be mathematically expressed as follows:

$$R_p(t) = 1 - \prod_{i=1}^{n} [1 - R_i(z)] \qquad (36)$$

3. The Hybrid System: It is a system made up of many partial systems, with each having many components within. Now when a system is composed of a number of partial systems, its components could be linked to each other serial or parallel and then within each of the partial systems we could also see components linked serials or parallel. This research would be dealing with the Hybrid parallel-series system.

4.1. The hybrid parallel-series system

This kind of hybrid system is made of n of partial systems linked serial to one another, and then within each partial system there are m partial systems that are linked to one another parallel. In this way, the system would still function even if there is only one component working, Whereas, the entire system would fail if one partial system stops working. In [1], the reliability function of the system was defined with n partial systems linked serially according to the following formula:

$R(z) = Pr\{\varphi\,(Z) > z\}$

Since φ (Z): represents a function identified by random time variables within the components of the system. Hence:

$R(z) = Pr(Z_1 > z\ , Z_2 > z\ , \dots , Z_m > z\ )$

If the partial systems are independent then:

$R(z) = Pr(Z_1 > z)\ Pr(\ Z_2 > z\ )\ \dots\ Pr(Z_n > z)$

$$R(z) = [R_1(z)\ R_2(z) \dots\ R_n(z)] = \prod_{i=1}^{n} R_i\ (z) \qquad (37)$$

Since $R_i(z)$ represents the reliability function of the partial system i and is calculated as below if then systems are independent and symmetrical:

$R_i(z)\ = 1\ -\ Pr\left(Z_j\ \le\ z\right)\ , \forall_j\ =\ 1\,, 2\,, \dots , m$

$R_i(z)\ \ =\ \ 1\ -\ \ Pr\ (Z_{i1}\ \le\ z\ , Z_{i2}\ \le\ z\ \dots .\ Z_{im}\ \le\ z\ )$

$R_i(z) = 1\ - [Pr\ (Z_{i1}\ \le\ z)\ .Pr\ (\ Z_{i2}\ \le\ z\ ).\ \dots\ .Pr\ (Z_{im}\ \le\ z)]$

$R_i(z) =\ 1\ \ - [\{1 - R_{i1}(z)\}\ .\ \{1 -\ R_{i2}(z)\}.\ \dots\ .\{1 -\ R_{im}(z)\}]$

$$R_i(z) = 1 - \prod_{j=1}^{m} \left[\left\{1 -\ R_{ij}(z)\right\}\right] \qquad (38)$$

Since R_{ij} represents the reliability function of the component j that is connected parallel within the partial system i .

Accordingly, if the system is made of two partial systems serially connected to each other and if each partial system is made of two components where the time variable in the first component is based on the exponential distribution for the parameter α and the second component following the Pareto distribution for the parameter ω , then the reliability function of each component within the partial system would be identified according to the equations (3) and (6) respectively, and the reliability function for the partial system i according to the equation (38) would be as follows:

$$R_i(z) =\ \left[1 - \left\{\left(1 -\ e^{-\frac{y}{\alpha}}\right)\ \left(1 - e^{-\frac{v^2}{2\omega^2}}\right)\right\}\right] \qquad (39)$$

Similarly, the reliability function for the hybrid parallel-series system according to the equation (37) would be:

$$R(z) = \prod_{i=1}^{n} \left[1\ \ - \prod_{j=1}^{m} \left\{1 -\ \ R_{ij}(z)\right\}\right]$$

$$R(z) = \prod_{i=1}^{2} \left[1 - \left\{(1 - R_{i1}(z)\ )\ \left(1 - R_{i2}(z)\right)\right\}\right]$$

$$R(z) = \left[1 - \left\{\left(1 -\ R1(z)\right)\ \left(1 - R2(z)\right)\right\}\right]^2 \qquad (40)$$

4.2. Maximum likelihood method estimator for the reliability function of the hybrid parallel – series system

The reliability function for the hybrid parallel-series system is estimated by means of the maximum likelihood method by substituting equations (10) and (13) into equation (40), so we get:

$$\hat{R}(Z)_{ML} = \left[1 - \left\{\left(1 -\ e^{-\frac{y\ \sum_{i=1}^{n} y_i}{n}}\ \right)\ \left(1 - e^{\ -\frac{n\,v^{\,2}}{\sum_{i=1}^{n} v_i^2}}\right)\right\}\right]^2 \qquad (41)$$

4.3. The Standard Bayes Method Estimator for the Reliability Function of Hybrid Parallel-Series System

Based on the Bayes method and the square loss function, we obtain the reliability function estimator of the hybrid parallel-series system by substituting the equations (31) and (33) into the equation (40) as follows:

$$\hat{R}(Z)_{BS} = \left[1 - \left\{\left\{1 - \left\{\frac{\sigma + \sum_{i=1}^{n} y_i}{(\sigma + y + \sum_{i=1}^{n} y_i)}\right\}^{(n+\mu)}\right\}\left\{1 - \left(1 + \frac{v^2}{\sigma + \sum_{i=1}^{n} v_i^2}\right)^{-\frac{2n+\mu}{2}}\right\}\right\}\right]^2 \quad (42)$$

Based on Bayes method and the Entropy loss function, we obtain the estimator of the reliability function of the hybrid parallel-series system by substituting the equations (32) and (34) into the equation (40) as follows:

$$\hat{R}(Z)_{BE} = \left[1 - \left\{\left\{1 - \left(\frac{\sigma + \sum_{i=1}^{n} y_i}{(\sigma - y + \sum_{i=1}^{n} y_i)}\right)^{-(n+\mu)}\right\}\left\{1 - \left(1 - \frac{v^2}{\sigma + \sum_{i=1}^{n} v_i^2}\right)^{\frac{2n+\mu}{2}}\right\}\right\}\right]^2 \quad (43)$$

5. Experimental sides

In this section, relying on the R.3.5.1 program, the Monte Carlo Simulation experiments were employed in estimating the reliability function of the hybrid parallel-series system as per the following steps reported in [13, 14]:

1. Selecting the default values for the exponential distribution parameter and the Rayleigh distribution parameter as well as for the parameters of the conjugate prior as is shown in the table 1 below:

Table 1. Different Simulation Experiments

Experiment	α	ω	μ	σ
1	2	2	2	2
2	5	5	2	5
3	2	5	2	3
4	5	2	2	3

2. Selecting the sizes of the default samples of the four components (10,25,50,100), in addition to selecting five times of running the hybrid parallel-series system that are Z=1,2,3,4,5 , with the recurrence of experiments being $B = 1000$.

3. Generating the random variables $y_i, v_i, i = 1,2,3,4$ that follow the exponential distribution and the Rayleigh distribution respectively, and that according to the values of the parameters default in step 1 , as well as generating the variables that follow the inverted Gamma distribution which in turn represents the natural conjugate prior by depending on the Inverse Transformation Method.

4. Estimating the reliability function of the hybrid parallel-series system according to the methods, equations and formulas mentioned in the theoretical side of the study, since

$$\hat{R}(z) = \frac{1}{B}\sum_{b=1}^{B}\hat{R}_b(z) \quad (44)$$

Also, estimating the Mean Square Error (MSE) for those estimators as follows:

$$MSE[\hat{R}(z)] = \frac{1}{B}\sum_{b=1}^{B}[\hat{R}_b(z) - R(z)]^2 \quad (45)$$

5.1. Simulation result

Through the results of the simulation experiments aimed at finding and comparing the reliability function of the hybrid parallel-series system that were gained through the application of the maximum likelihood method, and the standard Bayes method obtained by the Square Loss function and the Entropy function shown in tables 2-5 , the following became evident:

1- It was shown by the estimated values of the reliability function of the hybrid parallel-series system that the estimated values of the standard Bayes method through the Entropy Loss function $\hat{R}_{BE}$ was closest to the default values compared to the standard Bayes method via the square loss function $\hat{R}_{BS}$ and the maximum likelihood method $\hat{R}_{MLE}$ at sample sizes of (10,25,50). Whereas, the maximum likelihood method $\hat{R}_{MLE}$ was excelled on the Bayes standard method by virtue of the two loss functions $\hat{R}_{BS}$,$\hat{R}_{BE}$ at a sample size of 100 , noting that the estimated values are close to the default values ,as the sample size increases for all methods.

2- At sample sizes of (10, 25, 50), the estimated values of the means square error (MSE)for the reliability function of the hybrid parallel-series system under the Bayes standard method and through the Entropy Loss function $\hat{R}_{BE}$ were the least, followed by the valued of Bayes standard method through the square loss function $\hat{R}_{BS}$ which were followed by the values pertinent to the maximum likelihood method $\hat{R}_{MLE}$. Whereas, the estimated values of the means square error (MSE) of the maximum likelihood method $\hat{R}_{MLE}$ were less compared to the Bayes standard method at a sample size of 100.
3- When the default values of the parameters of the exponential distribution and of the Rayleigh distribution are different then the estimated values for the reliability function of the hybrid parallel-series system are far from the default values compared to the estimated values obtained in case the parameters of the two distributions are equal. This case holds true to the estimated value of the means square error which get less when the default parameters of the two distributions are equal.

Table 2. Estimated values of the reliability function and means square error for the hybrid system($\alpha = 2, \omega = 2$)

(α,ω)	n	z	$REAL$	$\hat{R}_{MLE}$	MSE	$\hat{R}_{BS}$	MSE	$\hat{R}_{BE}$	MSE
(2,2)	10	1	0.909669838	0.941318959	0.001252084	0.948461596	0.001881001	0.957470515	0.002856131
		2	0.564421549	0.682587923	0.014213709	0.717833218	0.02391134	0.753137559	0.036184959
		3	0.225950845	0.370816088	0.021236355	0.42893185	0.041577489	0.474156015	0.062177033
		4	0.063683009	0.170477545	0.01165549	0.225830407	0.026667979	0.268165233	0.042384206
		5	0.014985523	0.078202474	0.0042468	0.11775622	0.010938016	0.158801926	0.021254384
	25	1	0.909669838	0.928493617	0.000442918	0.936634969	0.000908898	0.949812273	0.002014269
		2	0.564421549	0.630305472	0.004429275	0.664417465	0.010180963	0.718428527	0.024121003
		3	0.225950845	0.300222164	0.005604813	0.347051781	0.014847216	0.420176619	0.038126505
		4	0.063683009	0.112709547	0.002492185	0.149943271	0.007622612	0.212972501	0.022690206
		5	0.014985523	0.040189808	0.00072384	0.062679429	0.002456488	0.109588718	0.009352618
	50	1	0.909669838	0.92098359	0.000160001	0.923423111	0.000236441	0.938026041	0.001005093
		2	0.564421549	0.577570609	0.000204898	0.612445505	0.002353588	0.66842314	0.011017349
		3	0.225950845	0.242484646	0.000305367	0.283438743	0.003352147	0.352722202	0.016271996
		4	0.063683009	0.07738521	0.000219751	0.105100463	0.001762694	0.158272682	0.009148225
		5	0.014985523	0.023324842	0.000101544	0.037880986	0.00057149	0.072457137	0.003504005
	100	1	0.909669838	0.912116149	7.48055E-06	0.921954075	0.000188628	0.936660558	0.000910624
		2	0.564421549	0.572142919	6.11157E-05	0.608781258	0.002005509	0.666692515	0.010641475
		3	0.225950845	0.245289449	0.000375478	0.288498498	0.003949935	0.36304446	0.018976784
		4	0.063683009	0.087807062	0.000583466	0.118762496	0.003071475	0.180253605	0.013770828
		5	0.014985523	0.033108869	0.000329952	0.051697124	0.001385467	0.095664762	0.006691264

Table 2 above showed that at sample sizes (10,25,50,100), $\hat{R}_{MLE}$ was closest to the default values from $\hat{R}_{BS}$, $\hat{R}_{BE}$. Also, it is generally observed from table 2 at sample sizes (10,25,50,100) that the estimated values of MSE for $\hat{R}_{MLE}$ were the least followed by the estimated values of MSE for $\hat{R}_{BS}$ and then the estimated values for MSE of $\hat{R}_{BE}$.

Table 3. Estimated values of the reliability function and mean square error for the hybrid system ($\alpha = 5, \omega = 5$)

(α,ω)	n	z	$REAL$	$\hat{R}_{MLE}$	MSE	$\hat{R}_{BS}$	MSE	$\hat{R}_{BE}$	MSE
(5,5)	10	1	0.99283414	0.952092217	0.00207488	0.960532214	0.001304268	0.962215798	0.001171854
		2	0.949948472	0.814202015	0.018842077	0.822827422	0.016420615	0.824139603	0.016062242
		3	0.856875754	0.629096272	0.052298468	0.635638961	0.049206572	0.63086061	0.051317216
		4	0.721137594	0.432847234	0.083345702	0.446912095	0.075460478	0.438657992	0.080209702
		5	0.564421549	0.264740133	0.090043322	0.287360098	0.077023901	0.275942731	0.083635005
	25	1	0.99283414	0.9613301	0.001240631	0.977433088	0.000296491	0.968073877	0.000766338
		2	0.949948472	0.848961441	0.010446506	0.877317105	0.005334614	0.854772823	0.009211672
		3	0.856875754	0.691465812	0.027608575	0.716472826	0.01977228	0.693005581	0.027006701
		4	0.721137594	0.517454053	0.041640253	0.53639775	0.034188108	0.518957657	0.041124853
		5	0.564421549	0.355215702	0.043920354	0.370558645	0.037642124	0.3588332	0.042514696

	50	1	0.99283414	0.965945928	0.00090372	0.996353417	1.54816E-05	0.966183892	0.000887795
		2	0.949948472	0.852498256	0.009677288	0.97315125	0.000541465	0.85286811	0.009602156
		3	0.856875754	0.693234848	0.02695909	0.918858621	0.003844972	0.693815899	0.026766075
		4	0.721137594	0.520266282	0.040530028	0.832474084	0.01239891	0.521407777	0.040069559
		5	0.564421549	0.360061273	0.041943867	0.722260819	0.024916331	0.361939826	0.041176407
	100	1	0.99283414	0.99461812	3.97823E-06	0.989334745	1.53072E-05	0.981203347	0.000169094
		2	0.949948472	0.961184152	0.000127036	0.928374343	0.000468504	0.887527908	0.003930146
		3	0.856875754	0.885639016	0.000828121	0.804082346	0.002790205	0.722611271	0.01806077
		4	0.721137594	0.770886583	0.002475758	0.636213191	0.007215216	0.528900605	0.036988879
		5	0.564421549	0.632749054	0.004669443	0.459135289	0.011088258	0.349314028	0.046305064

From Table 3, it is generally observed that at sample sizes (10,25,50), $\hat{R}_{BS}$ was closest to the default values compared to $\hat{R}_{BE}$, $\hat{R}_{MLE}$, whereas at sample size 100 , the $\hat{R}_{MLE}$ was closest to the default values from $\hat{R}_{BS}$, $\hat{R}_{BE}$. Also from table 3 it is observed that at sample sizes (10,25,50), the estimated values of MSE for $\hat{R}_{BS}$ were the least followed by the estimated values of MSE for $\hat{R}_{BE}$ and then the estimated values of MSE for $\hat{R}_{MLE}$, while at sample size 100 the estimated values of MSE for $\hat{R}_{MLE}$ were the least followed by $\hat{R}_{BS}$ and then $\hat{R}_{BE}$.

Table 4. Estimated values of the reliability function and mean square error for the hybrid system ($\alpha = 2, \omega = 5$)

(α,ω)	n	z	$REAL$	$\hat{R}_{MLE}$	MSE	$\hat{R}_{BS}$	MSE	$\hat{R}_{BE}$	MSE
(2,5)	10	1	0.957854281	0.949816621	0.001501788	0.951178602	0.001386085	0.779340587	0.039833924
		2	0.757389099	0.812956424	0.00880231	0.813961302	0.008594868	0.35845319	0.167116644
		3	0.483429647	0.625641211	0.018468384	0.629070942	0.017532428	0.106010182	0.150412237
		4	0.274423207	0.431064684	0.023230173	0.438656219	0.020965553	0.02168032	0.071845751
		5	0.156541202	0.264245396	0.020995071	0.275942634	0.017743306	0.00333797	0.031438015
	25	1	0.957854281	0.960215227	0.000735869	0.968834415	0.000305913	0.960231198	0.000734901
		2	0.757389099	0.848769312	0.003327361	0.855976055	0.002480485	0.848997062	0.003301532
		3	0.483429647	0.691451118	0.004905265	0.693760952	0.004505949	0.692388445	0.004776638
		4	0.274423207	0.518956852	0.004183714	0.521175054	0.00390658	0.517768109	0.004250187
		5	0.156541202	0.358833164	0.002574589	0.362571007	0.002220048	0.366444274	0.002850185
	50	1	0.957854281	0.973464652	0.000151625	0.973469981	0.000151478	0.970266456	0.00025247
		2	0.757389099	0.865964972	0.001566768	0.866066214	0.001558812	0.859058198	0.002176097
		3	0.483429647	0.703263063	0.003298388	0.703715498	0.003246834	0.697613319	0.003996434
		4	0.274423207	0.525419151	0.003287478	0.526517942	0.003163237	0.52298039	0.003591962
		5	0.156541202	0.362194074	0.00213786	0.36406203	0.001969812	0.355317194	0.002847415
	100	1	0.957854281	0.983114608	9.75506E-06	0.988533131	2.05523E-05	0.979997128	2.51008E-05
		2	0.757389099	0.89615242	0.000241066	0.923837493	0.000352867	0.88249255	0.000518945
		3	0.483429647	0.793912219	0.001125167	0.738040562	0.001144921	0.714476411	0.002116756
		4	0.274423207	0.621234864	0.001505226	0.547229111	0.002349384	0.520505249	0.003847207
		5	0.156541202	0.442110733	0.001160707	0.362455842	0.002134063	0.34290622	0.004255506

In Table 4, it is generally observed that $\hat{R}_{MLE}$ was the closest to the default values from $\hat{R}_{BS}$, $\hat{R}_{BE}$ at sample sizes (10,25,50), whereas $\hat{R}_{BE}$ was closest to the default values at sample size 100 . As for the estimated values of MSE , it can be seen from table 4 at sample sizes (10,25,50) that the estimated values of MSE for $\hat{R}_{BS}$ were the least followed by the estimated values of MSE for $\hat{R}_{MLE}$ and then $\hat{R}_{BE}$, while the estimated values of MSE for $\hat{R}_{MLE}$were the least at sample size of 100.

Table 5. Estimated values of the reliability function and mean square error for the hybrid system ($\alpha = 5, \omega = 2$)

(α, ω)	n	z	$REAL$	$\hat{R}_{MLE}$	MSE	$\hat{R}_{BS}$	MSE	$\hat{R}_{BE}$	MSE
(5,2)	10	1	0.984478273	0.863161025	0.011208516	0.890306842	0.005703321	0.879624925	0.00764979
		2	0.905162458	0.47105083	0.083519563	0.51181371	0.061447936	0.494671778	0.071262094
		3	0.760430034	0.13873677	0.120343137	0.186736721	0.089167356	0.194005069	0.086008289
		4	0.582490626	0.022586312	0.064951779	0.048449807	0.052204642	0.052905265	0.051311902
		5	0.408101967	0.0024144	0.025285029	0.010263758	0.022537755	0.010026261	0.023708331
	25	1	0.984478273	0.868953969	0.009879082	0.920316968	0.001761312	0.903238967	0.003728541
		2	0.905162458	0.45521696	0.093283818	0.60207179	0.024475729	0.541119258	0.047518352
		3	0.760430034	0.137875218	0.12138368	0.269784174	0.04599665	0.200847189	0.080598553
		4	0.582490626	0.025405204	0.063985782	0.092774137	0.033348647	0.050166737	0.051036672
		5	0.408101967	0.003076669	0.025527179	0.029077224	0.016599328	0.010345466	0.022118901
	50	1	0.984478273	0.93030517	0.000948692	0.931362518	0.000877267	0.93031654	0.000947909
		2	0.905162458	0.637851521	0.014478971	0.643016198	0.013256614	0.639087601	0.014184826
		3	0.760430034	0.313386159	0.029104526	0.321395638	0.026430473	0.317341862	0.027774734
		4	0.582490626	0.126836595	0.021971546	0.133442659	0.020050968	0.131367999	0.020654374
		5	0.408101967	0.051210655	0.011284262	0.055036098	0.010478739	0.05435012	0.010632599
	100	1	0.984478273	0.945680515	0.000185251	0.943321104	0.000264017	0.938840942	0.000451884
		2	0.905162458	0.70651293	0.002625435	0.696438394	0.003767792	0.673952533	0.007052037
		3	0.760430034	0.421748774	0.00384158	0.407119547	0.005876035	0.364141612	0.014320012
		4	0.582490626	0.235052528	0.0015871	0.220822526	0.002925836	0.169912249	0.011012917
		5	0.408101967	0.138894285	0.000348464	0.126747601	0.000940462	0.080429833	0.005883317

Table 5 depicted that at sample size (10,25,50), $\hat{R}_{BS}$ were closest to the default values compared to $\hat{R}_{BE}$, $\hat{R}_{MLE}$, while $\hat{R}_{MLE}$ was closest at sample size 100 , also from table5 it was found that at sample sizes (10,25,50), the estimated values of MSE for $\hat{R}_{BS}$ were the least followed by the estimated values of MSE for $\hat{R}_{BE}$ and then the estimated values of MSE for $\hat{R}_{MLE}$,whereas at sample size 100 the estimated values of MSE for $\hat{R}_{MLE}$ were the least.

6. Applications

The State Company for Vegetable Oils Industry was chosen to collect the data related to the research, specifically the soap section where "De Lux" soap is made and where the production process passes through the five stages: saponification, color mixing, cutting, printing -wrapping and packing. The time of running the two machines (1,3) were registered starting from the cutting stage as well as the time of running machines (2,4) that involved the printing - wrapping stage. All machines were made in Italy in 2007. The connection system linking those machines was a hybrid parallel-series asymmetrical-component one. The study included the data available in the company's records representing the times of running the four machines covering the period from June 5th 2017 to September 5th 2017, with excluding the stoppages occurred for maintenance purposes, and with an average of a daily working hours of 8. The working hours between a faulty-driven stoppage and the next are shown (in hours) in Table 6. Working hours belonging to old periods were collected as well and are presented in Table 7.

Table 6. Times of running the soap producing machines (in hours)

obs	Machine				obs	Machine			
	1	2	3	4		1	2	3	4
1	26.4	3.86	4.24	13.03	13	6.16	1.25	6.31	0.4

2	1	6.16	8.62	1.57	14	9.29	0.12	5.34	3.5
3	0.3	6.16	4.12	0.14	15	8.23	1.41	7.54	0.85
4	2.7	5.48	0.43	0.61	16	3.06	3.77	6.38	4.5
5	16.5	1.58	7.87	2.14	17	7.32	6.28	1.71	5.4
6	12	0.64	10.07	1.41	18	4.28	1.73	1.44	5.2
7	1.5	5.06	3.96	1.98	19	1.15	0.74	1.26	0.2
8	3.1	2.4	8.92	1.03	20	9.4	2.82	3.62	2.5
9	2.2	3.79	1.72	1.23	21	15.54	0.71	5.81	8.5
10	0.9	3.55	4.91	1.15	22	8.41	0.41	1.72	11.6
11	6	2.84	7.55	3.09	23	4.2	2.19	2.5	4
12	2.2	5.3	9.64	0.02	24	4.49	0.79	3.69	6.8

Table 7. Times of running the soap producing machines (in hours) for previous period

obs	Machine 1	obs	Machine 2	obs	Machine 3	obs	Machine 4
1	6.11	1	1.92	1	14.56	1	3.73
2	16.15	2	2.13	2	11.17	2	0.03
3	1.71	3	8.16	3	1.25	3	10
4	0.1	4	3.04	4	4	4	9.84
5	2.66	5	4.12	5	19.8	5	3.77
6	5.78	6	2.92	6	5.37	6	8.37
7	0.57	7	1.07	7	2.45	7	0.95
8	0.73	8	4.56	8	0.6	8	0.07
9	1.87	9	0.15	9	2.27	9	1.88
10	0.22	10	2.35	10	31.22	10	2.57
11	0.47	11	8.41	11	5.42	11	9.1
12	1.75	12	2.14	12	0.07	12	1.85

The goodness of fit test was carried out to know the distribution of the previous as well as the current data related to the running hours of the four machines and that depending on the Kolmogorov Smirnov Test and that at a significance level of (0.05) , and the hypotheses wear as follow:
Data of first and third machines following the exponential distribution: H_0
Data of second and fourth machines following the Rayleigh distribution: H_0
Previous data for the four machines following the Inverse Gamma distribution: H_0
The results were as shown in tables (8) and (9) below:

Table 8. K-S tests on machines running data

Machine	1	2	3	4
K- S	0.0793322	0.138382	0.154076	0.1436
P-Value	0.99817	0.747588	0.619063	0.705408

Table 9. K-S tests on previous machine-running data

Machine	1	2	3	4
K- S	0.114	0.15743	0.7583	0.18726
P-Value	0.9925	0.8831	0.18202	0.7283

The P-Value shows the validity of the null hypotheses above since they are greater than the level of significance, which means that the working-hours data of the first and third machines followed the exponential distribution and that the second and fourth machines followed the Rayleigh distribution. The same was true with the running hours data belonging to previous periods as they followed the Inverted Gamma distribution.
Now we will calculate the reliability function of the asymmetrical hybrid parallel-series system for the machine-running times (in hours) during the two stages of cutting and printing - wrapping and that by applying the Bayes standard method through the Entropy Loss function via equation (43), noting that the values for the previous

distribution parameters were ($\mu \approx 2, \sigma \approx 5$) that were approximately found by means of the (R) program. The obtained results are shown in table (10) below:

Table 10. Values of the reliability function of the hybrid system

Z	$\hat{R}(Z)_{BE}$
1	0.979361
2	0.867928
3	0.66495
4	0.43462
5	0.242837
6	0.117287
7	0.049977
8	0.019406
9	0.007192
10	0.002689

6. Conclusions

Through the results gained in both the experimental and applied sides, we can conclude the following:

1- All the estimations for all the estimation methods used in estimating the reliability function of the hybrid parallel-series system were identical with the statistical theory. The greater the size of the sample the closer the estimated values of the function to the default values and the lesser the values of the estimated mean square error. Also, the values of the reliability function were diminishing over the time (Z).

2-The Bayes Standard Method carried out through the Entropy Loss function excelled in estimating the reliability function of the parallel-series system when the sample size was small or medium. Whereas the Maximum Likelihood method was the best when the sample size was big.

3-The values calculated for the reliability function of the hybrid system of the soap producing machines during the two stages of cutting and printing - wrapping indicate that the efficiency of those machines was declining over time despite their modernity. This could also be noticed by comparing the previous running times with the current ones for the four machines involved.

4-The company lacks the new electronic and engineering systems needed to help in identifying the faults accumulatively. That was evident in the occurrence of the long stoppages of the majority of the machines and across all stages as we noticed.

References

[1] 1. H. K. Raheem, Comparing the Reliability Function Estimators of the Hybrid and Enhanced Systems through the Application of some of Bayes Methods, PhD thesis, College of Management and Economic, Mustansiriyah University, Iraq, 2014.

[2] A. H. Al-Saaidi, Modeling and Estimating the Reliability Function for some Systems with asymmetrical Components, PhD thesis, College of Management and Economic, Mustansiriyah University, Iraq, 2016.

[3] F. Merovci, and Ibrahim Elbatal, "Weibull-Rayleigh distribution: theory and applications." *Appl. Math. Inf. Sci*, vol.9, no. 5, pp. 1-11, 2015.

[4] G. R. Uwaid, Comparing the Bayes Estimators for the Reliability Function, Parameter, and Failure Rate through the Use of Balanced and Unbalanced Loss Functions, PhD thesis, College of Management and Economic, Mustansiriyah University, Iraq, 2012.

[5] S. Dey, "Bayesian Estimation of the Parameter and Reliability Function of an Inverse Rayleigh Distribution", *Malaysian Journal of Mathematical Sciences,* vol. 6, no.1, pp.113-124, 2012.

[6] N. Sha, R. Wang, P. Hu, and X. Xu, "Statistical inference in dependent component hybrid systems with masked data." *Advances in Statistics, vol.* 2015, pp.1-11, 2015.

[7] 7. A. Luguterah, Odd Generalized Exponential Rayleigh Distribution, Advances *and Application in Statistics*, vol.48, no. 1, pp. 33-48, 2016 .

[8] M. J. Ali and H. M. Gorgees, "The Comparison Between the MLE and Standard Bayes Estimation of the Reliability Function of Exponential Distribution", *Ibn Al-Haitham Journal for Pure and Applied Science*, vol. 30, no.2, pp.224-232, 2019.

[9] H.F. Martz and R.A. Waller, Bayesian Reliability Analysis. Wiley, New York, 1982.

[10] G.J. Andrew, B.C. John, S.S. Hal and B.R. Donald, Bayesian Data Analysis. Champan and Hall/CRC Publisher, 2004.

[11] J.O. Bergor, Statistics of Decision Theory and Bayesian Analysis, 2nd Edition, Springer Verging, New York, 1985.

[12] M. Rausand and A. Hoylan, System Reliability theory.,models, Statistical Methods and applications. John wiley and sone, 2nd edition, 2004.

[13] H.S. Al-Kutubi, On Comparison estimation, procedures for parameter and survival function exponential distribution using simulation, Ph.D. Thesis, College of Ibn Al-Hatham, Baghdad University, Iraq, 2005.

[14] P. R. Christian and Gorge Casella, Introduction Monte Carlo Methods With R, Springer, 2010.