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Adapting some statistical methods to analyze TDS in drinking water

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ABSTRACT

In this study, numerous statistical models were used including the Box-Jenkins models with several stages to build and forecasting the best model in the analysis of time series. Modern methods in time series analysis including fuzzy logic and fuzzy sets, have appeared as the most important alternatives to classical statistical methods. They have a mechanical ability to find solutions because they do not require the availability of classical model conditions, which are difficult to achieve in most cases.

This paper aims to find the best method to analyze the behavior of pollution rates by studying Box-Jenkins and high order fuzzy time series methods. Then, an adaptation has conducted between the two methods as a proposed procedure on chemical examined data for total dissolved solids in drinking water for Baghdad city. The data are recorded from January 2004 to December 2018. These methods are compared in details through statistical criteria RMSE, MAE, MAPE.

Keywords: ARIMA, high order fuzzy, TMF, PSO, prediction, criterion.

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1. Introduction

Statistics science has prioritized the forecasting of time series for foreseeing the future through prediction theory and methods of forecasting techniques. The idea of documenting the historical data for phenomena in all fields using these data in forecasting is one of the most important statistical bases. A lot of statistical techniques have been used in this field, such as regression analysis, Box-Jenkins models etc., which were prevalent. Due to the great interest in the field of inaccuracies existing in various fields, the fuzzy has emerged to mimic human thinking, which depends on contrastive degrees. Fuzzy groups have found successful applications in many fields.

Fuzzy logic is a technique with the ability to find solutions for various scientific and human problems. The most important modern techniques for the application of this logic is the idea of fuzzy time series. It was proposed by the researchers' Song and Chissom in 1993 as an alternative to classical methods, which gives effective results as compared with classical statistical techniques [1].

The goal of the paper is to analyze the behavior of pollution rates in drinking water for Baghdad city by using the Box-Jenkins and high order fuzzy time series methods. An adaptation the two methods as a modified technique was employed to improve the adequacy of the model. It has taken a series of monthly chemical examinations for the total dissolved solids (TDS) from January 2004 to December 2018 for drinking water in Baghdad city. The best method is found in several comparative criteria based on RMSE, MAE, and MAPE.

2. Methodology

2.1. Box-Jenkins method

The autoregressive model (AR) represents the correlation of the current observations of the time series with previous observations of the same series and can be written as follows:

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \ldots + \phi_p Z_{t-p} + a_t \qquad \ldots (1)$$

It can be denoted as AR(p) and p represents the model's order. Accordingly, Z represents the series observations and ϕ represents the parameters of the model.
The moving average model (MA) is a correlation of the current observations of the time series with the same series error as previous observations and the general equation:

$$Z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \qquad \ldots (2)$$

It is denoted as MA(q) and q represents the model's order. Accordingly, θ represents the parameters of the model.
The mixed model autoregressive-moving average denoted as ARMA(p,q) is the correlation of the current time series values with previous values of the same series. Also, it includes the correlation of the series values with the same series error of previous observations as follows:

$$Z_t - \phi_1 Z_{t-1} - \phi_2 Z_{t-2} - \ldots - \phi_p Z_{t-p} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \qquad \ldots (3)$$

Where (p, q) stands for the model order, and the model above represents the non-seasonal part or the so-called regular part for the time series.
Most studies have proven that autoregressive integrated moving average models (ARIMA) have superiority in all applied fields in model identification and time series prediction. For these models transformation, the time series has the stationarity feature. It is realized by taking a number of differences d for non-stationary time series where the degree of differences d is given, and $w_t = \nabla^d Z_t$ for transforming it into a stationary series. The ARIMA(p, d, q) model can be written as [2, 3]:

$$\left.\begin{aligned} &\phi_p(B) w_t = \phi_p(B)\nabla^d Z_t = \theta_q(B) a_t \qquad , a_t \sim WN(0,\sigma^2) \\ OR \quad &\phi_p(B)(1-B)^d Z_t = \theta_q(B) a_t \qquad , a_t \sim WN(0,\sigma^2) \end{aligned}\right\} \ldots (4)$$

2.2. Building the Box-Jenkins method

The Box-Jenkins (B-J) method is used to represent and analyse stationary or non-stationary time series, for seasonal and non-seasonal types. It can predict future values of the phenomenon by applying models of this method in the process of building the model of the time series. It includes identification the series and determines the appropriate model using some suitable criteria for this stage with estimation and prediction processes [2, 4].

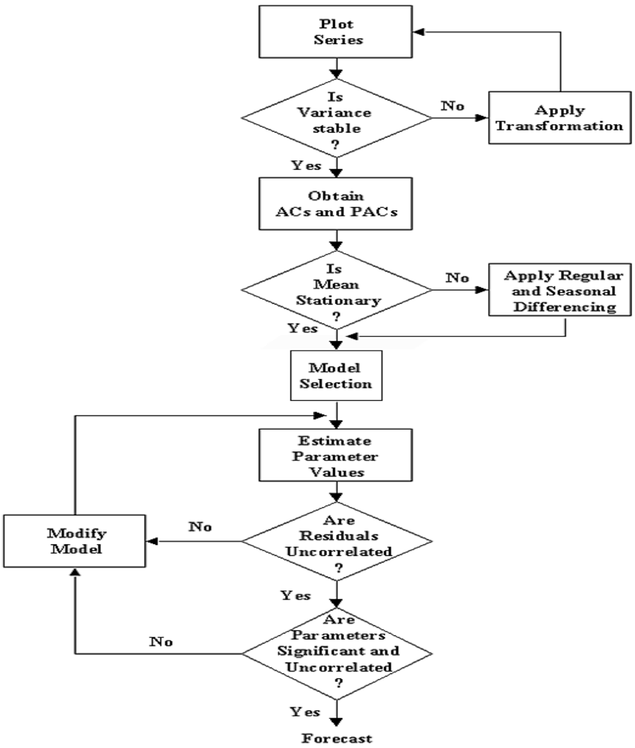

Figure1. Box-Jenkins model building stages algorithm

The Box Jenkins methodology is one of the most important methods for predicting time series. It was presented by researchers Box & Jenkins in 1970, as one of the most used methods in the analysis of the time series. This method is based on several stages:

- Stationary check stage of time series and application of transformation conversions needed to make them stationary.
- Determination of the appropriate model within the ARIMA models.
- Estimation of selected model parameters by one of the estimation methods.
- Diagnosis checking to investigate its feasibility for the time series.
- Forecasting using the selected model.

Figure 1 illustrates the model-building algorithm for B-J method [5].

2.3. Statistical criteria

A number of statistical criteria is used to differentiate between time series models. Then, it has selected the model order that corresponds to the lowest value for each criterion, including [4, 6]:

- **Root square for mean square error**

The mathematical equation of root square for mean square error denoted by RMSE is:

$$RMSE = \sqrt{\frac{\sum \hat{a}_t^2}{n}} = \sqrt{MSE} \qquad , \hat{a}_t^2 = (Z_t - \hat{Z}_t)^2 \qquad \ldots (5)$$

Where,
MSE represent mean square error.

- **The mean absolute error (MAE)**

The mean absolute error can be written as:

$$MAE = \frac{\sum |\hat{a}_t|}{n} \qquad , \hat{a}_t = Z_t - \widehat{Z}_t \qquad \ldots (6)$$

- **Mean absolute percentage error**

The mathematical equation for mean absolute percentage error (MAPE) is determined by:

$$MAPE = \frac{\sum (|\hat{a}_t| / Z_t)}{n} \times 100 \qquad \ldots (7)$$

2.4. Fuzzy time series

Fuzzy time series are modern methods of prediction that were proposed by Song and Chissom in 1993 through their research reported [7]. They introduced the definition of fuzzy time series models and their most important features. Some definitions for fuzzy time series models are based on the following [8, 9]:

- Definition (1): Fuzzy time series

If we had $Z(t)$ and $t = (....,0,1,2,3,....)$ as sets of real number, a universe of discourse can be defined with the fuzzy sets $f_i(t)$, $i = (1,2,3,....)$. However, $F(t)$ is a time series defined within the specified period $f_i(t)$ .In this case, $F(t)$ is a fuzzy time series on $Z(t)$ and $f_i(t)$, $i = (1,2,3,....)$ which are linguistic values of the linguistic variable are $F(t)$.

- Definition (2): To establish fuzzy logic relationship (FLR), suppose that *F(t-1)=Ai* . So, the relationship between the two consecutive observations *F(t)* and *F(t-1)* gives us a fuzzy logic relationship (FLR) $A_i \rightarrow A_j$.

The fuzzy logic relationships can be written in the form that each fuzzy relationship has the same fuzzy variables on the left (*Ai*). For example, if we have the two relationships:

$A_i \rightarrow A_{j1}$, $A_i \rightarrow A_{j2}$

So, it’s written as:

$A_i \rightarrow A_{j1}, A_{j2}$

- Definition (3): If we suppose that *F(t)* is a result of *F(t-1), F(t-2),…,F(t-n)*

So, the fuzzy logic relationship can be written as follows:

$F(t-2), F(t-1), \ldots, \rightarrow F(t)$

This term is called the fuzzy time series prediction model of (n) degree so that n≥2. Several fuzzy time series models were projected in recent years that have manipulated many problems for different fields, including the high order fuzzy time series model.

2.5. High order fuzzy time series model

The high order fuzzy time series prediction model requires a number of steps to obtain predictive values. The process of predicting by using this model has mainly divided into dual parts of fuzzification and defuzzification [10].

The algorithm fuzzification depends on the generation of a time series of trapezoid fuzzy sets [11] for the original time series data and formation the relationships between the original data and the generated fuzzy aggregated. The fuzzification data process can be divided into six main steps [12,13,14]:

I. Arrangement of time series data in ascending order.

II. Calculating the average distance which denoted by (AD) between any two consecutive values (x_p) in the resulting data from the previous step as follows:

$$AD(x_i,...,x_n) = \frac{1}{n-1}\sum_{i=1}^{n-1}\left|x_{p(i)} - x_{p(i+1)}\right| \quad \ldots (8)$$

and though $x_{p(i)} \leq x_{p(i+1)}$.

A standard deviation (SD) of the AD is calculated by:

$$SD_{AD} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - AD)^2} \quad \ldots (9)$$

III. Removing the AD values that lie outside the limits of the following defined period:

$$AD - SD \leq m \leq AD + SD \quad \ldots (10)$$

Here, (m) represents the set of values that lie within this period.

IV. Calculating the average distance rate (AD_R) by taking the average for the values specified in the previous step (m).

V. Definition of the limits of the universe of discourse (U) as follows:

$$U = [D_{\min} - AD_R \quad , \quad D_{\max} + AD_R] \quad \ldots (11)$$

Here, $(D_{\min}, D_{\max})$ stand for lowest and the highest values in time series data respectively.

Fuzzification of time series data has been by using trapezoidal membership function (TMF). This process requires knowing the number of fuzzy sets (L) that must be based on (U). A number of these sets have calculated by:

$$L = \frac{R - AD_R}{2 \times AD_R} \quad \ldots (12)$$

Here, R is defined the domain of universe of discourse and calculated as a follows:

$R = UB - LB$

The (UB, LB) are the upper and lower limit that were defined in equation (11).

The second section of this model is the process of defuzzification and the calculation of predictive values. To know how to calculate the output of the defuzzification process, use the defuzzification operator that defined with the following equation:

$$X_t = \sum_{i=1}^{n} x_{t-i} . w_i \quad \ldots (13)$$

Namely, $w_i \in [0,1]$ and (x_{t-i}) is the actual value for the time series in time $(t-i)$. Accordingly, L is depending on the range of time series. Assuming that (L = 2), the defuzzification factor can be computed by:

$X_t = (x_{t-1}.w_1) + (x_{t-2}.w_2)$

The defuzzification factor (X_t) can conclude a fuzzy logic view since weights (w_i) are fuzzy relationships between the previous and subsequent values in the time series. Each of these weights (w_i) stands for the strong point of the causal association between the input values to the series and the resulting undefined values. The strongest causal relationship is being when the weight value (w_i) is near to (1).

The process of defuzzification can be summarized as follows:

- **Establishing fuzzy set groups**

In classical fuzzy time series models [15, 16], fuzzy logic relationships can be defined after fuzzification of time series data. But, in this model, the right side of fuzzy logic relationships is undefined until weights (w_i) are calculated. Therefore, we established the fuzzy set groups (FSGS) instead of fuzzy logic relationships by inserting fuzzy groups into arranged pairs. The aim of listing fuzzy sets in this shape is to find out if dual or more FSGS have the identical group components (fuzzy groups). Its aim is also to obtain a series of FSGS that do not contain a similar frequency for more than once in its values. FSGS items are taken from the third order to obtain a FSGS series that does not have frequencies groups.

- **Transforming groups for fuzzy set into (If) rule**

The rules (If) for fuzzy set groups FSGS can be evaluated using the following:

$$If\,(F(t-1) = A_{i,t-1} \wedge F(t-2) = A_{i,t-2} \wedge ... \wedge F(t-n) = A_{i,t-n}) \quad \text{… (14)}$$

- **Evaluating the rules of $(If - then)$ by using particle swarm optimization**

In this step we use particle swarm optimization which denoted (PSO) by using the following equation [17,18]:

$$v_{i+1} = w.v_1 + c_1 r_1 (\hat{x}_i - x_j) + c_2 r_2 (\hat{g} - x_j) \quad \text{… (15)}$$

$$x_{j+1} = x_j + v_{i+1} \quad \text{… (16)}$$

To obtain the necessary weights (w_i) to calculate the value of the coefficient of defuzzification (X_i), the coefficients of the PSO function can be determined as follows:

- The value of the weight coefficient w is equal 1.4.
- The value of the velocity coefficients of elements c_1, c_2 equal to 2.
- The value of $[-v_{\max}, v_{\max}]$ are defined within the interval $[-0.01, 0.01]$.
- The highest and the lowest values of the function elements are defined within the interval $[0,1]$.
- The number of the elements of the PSO function is equal 5.

It is conventional to select the values of the function's coefficients of such form as they give the best results. The best global fitness value can be expressed in terms of the square error (SE) value, which is defined according to the following equation:

$$SE = [Forecast - Actual]^2 \quad \text{… (17)}$$

The basic idea for calculating the best global fitness value is to reduce the square error between the defuzzilied value and the real value in the corresponding time series in time (*t*). When getting to this value, the element coordinates in the PSO function, which gives the optimal value, represent the value of the weights (w_i).

- **Estimating the forecasting value**

After calculating the optimal weights, the prediction values have also calculated based on the evaluation of the rules (*If- then*) measured before in the previous step for each time series values.

2.6. Proposed procedure

The adaptation procedure for the Box-Jenkins series was applied using the high order fuzzy time series method as a proposed procedure to improve the adequacy of the model. This procedure was done by taking predictive data for the best ARIMA model as a new time series and processing it with a high order fuzzy time series method. The proposed procedure was compared with dual research methods used to find the best in the prediction process by some statistical criteria.

3. Analysis of the total dissolve solid data for drinking water

Water pollution is one of the first issues that attract the interest of researchers and specialists in the field of pollution because of the importance of water and its necessity in all biological processes. Drinking water is the basic element for the formation of communities and the emergence and prosperity of cities. Consequently, water processing is very important in daily life to obtain safe water for human health. The quality of water depends on its physical, chemical and biological properties and the extent to which humans can use it. This requires a specific approach to modify one or more of these properties.
On this basis, standard specifications for water quality have built through several examinations to determine the quality of water. The most important one, which has the total dissolved solids (TDS), had taken from drinking water in the city of Baghdad from January 2004 to December 2018. The examinations are measured in (mg/l) units. The used data and methods have been analysed according to SPSS Var.20, STATISTICA Var.7, and MATLAB 2009b programs.

3.1 Box-Jenkins method

One of the steps that have used in analysing any time series is to draw it to see the trend and stationarity of the series. The seasonality of the time series was tested by using the Jonckheer-Terpstra test of equal value with J-T=1.104 when p-value=0.270. This is not to reject the null hypothesis, and reaching that data is not seasonal at the significant level of 0.05. Figure 2 shows that time series have a trend and non-stationary, as illustrated in Figure 3 of the two functions of autocorrelation (AC) and partial autocorrelation (PAC), which shows by drawing the functions that the series is non stationary in mean.

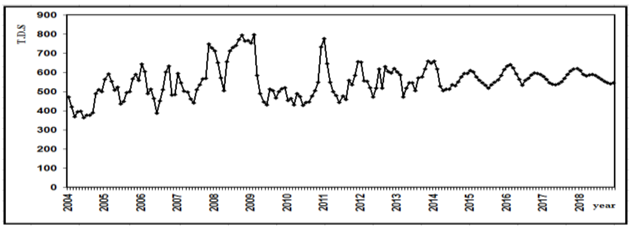

Figure 2. TDS data for drinking water in Baghdad city

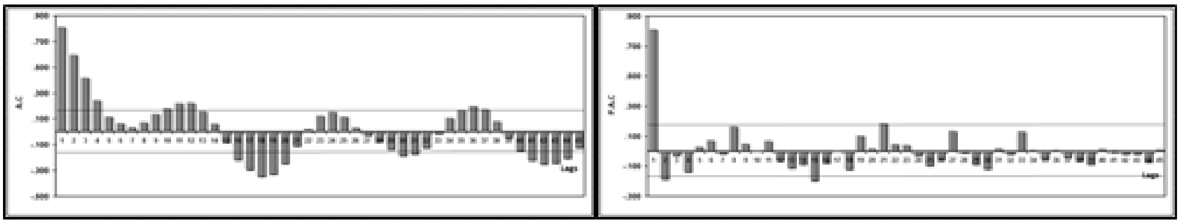

Figure 3. AC and PAC for TDS series

A Dickey-Fuller test for data with DF=0.534 was calculated. This value is less than the tabular value of 1.943 with a level of 0.05. This refers that the null hypothesis is not rejected and the series is non stationary. Thus, the first differences of data were taken. Figure 4 shows the coefficients of AC and PAC. It shows the stationary in the series with DF = 12.578, which is greater than the tabular value of 1.943 at a level of 0.05. This refers to the rejection of the null hypothesis and that the data has stationary.
Based on the stationary data, a number of combinations of statistical models have been applied. Table 1 shows the representation of models. It is clear that the model ARIMA (3,1,3) is appropriate for the data, which corresponds to the lowest values in the following criteria: RMSE, MAE, MAPE with the significance of its parameters.

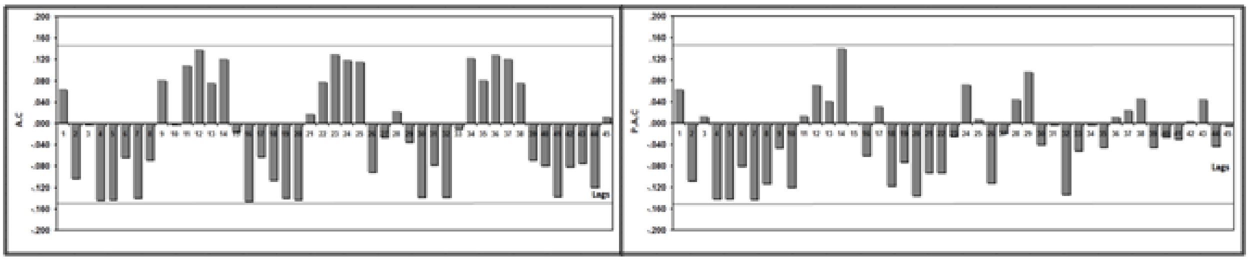

Figure 4. AC and PAC for TDS series after taking the first difference

Table 1. A number of combinations of statistical models to represent the TDS

Model \ Criteria	RMSE	MAPE	MAE
ARIMA(0,1,1)	53.100	6.938	38.035
ARIMA(0,1,2)	52.876	6.940	38.059
ARIMA(0,1,3)	52.814	7.065	38.657
ARIMA(1,1,0)	53.129	6.943	38.064
ARIMA(1,1,1)	53.050	6.884	37.748
ARIMA(1,1,2)	50.855	6.655	36.937
ARIMA(1,1,3)	50.880	6.656	36.942
ARIMA(2,1,0)	52.977	6.952	38.102
ARIMA(2,1,1)	53.079	6.897	37.842
ARIMA(2,1,2)	50.876	6.657	36.961
ARIMA(2,1,3)	51.006	6.650	36.924
ARIMA(3,1,0)	53.123	6.939	38.046
ARIMA(3,1,1)	53.230	6.900	37.854
ARIMA(3,1,2)	50.972	6.638	36.854
ARIMA(3,1,3)	50.277	6.628	36.830

The estimation of the model parameters ARIMA (3,1,3) is equal to:

	$\phi_1 = -0.536$	$\phi_2 = 0.195$	$\phi_3 = 0.724$	$\theta_1 = -0.486$	$\theta_2 = 0.470$	$\theta_3 = 0.986$
$s.e$:	0.069	0 .078	0.068	0.105	0.062	0.138
sig:	0.000	0.013	0.000	0.000	0.000	0.000

Accordingly, the selected model will be written in the following form:

$$(1 + 0.536B - 0.195B^2 - 0.724B^3)(1 - B)Z_t \quad = (1 + 0.486B - 0.470B^2 - 0.986B^3)a_t$$

To compute the accuracy of the ARIMA(3,1,3), the Q-tests for Box-Pierce and Ljung-Box were established for a series of errors in the model in the case of $Q_{B\text{-}P}$ = 48.775, $Q^*_{L\text{-}B}$ = 50.872. The value of χ^2 table value based on d.f=39 and $\alpha = 0.05$ is equal to 54.561 obviously. The null hypothesis is not rejected. This refers that the selected model is appropriate for representing the TDS data in for the period of study and that the model errors have a random characteristic. The AC and PAC coefficients for the residuals series of the model were plotted after checking their coefficients within the confidence intervals $\left\{\pm 1.96(n)^{-\frac{1}{2}}\right\}$ with 95% confidence level, Figure 5 shows the significance of some correlation coefficients at a number of lags and equal to 5. This is not effective as total numbers of lags equal to 45, because there are high fluctuations and few series of errors. Thus, the errors of the model can be considered to have a white noise.

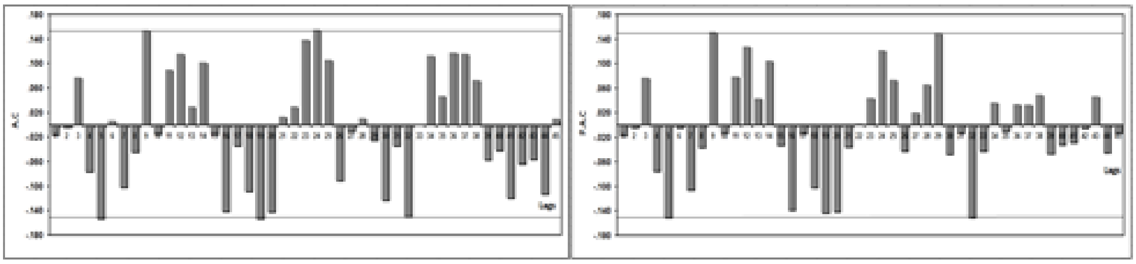

Figure 5. AC and PAC for the residual's model of the ARIMA(3,1,3)

Fig.6 shows the series observations and predictive values according to the selected model.

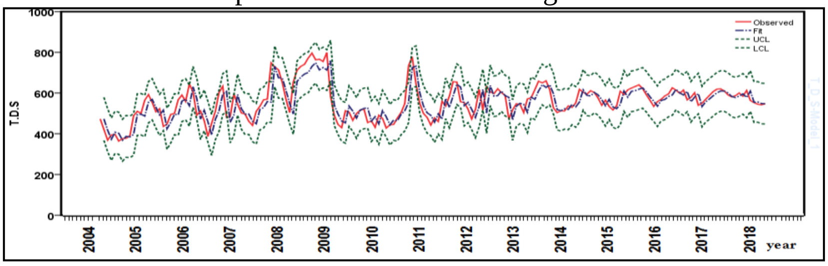

Figure 6. Observational and predictive values of the ARIMA(3,1,3) model for TDS series

3.2. High order fuzzy time series method

Monthly data for the TDS has taken in to consideration. The steps of the high order fuzzy time series model prediction method have applied through the fuzzification and defuzzification stages as follows:

- **Fuzzification stage**

Time series data have been fuzzified by using the trapezoidal membership functon (TMF)in fuzzification data. It requires to know the number of fuzzy sets and the length of each interval for this sets. The following steps have been applied:

• Average distance is equal to AD = 2.4246
• Average distance rate (ADR)= 1.7535
• The limits of the universe of discourse (U)= [362.2465, 799.7535]
• Number of fuzzy sets (n) = 124

Therefore, the fuzzy sets of series data, which are 124 fuzzy sets, can be defined by using the (TMF) as shown in Table 2.

Table 2. Fuzzy sets of the high order fuzzy model for TDS series

Fuzzy Sets	Fuzzy Numbers				Fuzzy Sets	Fuzzy Numbers			
A_{1}	362.2465	364	365.7535	367.507	A_{63}	579.6805	581.434	583.1875	584.941
A_{2}	365.7535	367.507	369.2605	371.014	A_{64}	583.1875	584.941	586.6945	588.448
A_{3}	369.2605	371.014	372.7675	374.521	A_{65}	586.6945	588.448	590.2015	591.955
A_{4}	372.7675	374.521	376.2745	378.028	A_{66}	590.2015	591.955	593.7085	595.462
A_{5}	376.2745	378.028	379.7815	381.535	A_{67}	593.7085	595.462	597.2155	598.969
A_{6}	379.7815	381.535	383.2885	385.042	A_{68}	597.2155	598.969	600.7225	602.476
A_{7}	383.2885	385.042	386.7955	388.549	A_{69}	600.7225	602.476	604.2295	605.983
A_{8}	386.7955	388.549	390.3025	392.056	A_{70}	604.2295	605.983	607.7365	609.49
A_{9}	390.3025	392.056	393.8095	395.563	A_{71}	607.7365	609.49	611.2435	612.997
A_{10}	393.8095	395.563	397.3165	399.07	A_{72}	611.2435	612.997	614.7505	616.504
A_{11}	397.3165	399.07	400.8235	402.577	A_{73}	614.7505	616.504	618.2575	620.011
A_{12}	400.8235	402.577	404.3305	406.084	A_{74}	618.2575	620.011	621.7645	623.518
A_{13}	404.3305	406.084	407.8375	409.591	A_{75}	621.7645	623.518	625.2715	627.025
A_{14}	407.8375	409.591	411.3445	413.098	A_{76}	625.2715	627.025	628.7785	630.532
A_{15}	411.3445	413.098	414.8515	416.605	A_{77}	628.7785	630.532	632.2855	634.039
A_{16}	414.8515	416.605	418.3585	420.112	A_{78}	632.2855	634.039	635.7925	637.546
A_{17}	418.3585	420.112	421.8655	423.619	A_{79}	635.7925	637.546	639.2995	641.053
A_{18}	421.8655	423.619	425.3725	427.126	A_{80}	639.2995	641.053	642.8065	644.56
A_{19}	425.3725	427.126	428.8795	430.633	A_{81}	642.8065	644.56	646.3135	648.067
A_{20}	428.8795	430.633	432.3865	434.14	A_{82}	646.3135	648.067	649.8205	651.574
A_{21}	432.3865	434.14	435.8935	437.647	A_{83}	649.8205	651.574	653.3275	655.081
A_{22}	435.8935	437.647	439.4005	441.154	A_{84}	653.3275	655.081	656.8345	658.588
A_{23}	439.4005	441.154	442.9075	444.661	A_{85}	656.8345	658.588	660.3415	662.095
A_{24}	442.9075	444.661	446.4145	448.168	A_{86}	660.3415	662.095	663.8485	665.602
A_{25}	446.4145	448.168	449.9215	451.675	A_{87}	663.8485	665.602	667.3555	669.109
A_{26}	449.9215	451.675	453.4285	455.182	A_{88}	667.3555	669.109	670.8625	672.616
A_{27}	453.4285	455.182	456.9355	458.689	A_{89}	670.8625	672.616	674.3695	676.123
A_{28}	456.9355	458.689	460.4425	462.196	A_{90}	674.3695	676.123	677.8765	679.63
A_{29}	460.4425	462.196	463.9495	465.703	A_{91}	677.8765	679.63	681.3835	683.137
A_{30}	463.9495	465.703	467.4565	469.21	A_{92}	681.3835	683.137	684.8905	686.644
A_{31}	467.4565	469.21	470.9635	472.717	A_{93}	684.8905	686.644	688.3975	690.151
A_{32}	470.9635	472.717	474.4705	476.224	A_{94}	688.3975	690.151	691.9045	693.658
A_{33}	474.4705	476.224	477.9775	479.731	A_{95}	691.9045	693.658	695.4115	697.165
A_{34}	477.9775	479.731	481.4845	483.238	A_{96}	695.4115	697.165	698.9185	700.672
A_{35}	481.4845	483.238	484.9915	486.745	A_{97}	698.9185	700.672	702.4255	704.179
A_{36}	484.9915	486.745	488.4985	490.252	A_{98}	702.4255	704.179	705.9325	707.686
A_{37}	488.4985	490.252	492.0055	493.759	A_{99}	705.9325	707.686	709.4395	711.193
A_{38}	492.0055	493.759	495.5125	497.266	A_{100}	709.4395	711.193	712.9465	714.7
A_{39}	495.5125	497.266	499.0195	500.773	A_{101}	712.9465	714.7	716.4535	718.207
A_{40}	499.0195	500.773	502.5265	504.28	A_{102}	716.4535	718.207	719.9605	721.714
A_{41}	502.5265	504.28	506.0335	507.787	A_{103}	719.9605	721.714	723.4675	725.221
A_{42}	506.0335	507.787	509.5405	511.294	A_{104}	723.4675	725.221	726.9745	728.728
A_{43}	509.5405	511.294	513.0475	514.801	A_{105}	726.9745	728.728	730.4815	732.235
A_{44}	513.0475	514.801	516.5545	518.308	A_{106}	730.4815	732.235	733.9885	735.742
A_{45}	516.5545	518.308	520.0615	521.815	A_{107}	733.9885	735.742	737.4955	739.249
A_{46}	520.0615	521.815	523.5685	525.322	A_{108}	737.4955	739.249	741.0025	742.756
A_{47}	523.5685	525.322	527.0755	528.829	A_{109}	741.0025	742.756	744.5095	746.263
A_{48}	527.0755	528.829	530.5825	532.336	A_{110}	744.5095	746.263	748.0165	749.77
A_{49}	530.5825	532.336	534.0895	535.843	A_{111}	748.0165	749.77	751.5235	753.277
A_{50}	534.0895	535.843	537.5965	539.35	A_{112}	751.5235	753.277	755.0305	756.784
A_{51}	537.5965	539.35	541.1035	542.857	A_{113}	755.0305	756.784	758.5375	760.291
A_{52}	541.1035	542.857	544.6105	546.364	A_{114}	758.5375	760.291	762.0445	763.798

A_{53}	544.6105	546.364	548.1175	549.871	A_{115}	762.0445	763.798	765.5515	767.305
A_{54}	548.1175	549.871	551.6245	553.378	A_{116}	765.5515	767.305	769.0585	770.812
A_{55}	551.6245	553.378	555.1315	556.885	A_{117}	769.0585	770.812	772.5655	774.319
A_{56}	555.1315	556.885	558.6385	560.392	A_{118}	772.5655	774.319	776.0725	777.826
A_{57}	558.6385	560.392	562.1455	563.899	A_{119}	776.0725	777.826	779.5795	781.333
A_{58}	562.1455	563.899	565.6525	567.406	A_{120}	779.5795	781.333	783.0865	784.84
A_{59}	565.6525	567.406	569.1595	570.913	A_{121}	783.0865	784.84	786.5935	788.347
A_{60}	569.1595	570.913	572.6665	574.42	A_{122}	786.5935	788.347	790.1005	791.854
A_{61}	572.6665	574.42	576.1735	577.927	A_{123}	790.1005	791.854	793.6075	795.361
A_{62}	576.1735	577.927	579.6805	581.434	A_{124}	793.6075	795.361	797.1145	798.868

By defining the fuzzy sets, the time series data has been fuzzified by converting their observations into linguistic variables. Table 3 explains the fuzzification of data to determine the membership function for each linguistic variable. For example, it is noted that the value of the test in January 2004, which is equal to 473, lies within the interval of the linguistic variable A_{32}. It is based on the following fuzzy numbers from Table 2: (470.9635, 472.717, 474.4705, 476.224).

Table 3. Fuzzification the data of TDS for the high order fuzzy model

Linguistic Variable	Month	Year	Linguistic Variable	Month	Year	Linguistic Variable	Month	Year	Linguistic Variable	Month	Year
A_{32}	Jan	2004	A_{100}	Jan	2008	A_{83}	Jan	2012	A_{77}	Jan	2016
A_{17}	Feb		A_{83}	Feb		A_{56}	Feb		A_{79}	Feb	
A_{2}	Mar		A_{60}	Mar		A_{54}	Mar		A_{74}	Mar	
A_{9}	Apr		A_{40}	Apr		A_{45}	Apr		A_{65}	Apr	
A_{10}	May		A_{84}	May		A_{32}	May		A_{58}	May	
A_{1}	June		A_{100}	June		A_{45}	June		A_{49}	June	
A_{4}	July		A_{105}	July		A_{73}	July		A_{56}	July	
A_{4}	Aug		A_{108}	Aug		A_{44}	Aug		A_{59}	Aug	
A_{8}	Sep		A_{117}	Sep		A_{77}	Sep		A_{64}	Sep	
A_{36}	Oct		A_{124}	Oct		A_{70}	Oct		A_{67}	Oct	
A_{42}	Nov		A_{114}	Nov		A_{67}	Nov		A_{75}	Nov	
A_{39}	Dec		A_{115}	Dec		A_{74}	Dec		A_{72}	Dec	
A_{58}	Jan	2005	A_{112}	Jan	2009	A_{68}	Jan	2013	A_{68}	Jan	2017
A_{66}	Feb		A_{124}	Feb		A_{64}	Feb		A_{72}	Feb	
A_{55}	Mar		A_{63}	Mar		A_{31}	Mar		A_{58}	Mar	
A_{42}	Apr		A_{37}	Apr		A_{44}	Apr		A_{62}	Apr	
A_{46}	May		A_{24}	May		A_{53}	May		A_{68}	May	
A_{21}	June		A_{19}	June		A_{52}	June		A_{51}	June	
A_{24}	July		A_{43}	July		A_{40}	July		A_{54}	July	
A_{38}	Aug		A_{41}	Aug		A_{60}	Aug		A_{59}	Aug	
A_{39}	Sep		A_{30}	Sep		A_{61}	Sep		A_{65}	Sep	
A_{58}	Oct		A_{39}	Oct		A_{73}	Oct		A_{70}	Oct	
A_{65}	Nov		A_{44}	Nov		A_{84}	Nov		A_{73}	Nov	
A_{56}	Dec		A_{45}	Dec		A_{82}	Dec		A_{74}	Dec	
A_{80}	Jan	2006	A_{26}	Jan	2010	A_{85}	Jan	2014	A_{71}	Jan	2018
A_{70}	Feb		A_{29}	Feb		A_{73}	Feb		A_{65}	Feb	
A_{37}	Mar		A_{20}	Mar		A_{47}	Mar		A_{62}	Mar	
A_{43}	Apr		A_{37}	Apr		A_{40}	Apr		A_{64}	Apr	
A_{29}	May		A_{32}	May		A_{43}	May		A_{68}	May	
A_{7}	June		A_{19}	June		A_{43}	June		A_{63}	June	
A_{25}	July		A_{23}	July		A_{50}	July		A_{71}	July	
A_{42}	Aug		A_{24}	Aug		A_{48}	Aug		A_{57}	Aug	
A_{69}	Sep		A_{33}	Sep		A_{54}	Sep		A_{54}	Sep	
A_{77}	Oct		A_{40}	Oct		A_{73}	Oct		A_{52}	Oct	
A_{34}	Nov		A_{53}	Nov		A_{69}	Nov		A_{51}	Nov	
A_{35}	Dec		A_{106}	Dec		A_{66}	Dec		A_{53}	Dec	
A_{66}	Jan	2007	A_{119}	Jan	2011	A_{71}	Jan	2015			
A_{52}	Feb		A_{81}	Feb		A_{69}	Feb				
A_{40}	Mar		A_{54}	Mar		A_{61}	Mar				
A_{38}	Apr		A_{39}	Apr		A_{51}	Apr				
A_{28}	May		A_{34}	May		A_{58}	May				
A_{22}	June		A_{23}	June		A_{49}	June				
A_{42}	July		A_{33}	July		A_{44}	July				
A_{49}	Aug		A_{28}	Aug		A_{50}	Aug				
A_{58}	Sep		A_{56}	Sep		A_{70}	Sep				
A_{59}	Oct		A_{50}	Oct		A_{66}	Oct				

Linguistic Variable	Month	Year	Linguistic Variable	Month	Year	Linguistic Variable	Month	Year	Linguistic Variable	Month	Year
A_{110}	Nov		A_{64}	Nov		A_{72}	Nov				
A_{104}	Dec		A_{84}	Dec		A_{75}	Dec				

- **Defuzzification stage**

This stage involves a number of steps to arrive at the prediction process. The FSGS fuzzy sets are established as in Table 4.

Table 4. FSGs for TDS series (high order fuzzy model)

#	FSGS	#	FSGS	#	FSGS	#	FSGS	#	FSGS
1	$\{A_{32},A_{17}\}$	37	$\{A_{66},A_{52}\}$	73	$\{A_{26},A_{29}\}$	109	$\{A_{68},A_{64}\}$	145	$\{A_{77},A_{79}\}$
2	$\{A_{17},A_{2}\}$	38	$\{A_{52},A_{40}\}$	74	$\{A_{29},A_{20}\}$	110	$\{A_{64},A_{31}\}$	146	$\{A_{79},A_{74}\}$
3	$\{A_{2},A_{9}\}$	39	$\{A_{40},A_{38}\}$	75	$\{A_{20},A_{37}\}$	111	$\{A_{31},A_{44}\}$	147	$\{A_{74},A_{65}\}$
4	$\{A_{9},A_{10}\}$	40	$\{A_{38},A_{28}\}$	76	$\{A_{37},A_{32}\}$	112	$\{A_{44},A_{53}\}$	148	$\{A_{65},A_{58}\}$
5	$\{A_{10},A_{1}\}$	41	$\{A_{28},A_{22}\}$	77	$\{A_{32},A_{19}\}$	113	$\{A_{53},A_{52}\}$	149	$\{A_{58},A_{49}\}$
6	$\{A_{1},A_{4}\}$	42	$\{A_{22},A_{42}\}$	78	$\{A_{19},A_{23}\}$	114	$\{A_{52},A_{40}\}$	150	$\{A_{49},A_{56}\}$
7	$\{A_{4},A_{4}\}$	43	$\{A_{42},A_{49}\}$	79	$\{A_{23},A_{24}\}$	115	$\{A_{40},A_{60}\}$	151	$\{A_{56},A_{59}\}$
8	$\{A_{4},A_{8}\}$	44	$\{A_{49},A_{58}\}$	80	$\{A_{24},A_{33}\}$	116	$\{A_{60},A_{61}\}$	152	$\{A_{59},A_{64}\}$
9	$\{A_{8},A_{36}\}$	45	$\{A_{58},A_{59}\}$	81	$\{A_{33},A_{40}\}$	117	$\{A_{61},A_{73}\}$	153	$\{A_{64},A_{67}\}$
10	$\{A_{36},A_{42}\}$	46	$\{A_{59},A_{110}\}$	82	$\{A_{40},A_{53}\}$	118	$\{A_{73},A_{84}\}$	154	$\{A_{67},A_{75}\}$
11	$\{A_{42},A_{39}\}$	47	$\{A_{110},A_{104}\}$	83	$\{A_{53},A_{106}\}$	119	$\{A_{84},A_{82}\}$	155	$\{A_{75},A_{72}\}$
12	$\{A_{39},A_{58}\}$	48	$\{A_{104},A_{100}\}$	84	$\{A_{106},A_{119}\}$	120	$\{A_{82},A_{85}\}$	156	$\{A_{72},A_{68}\}$
13	$\{A_{58},A_{66}\}$	49	$\{A_{100},A_{83}\}$	85	$\{A_{119},A_{81}\}$	121	$\{A_{85},A_{73}\}$	157	$\{A_{68},A_{72}\}$
14	$\{A_{66},A_{55}\}$	50	$\{A_{83},A_{60}\}$	86	$\{A_{81},A_{54}\}$	122	$\{A_{73},A_{47}\}$	158	$\{A_{72},A_{58}\}$
15	$\{A_{55},A_{42}\}$	51	$\{A_{60},A_{40}\}$	87	$\{A_{54},A_{39}\}$	123	$\{A_{47},A_{40}\}$	159	$\{A_{58},A_{62}\}$
16	$\{A_{42},A_{46}\}$	52	$\{A_{40},A_{84}\}$	88	$\{A_{39},A_{34}\}$	124	$\{A_{40},A_{43}\}$	160	$\{A_{62},A_{68}\}$
17	$\{A_{46},A_{21}\}$	53	$\{A_{84},A_{100}\}$	89	$\{A_{34},A_{23}\}$	125	$\{A_{43},A_{43}\}$	161	$\{A_{68},A_{51}\}$
18	$\{A_{21},A_{24}\}$	54	$\{A_{100},A_{105}\}$	90	$\{A_{23},A_{33}\}$	126	$\{A_{43},A_{50}\}$	162	$\{A_{51},A_{54}\}$
19	$\{A_{24},A_{38}\}$	55	$\{A_{105},A_{108}\}$	91	$\{A_{33},A_{28}\}$	127	$\{A_{50},A_{48}\}$	163	$\{A_{54},A_{59}\}$
20	$\{A_{38},A_{39}\}$	56	$\{A_{108},A_{117}\}$	92	$\{A_{28},A_{56}\}$	128	$\{A_{48},A_{54}\}$	164	$\{A_{59},A_{65}\}$
21	$\{A_{39},A_{58}\}$	57	$\{A_{117},A_{124}\}$	93	$\{A_{56},A_{50}\}$	129	$\{A_{54},A_{73}\}$	165	$\{A_{65},A_{70}\}$
22	$\{A_{58},A_{65}\}$	58	$\{A_{124},A_{114}\}$	94	$\{A_{50},A_{64}\}$	130	$\{A_{73},A_{69}\}$	166	$\{A_{70},A_{73}\}$
23	$\{A_{65},A_{56}\}$	59	$\{A_{114},A_{115}\}$	95	$\{A_{64},A_{84}\}$	131	$\{A_{69},A_{66}\}$	167	$\{A_{73},A_{74}\}$
24	$\{A_{56},A_{80}\}$	60	$\{A_{115},A_{112}\}$	96	$\{A_{84},A_{83}\}$	132	$\{A_{66},A_{71}\}$	168	$\{A_{74},A_{71}\}$
25	$\{A_{80},A_{70}\}$	61	$\{A_{112},A_{124}\}$	97	$\{A_{83},A_{56}\}$	133	$\{A_{71},A_{69}\}$	169	$\{A_{71},A_{65}\}$
26	$\{A_{70},A_{37}\}$	62	$\{A_{124},A_{63}\}$	98	$\{A_{56},A_{54}\}$	134	$\{A_{69},A_{61}\}$	170	$\{A_{65},A_{62}\}$
27	$\{A_{37},A_{43}\}$	63	$\{A_{63},A_{37}\}$	99	$\{A_{54},A_{45}\}$	135	$\{A_{61},A_{51}\}$	171	$\{A_{62},A_{64}\}$
28	$\{A_{43},A_{29}\}$	64	$\{A_{37},A_{24}\}$	100	$\{A_{45},A_{32}\}$	136	$\{A_{51},A_{58}\}$	172	$\{A_{64},A_{68}\}$
29	$\{A_{29},A_{7}\}$	65	$\{A_{24},A_{19}\}$	101	$\{A_{32},A_{45}\}$	137	$\{A_{58},A_{49}\}$	173	$\{A_{68},A_{63}\}$
30	$\{A_{7},A_{25}\}$	66	$\{A_{19},A_{43}\}$	102	$\{A_{45},A_{73}\}$	138	$\{A_{49},A_{44}\}$	174	$\{A_{63},A_{71}\}$
31	$\{A_{25},A_{42}\}$	67	$\{A_{43},A_{41}\}$	103	$\{A_{73},A_{44}\}$	139	$\{A_{44},A_{50}\}$	175	$\{A_{71},A_{57}\}$
32	$\{A_{42},A_{69}\}$	68	$\{A_{41},A_{30}\}$	104	$\{A_{44},A_{77}\}$	140	$\{A_{50},A_{70}\}$	176	$\{A_{57},A_{54}\}$
33	$\{A_{69},A_{77}\}$	69	$\{A_{30},A_{39}\}$	105	$\{A_{77},A_{70}\}$	141	$\{A_{70},A_{66}\}$	177	$\{A_{54},A_{52}\}$
34	$\{A_{77},A_{34}\}$	70	$\{A_{39},A_{44}\}$	106	$\{A_{70},A_{67}\}$	142	$\{A_{66},A_{72}\}$	178	$\{A_{52},A_{51}\}$
35	$\{A_{34},A_{35}\}$	71	$\{A_{44},A_{45}\}$	107	$\{A_{67},A_{74}\}$	143	$\{A_{72},A_{75}\}$	179	$\{A_{51},A_{53}\}$
36	$\{A_{35},A_{66}\}$	72	$\{A_{45},A_{26}\}$	108	$\{A_{74},A_{68}\}$	144	$\{A_{75},A_{77}\}$		

From Table 4, it is clear that there are frequencies in some elements of fuzzy set group sets (FSGS). The group $\{A_{39}, A_{58}\}$ and group $\{A_{52}, A_{40}\}$ were frequented twice. Therefore, a higher order is taken for these frequent groups to obtain FSGs that do not contain frequent sets as in Table 5.

Table 5. FSGS for the TDS series after taking the higher order

#	FSGS	#	FSGS	#	FSGS	#	FSGS	#	FSGS
1	$\{A_{32},A_{17}\}$	37	$\{A_{66},A_{52}\}$	73	$\{A_{26},A_{29}\}$	109	$\{A_{68},A_{64}\}$	145	$\{A_{77},A_{79}\}$
2	$\{A_{17},A_{2}\}$	38	$\{A_{66},A_{52},A_{40}\}$	74	$\{A_{29},A_{20}\}$	110	$\{A_{64},A_{31}\}$	146	$\{A_{79},A_{74}\}$
3	$\{A_{2},A_{9}\}$	39	$\{A_{40},A_{38}\}$	75	$\{A_{20},A_{37}\}$	111	$\{A_{31},A_{44}\}$	147	$\{A_{74},A_{65}\}$
4	$\{A_{9},A_{10}\}$	40	$\{A_{38},A_{28}\}$	76	$\{A_{37},A_{32}\}$	112	$\{A_{44},A_{53}\}$	148	$\{A_{65},A_{58}\}$
5	$\{A_{10},A_{1}\}$	41	$\{A_{28},A_{22}\}$	77	$\{A_{32},A_{19}\}$	113	$\{A_{53},A_{52}\}$	149	$\{A_{58},A_{49}\}$
6	$\{A_{1},A_{4}\}$	42	$\{A_{22},A_{42}\}$	78	$\{A_{19},A_{23}\}$	114	$\{A_{53},A_{52},A_{40}\}$	150	$\{A_{49},A_{56}\}$
7	$\{A_{4},A_{4}\}$	43	$\{A_{42},A_{49}\}$	79	$\{A_{23},A_{24}\}$	115	$\{A_{40},A_{60}\}$	151	$\{A_{56},A_{59}\}$
8	$\{A_{4},A_{8}\}$	44	$\{A_{49},A_{58}\}$	80	$\{A_{24},A_{33}\}$	116	$\{A_{60},A_{61}\}$	152	$\{A_{59},A_{64}\}$
9	$\{A_{8},A_{36}\}$	45	$\{A_{58},A_{59}\}$	81	$\{A_{33},A_{40}\}$	117	$\{A_{61},A_{73}\}$	153	$\{A_{64},A_{67}\}$
10	$\{A_{36},A_{42}\}$	46	$\{A_{59},A_{110}\}$	82	$\{A_{40},A_{53}\}$	118	$\{A_{73},A_{84}\}$	154	$\{A_{67},A_{75}\}$
11	$\{A_{42},A_{39}\}$	47	$\{A_{110},A_{104}\}$	83	$\{A_{53},A_{106}\}$	119	$\{A_{84},A_{82}\}$	155	$\{A_{75},A_{72}\}$
12	$\{A_{42},A_{39},A_{58}\}$	48	$\{A_{104},A_{100}\}$	84	$\{A_{106},A_{119}\}$	120	$\{A_{82},A_{85}\}$	156	$\{A_{72},A_{68}\}$
13	$\{A_{58},A_{66}\}$	49	$\{A_{100},A_{83}\}$	85	$\{A_{119},A_{81}\}$	121	$\{A_{85},A_{73}\}$	157	$\{A_{68},A_{72}\}$
14	$\{A_{66},A_{55}\}$	50	$\{A_{83},A_{60}\}$	86	$\{A_{81},A_{54}\}$	122	$\{A_{73},A_{47}\}$	158	$\{A_{72},A_{58}\}$
15	$\{A_{55},A_{42}\}$	51	$\{A_{60},A_{40}\}$	87	$\{A_{54},A_{39}\}$	123	$\{A_{47},A_{40}\}$	159	$\{A_{58},A_{62}\}$

16	$\{A_{42},A_{46}\}$	52	$\{A_{40},A_{84}\}$	88	$\{A_{39},A_{34}\}$	124	$\{A_{40},A_{43}\}$	160	$\{A_{62},A_{68}\}$
17	$\{A_{46},A_{21}\}$	53	$\{A_{84},A_{100}\}$	89	$\{A_{34},A_{23}\}$	125	$\{A_{43},A_{43}\}$	161	$\{A_{68},A_{51}\}$
18	$\{A_{21},A_{24}\}$	54	$\{A_{100},A_{105}\}$	90	$\{A_{23},A_{33}\}$	126	$\{A_{43},A_{50}\}$	162	$\{A_{51},A_{54}\}$
19	$\{A_{24},A_{38}\}$	55	$\{A_{105},A_{108}\}$	91	$\{A_{33},A_{28}\}$	127	$\{A_{50},A_{48}\}$	163	$\{A_{54},A_{59}\}$
20	$\{A_{38},A_{39}\}$	56	$\{A_{108},A_{117}\}$	92	$\{A_{28},A_{56}\}$	128	$\{A_{48},A_{54}\}$	164	$\{A_{59},A_{65}\}$
21	$\{A_{38},A_{39},A_{58}\}$	57	$\{A_{117},A_{124}\}$	93	$\{A_{56},A_{50}\}$	129	$\{A_{54},A_{73}\}$	165	$\{A_{65},A_{70}\}$
22	$\{A_{58},A_{65}\}$	58	$\{A_{124},A_{114}\}$	94	$\{A_{50},A_{64}\}$	130	$\{A_{73},A_{69}\}$	166	$\{A_{70},A_{73}\}$
23	$\{A_{65},A_{56}\}$	59	$\{A_{114},A_{115}\}$	95	$\{A_{64},A_{84}\}$	131	$\{A_{69},A_{66}\}$	167	$\{A_{73},A_{74}\}$
24	$\{A_{56},A_{80}\}$	60	$\{A_{115},A_{112}\}$	96	$\{A_{84},A_{83}\}$	132	$\{A_{66},A_{71}\}$	168	$\{A_{74},A_{71}\}$
25	$\{A_{80},A_{70}\}$	61	$\{A_{112},A_{124}\}$	97	$\{A_{83},A_{56}\}$	133	$\{A_{71},A_{69}\}$	169	$\{A_{71},A_{65}\}$
26	$\{A_{70},A_{37}\}$	62	$\{A_{124},A_{63}\}$	98	$\{A_{56},A_{54}\}$	134	$\{A_{69},A_{61}\}$	170	$\{A_{65},A_{62}\}$
27	$\{A_{37},A_{43}\}$	63	$\{A_{63},A_{37}\}$	99	$\{A_{54},A_{45}\}$	135	$\{A_{61},A_{51}\}$	171	$\{A_{62},A_{64}\}$
28	$\{A_{43},A_{29}\}$	64	$\{A_{37},A_{24}\}$	100	$\{A_{45},A_{32}\}$	136	$\{A_{51},A_{58}\}$	172	$\{A_{64},A_{68}\}$
29	$\{A_{29},A_{7}\}$	65	$\{A_{24},A_{19}\}$	101	$\{A_{32},A_{45}\}$	137	$\{A_{58},A_{49}\}$	173	$\{A_{68},A_{63}\}$
30	$\{A_{7},A_{25}\}$	66	$\{A_{19},A_{43}\}$	102	$\{A_{45},A_{73}\}$	138	$\{A_{49},A_{44}\}$	174	$\{A_{63},A_{71}\}$
31	$\{A_{25},A_{42}\}$	67	$\{A_{43},A_{41}\}$	103	$\{A_{73},A_{44}\}$	139	$\{A_{44},A_{50}\}$	175	$\{A_{71},A_{57}\}$
32	$\{A_{42},A_{69}\}$	68	$\{A_{41},A_{30}\}$	104	$\{A_{44},A_{77}\}$	140	$\{A_{50},A_{70}\}$	176	$\{A_{57},A_{54}\}$
33	$\{A_{69},A_{77}\}$	69	$\{A_{30},A_{39}\}$	105	$\{A_{77},A_{70}\}$	141	$\{A_{70},A_{66}\}$	177	$\{A_{54},A_{52}\}$
34	$\{A_{77},A_{34}\}$	70	$\{A_{39},A_{44}\}$	106	$\{A_{70},A_{67}\}$	142	$\{A_{66},A_{72}\}$	178	$\{A_{52},A_{51}\}$
35	$\{A_{34},A_{35}\}$	71	$\{A_{44},A_{45}\}$	107	$\{A_{67},A_{74}\}$	143	$\{A_{72},A_{75}\}$	179	$\{A_{51},A_{53}\}$
36	$\{A_{35},A_{66}\}$	72	$\{A_{45},A_{26}\}$	108	$\{A_{74},A_{68}\}$	144	$\{A_{75},A_{77}\}$		

After obtaining a series for a FSG free of frequencies, the *If-then* rules for these sets have evaluated and Applied. The PSO function algorithm based on w_i weights is for computing the value of defuzzification coefficients X_i . Table 6 explains the process of evaluating *if-then* rules and the specific results of weight w_i.

Table 6. Evaluation of *If-then* rule with weights w_i results of TDS data using PSO function

#	Matching Measure *If - then*	Resultant Weights	#	Matching Measure *If - then*	Resultant Weights
1	F(t-1)= A_{17}^F(t-2)= A_{32}	w_1=0.734and w_2=0.1299	91	F(t-1)=A_{28}^F(t-2)=A_{33}	w_1=0.7497and w_2=0.4466
2	F(t-1)=A_{2}^F(t-2)=A_{17}	w_1=0.5321and w_2=0.4711	92	F(t-1)=A_{56}^F(t-2)=A_{28}	w_1=0.7803and w_2=0.2182
3	F(t-1)=A_{9}^F(t-2)=A_{2}	w_1=0.7126and w_2=0.3207	93	F(t-1)=A_{50}^F(t-2)=A_{56}	w_1=0.6962and w_2=0.3785
4	F(t-1)=A_{10}^F(t-2)=A_{9}	w_1=0.6526and w_2=0.2654	94	F(t-1)=A_{64}^F(t-2)=A_{50}	w_1=0.67and w_2=0.4922
5	F(t-1)=A_{1}^F(t-2)=A_{10}	w_1=0.773and w_2=0.2332	95	F(t-1)=A_{84}^F(t-2)=A_{64}	w_1=0.5981and w_2=0.4483
6	F(t-1)=A_{4}^F(t-2)=A_{1}	w_1=0.6879and w_2=0.3311	96	F(t-1)=A_{83}^F(t-2)=A_{84}	w_1=0.7505and w_2=0.1002
7	F(t-1)=A_{4}^F(t-2)=A_{4}	w_1=0.8904and w_2=0.1362	97	F(t-1)=A_{56}^F(t-2)=A_{83}	w_1=0.7379and w_2=0.2166
8	F(t-1)=A_{8}^F(t-2)=A_{4}	w_1=0.7593and w_2=0.5092	98	F(t-1)=A_{54}^F(t-2)=A_{56}	w_1=0.5612and w_2=0.3792
9	F(t-1)=A_{36}^F(t-2)=A_{8}	w_1=0.6553and w_2=0.4901	99	F(t-1)=A_{45}^F(t-2)=A_{54}	w_1=0.6782and w_2=0.2161
10	F(t-1)=A_{42}^F(t-2)=A_{36}	w_1=0.7927and w_2=0.1951	100	F(t-1)=A_{32}^F(t-2)=A_{45}	w_1=0.5584and w_2=0.4901
11	F(t-1)=A_{39}^F(t-2)=A_{42}	w_1=0.7452and w_2=0.3811	101	F(t-1)=A_{45}^F(t-2)=A_{32}	w_1=0.6944and w_2=0.5433
12	F(t-1)=A_{58}^F(t-2) =A_{39}^F(t-3)=A_{42}	w_1=0.6867and w_2=0.1625 and w_3=0.25	102	F(t-1)=A_{73}^F(t-2)=A_{45}	w_1=0.5438and w_2=0.3528
13	F(t-1)=A_{66}^F(t-2)=A_{58}	w_1=0.7326and w_2=0.2098	103	F(t-1)=A_{44}^F(t-2)=A_{73}	w_1=0.7311and w_2=0.2902
14	F(t-1)=A_{55}^F(t-2)=A_{66}	w_1=0.7506and w_2=0.1542	104	F(t-1)=A_{77}^F(t-2)=A_{44}	w_1=0.7017and w_2=0.2635
15	F(t-1)=A_{42}^F(t-2)=A_{55}	w_1=0.5805and w_2=0.4081	105	F(t-1)=A_{70}^F(t-2)=A_{77}	w_1=0.7708and w_2=0.2053
16	F(t-1)=A_{46}^F(t-2)=A_{42}	w_1=0.7362and w_2=0.1014	106	F(t-1)=A_{67}^F(t-2)=A_{70}	w_1=0.6275and w_2=0.4063
17	F(t-1)=A_{21}^F(t-2)=A_{46}	w_1=0.7927and w_2=0.1951	107	F(t-1)=A_{74}^F(t-2)=A_{67}	w_1=0.75and w_2=0.2276
18	F(t-1)=A_{24}^F(t-2)=A_{21}	w_1=0.7505and w_2=0.3677	108	F(t-1)=A_{68}^F(t-2)=A_{74}	w_1=0.7497and w_2=0.219
19	F(t-1)=A_{38}^F(t-2)=A_{24}	w_1=0.7386and w_2=0.2994	109	F(t-1)=A_{64}^F(t-2)=A_{68}	w_1=0.7536and w_2=0.05
20	F(t-1)=A_{39}^F(t-2)=A_{38}	w_1=0.6183and w_2=0.521	110	F(t-1)=A_{31}^F(t-2)=A_{64}	w_1=0.7694and w_2=0.2615

21	$F(t-1)=A_{58}$^$F(t-2)=A_{39}$^$F(t-3)=A_{38}$	$w_1=0.7491$and $w_2=0.1697$ and $w_3=0.1568$	111	$F(t-1)=A_{44}$^$F(t-2)=A_{31}$	$w_1=0.6855$and $w_2=0.4091$
22	$F(t-1)=A_{65}$^$F(t-2)=A_{58}$	$w_1=0.7282$and $w_2=0.2288$	112	$F(t-1)=A_{53}$^$F(t-2)=A_{44}$	$w_1=0.5218$and $w_2=0.501$
23	$F(t-1)=A_{56}$^$F(t-2)=A_{65}$	$w_1=0.6179$and $w_2=0.5041$	113	$F(t-1)=A_{52}$^$F(t-2)=A_{53}$	$w_1=0.5081$and $w_2=0.4152$
24	$F(t-1)=A_{80}$^$F(t-2)=A_{56}$	$w_1=0.7423$and $w_2=0.2315$	114	$F(t-1)=A_{40}$^$F(t-2)=A_{52}$^$F(t-3)=A_{53}$	$w_1=0.75$and $w_2=0.156$ and $w_3=0.1987$
25	$F(t-1)=A_{70}$^$F(t-2)=A_{80}$	$w_1=0.7085$and $w_2=0.098$	115	$F(t-1)=A_{60}$^$F(t-2)=A_{40}$	$w_1=0.6017$and $w_2=0.4628$
26	$F(t-1)=A_{37}$^$F(t-2)=A_{70}$	$w_1=0.5527$and $w_2=0.4007$	116	$F(t-1)=A_{61}$^$F(t-2)=A_{60}$	$w_1=0.7465$and $w_2=0.3255$
27	$F(t-1)=A_{43}$^$F(t-2)=A_{37}$	$w_1=0.4826$and $w_2=0.4416$	117	$F(t-1)=A_{73}$^$F(t-2)=A_{61}$	$w_1=0.75$and $w_2=0.3373$
28	$F(t-1)=A_{29}$^$F(t-2)=A_{43}$	$w_1=0.7592$and $w_2=0.068$	118	$F(t-1)=A_{84}$^$F(t-2)=A_{73}$	$w_1=0.5207$and $w_2=0.4981$
29	$F(t-1)=A_{7}$^$F(t-2)=A_{29}$	$w_1=0.7446$and $w_2=0.3489$	119	$F(t-1)=A_{82}$^$F(t-2)=A_{84}$	$w_1=0.6672$and $w_2=0.343$
30	$F(t-1)=A_{25}$^$F(t-2)=A_{7}$	$w_1=0.728$and $w_2=0.4657$	120	$F(t-1)=A_{85}$^$F(t-2)=A_{82}$	$w_1=0.7593$and $w_2=0.1791$
31	$F(t-1)=A_{42}$^$F(t-2)=A_{25}$	$w_1=0.7415$and $w_2=0.4977$	121	$F(t-1)=A_{73}$^$F(t-2)=A_{85}$	$w_1=0.6296$and $w_2=0.2092$
32	$F(t-1)=A_{69}$^$F(t-2)=A_{42}$	$w_1=0.6831$and $w_2=0.4292$	122	$F(t-1)=A_{47}$^$F(t-2)=A_{73}$	$w_1=0.6077$and $w_2=0.2972$
33	$F(t-1)=A_{77}$^$F(t-2)=A_{69}$	$w_1=0.5474$and $w_2=0.225$	123	$F(t-1)=A_{40}$^$F(t-2)=A_{47}$	$w_1=0.724$and $w_2=0.2818$
34	$F(t-1)=A_{34}$^$F(t-2)=A_{77}$	$w_1=0.6644$and $w_2=0.2625$	124	$F(t-1)=A_{43}$^$F(t-2)=A_{40}$	$w_1=0.5574$and $w_2=0.4486$
35	$F(t-1)=A_{35}$^$F(t-2)=A_{34}$	$w_1=0.7316$and $w_2=0.4958$	125	$F(t-1)=A_{41}$^$F(t-2)=A_{43}$	$w_1=0.5481$and $w_2=0.5$
36	$F(t-1)=A_{66}$^$F(t-2)=A_{35}$	$w_1=0.613$and $w_2=0.3715$	126	$F(t-1)=A_{50}$^$F(t-2)=A_{41}$	$w_1=0.7991$and $w_2=0.1969$
37	$F(t-1)=A_{52}$^$F(t-2)=A_{66}$	$w_1=0.5084$and $w_2=0.3776$	127	$F(t-1)=A_{48}$^$F(t-2)=A_{50}$	$w_1=0.5304$and $w_2=0.5038$
38	$F(t-1)=A_{40}$^$F(t-2)=A_{52}$^$F(t-3)=A_{66}$	$w_1=0.3487$and $w_2=0.2821$ and $w_3=0.2895$	128	$F(t-1)=A_{54}$^$F(t-2)=A_{48}$	$w_1=0.7432$and $w_2=0.3925$
39	$F(t-1)=A_{38}$^$F(t-2)=A_{40}$	$w_1=0.6396$and $w_2=0.2882$	129	$F(t-1)=A_{73}$^$F(t-2)=A_{54}$	$w_1=0.7681$and $w_2=0.2341$
40	$F(t-1)=A_{28}$^$F(t-2)=A_{38}$	$w_1=0.5925$and $w_2=0.3352$	130	$F(t-1)=A_{69}$^$F(t-2)=A_{73}$	$w_1=0.6691$and $w_2=0.3086$
41	$F(t-1)=A_{22}$^$F(t-2)=A_{28}$	$w_1=0.7039$and $w_2=0.435$	131	$F(t-1)=A_{66}$^$F(t-2)=A_{69}$	$w_1=0.7491$and $w_2=0.2739$
42	$F(t-1)=A_{42}$^$F(t-2)=A_{22}$	$w_1=0.6183$and $w_2=0.4986$	132	$F(t-1)=A_{71}$^$F(t-2)=A_{66}$	$w_1=0.5847$and $w_2=0.4135$
43	$F(t-1)=A_{49}$^$F(t-2)=A_{42}$	$w_1=0.7037$and $w_2=0.3772$	133	$F(t-1)=A_{69}$^$F(t-2)=A_{71}$	$w_1=0.7201$and $w_2=0.2319$
44	$F(t-1)=A_{58}$^$F(t-2)=A_{49}$	$w_1=0.5342$and $w_2=0.4988$	134	$F(t-1)=A_{61}$^$F(t-2)=A_{69}$	$w_1=0.748$and $w_2=0.1806$
45	$F(t-1)=A_{59}$^$F(t-2)=A_{58}$	$w_1=0.7814$and $w_2=0.5298$	135	$F(t-1)=A_{51}$^$F(t-2)=A_{61}$	$w_1=0.5276$and $w_2=0.4863$
46	$F(t-1)=A_{110}$^$F(t-2)=A_{59}$	$w_1=0.6211$and $w_2=0.4642$	136	$F(t-1)=A_{58}$^$F(t-2)=A_{51}$	$w_1=0.8162$and $w_2=0.1344$
47	$F(t-1)=A_{104}$^$F(t-2)=A_{110}$	$w_1=0.653$and $w_2=0.3165$	137	$F(t-1)=A_{49}$^$F(t-2)=A_{58}$	$w_1=0.439$and $w_2=0.5$
48	$F(t-1)=A_{100}$^$F(t-2)=A_{104}$	$w_1=0.6951$and $w_2=0.2109$	138	$F(t-1)=A_{44}$^$F(t-2)=A_{49}$	$w_1=0.7134$and $w_2=0.3151$
49	$F(t-1)=A_{83}$^$F(t-2)=A_{100}$	$w_1=0.8146$and $w_2=0.0573$	139	$F(t-1)=A_{50}$^$F(t-2)=A_{44}$	$w_1=0.7482$and $w_2=0.4013$
50	$F(t-1)=A_{60}$^$F(t-2)=A_{83}$	$w_1=0.5655$and $w_2=0.2749$	140	$F(t-1)=A_{70}$^$F(t-2)=A_{50}$	$w_1=0.7351$and $w_2=0.2708$
51	$F(t-1)=A_{40}$^$F(t-2)=A_{60}$	$w_1=0.7362$and $w_2=0.503$	141	$F(t-1)=A_{66}$^$F(t-2)=A_{70}$	$w_1=0.6791$and $w_2=0.3484$
52	$F(t-1)=A_{84}$^$F(t-2)=A_{40}$	$w_1=0.7017$and $w_2=0.4974$	142	$F(t-1)=A_{72}$^$F(t-2)=A_{66}$	$w_1=0.6969$and $w_2=0.3311$
53	$F(t-1)=A_{100}$^$F(t-2)=A_{84}$	$w_1=0.722$and $w_2=0.3317$	143	$F(t-1)=A_{75}$^$F(t-2)=A_{72}$	$w_1=0.5408$and $w_2=0.4781$
54	$F(t-1)=A_{105}$^$F(t-2)=A_{100}$	$w_1=0.5277$and $w_2=0.4992$	144	$F(t-1)=A_{77}$^$F(t-2)=A_{75}$	$w_1=0.8878$and $w_2=0.1262$
55	$F(t-1)=A_{108}$^$F(t-2)=A_{105}$	$w_1=0.5987$and $w_2=0.4511$	145	$F(t-1)=A_{79}$^$F(t-2)=A_{77}$	$w_1=0.5669$and $w_2=0.4104$

56	$F(t-1)=A_{117}\wedge F(t-2)=A_{108}$	w_1=0.7615and w_2=0.2801	146	$F(t-1)=A_{74}\wedge F(t-2)=A_{79}$	w_1=0.7468and w_2=0.199
57	$F(t-1)=A_{124}\wedge F(t-2)=A_{117}$	w_1=0.5502and w_2=0.4187	147	$F(t-1)=A_{65}\wedge F(t-2)=A_{74}$	w_1=0.6026and w_2=0.3335
58	$F(t-1)=A_{114}\wedge F(t-2)=A_{124}$	w_1=0.6178and w_2=0.3712	148	$F(t-1)=A_{58}\wedge F(t-2)=A_{65}$	w_1=0.4959and w_2=0.4313
59	$F(t-1)=A_{115}\wedge F(t-2)=A_{114}$	w_1=0.7718and w_2=0.2152	149	$F(t-1)=A_{49}\wedge F(t-2)=A_{58}$	w_1=0.7544and w_2=0.2732
60	$F(t-1)=A_{112}\wedge F(t-2)=A_{115}$	w_1=0.5934and w_2=0.4542	150	$F(t-1)=A_{56}\wedge F(t-2)=A_{49}$	w_1=0.7528and w_2=0.2798
61	$F(t-1)=A_{124}\wedge F(t-2)=A_{112}$	w_1=0.5126and w_2=0.2334	151	$F(t-1)=A_{59}\wedge F(t-2)=A_{56}$	w_1=0.7469and w_2=0.2868
62	$F(t-1)=A_{63}\wedge F(t-2)=A_{124}$	w_1=0.8095and w_2=0.0246	152	$F(t-1)=A_{64}\wedge F(t-2)=A_{59}$	w_1=0.5305and w_2=0.5004
63	$F(t-1)=A_{37}\wedge F(t-2)=A_{63}$	w_1=0.5201and w_2=0.3248	153	$F(t-1)=A_{67}\wedge F(t-2)=A_{64}$	w_1=0.7415and w_2=0.3142
64	$F(t-1)=A_{24}\wedge F(t-2)=A_{37}$	w_1=0.94and w_2=0.0227	154	$F(t-1)=A_{75}\wedge F(t-2)=A_{67}$	w_1=0.4368and w_2=0.5715
65	$F(t-1)=A_{19}\wedge F(t-2)=A_{24}$	w_1=0.7613and w_2=0.4193	155	$F(t-1)=A_{72}\wedge F(t-2)=A_{75}$	w_1=0.6571and w_2=0.3123
66	$F(t-1)=A_{43}\wedge F(t-2)=A_{19}$	w_1=0.5703and w_2=0.497	156	$F(t-1)=A_{68}\wedge F(t-2)=A_{72}$	w_1=0.3866and w_2=0.6246
67	$F(t-1)=A_{41}\wedge F(t-2)=A_{43}$	w_1=0.7731and w_2=0.1451	157	$F(t-1)=A_{72}\wedge F(t-2)=A_{68}$	w_1=0.6505and w_2=0.2767
68	$F(t-1)=A_{30}\wedge F(t-2)=A_{41}$	w_1=0.5769and w_2=0.455	158	$F(t-1)=A_{58}\wedge F(t-2)=A_{72}$	w_1=0.7454and w_2=0.2555
69	$F(t-1)=A_{39}\wedge F(t-2)=A_{30}$	w_1=0.6175and w_2=0.4501	159	$F(t-1)=A_{62}\wedge F(t-2)=A_{58}$	w_1=0.4072and w_2=0.6457
70	$F(t-1)=A_{44}\wedge F(t-2)=A_{39}$	w_1=0.572and w_2=0.4556	160	$F(t-1)=A_{68}\wedge F(t-2)=A_{62}$	w_1=0.5326and w_2=0.3795
71	$F(t-1)=A_{45}\wedge F(t-2)=A_{44}$	w_1=0.7394and w_2=0.1382	161	$F(t-1)=A_{51}\wedge F(t-2)=A_{68}$	w_1=0.7501and w_2=0.241
72	$F(t-1)=A_{26}\wedge F(t-2)=A_{45}$	w_1=0.5381and w_2=0.4195	162	$F(t-1)=A_{54}\wedge F(t-2)=A_{51}$	w_1=0.7499and w_2=0.2918
73	$F(t-1)=A_{29}\wedge F(t-2)=A_{26}$	w_1=0.5661and w_2=0.3769	163	$F(t-1)=A_{59}\wedge F(t-2)=A_{54}$	w_1=0.5706and w_2=0.4792
74	$F(t-1)=A_{20}\wedge F(t-2)=A_{29}$	w_1=0.6083and w_2=0.4925	164	$F(t-1)=A_{65}\wedge F(t-2)=A_{59}$	w_1=0.6608and w_2=0.3818
75	$F(t-1)=A_{37}\wedge F(t-2)=A_{20}$	w_1=0.7506and w_2=0.2416	165	$F(t-1)=A_{70}\wedge F(t-2)=A_{65}$	w_1=0.8178nd w_2=0.2086
76	$F(t-1)=A_{32}\wedge F(t-2)=A_{37}$	w_1=0.4859and w_2=0.4036	166	$F(t-1)=A_{73}\wedge F(t-2)=A_{70}$	w_1=0.5414and w_2=0.4692
77	$F(t-1)=A_{19}\wedge F(t-2)=A_{32}$	w_1=0.587and w_2=0.4051	167	$F(t-1)=A_{74}\wedge F(t-2)=A_{73}$	w_1=0.7095and w_2=0.2744
78	$F(t-1)=A_{23}\wedge F(t-2)=A_{19}$	w_1=0.522and w_2=0.4958	168	$F(t-1)=A_{71}\wedge F(t-2)=A_{74}$	w_1=0.6151and w_2=0.3478
79	$F(t-1)=A_{24}\wedge F(t-2)=A_{23}$	w_1=0.6045and w_2=0.4688	169	$F(t-1)=A_{65}\wedge F(t-2)=A_{71}$	w_1=0.6756and w_2=0.2981
80	$F(t-1)=A_{33}\wedge F(t-2)=A_{24}$	w_1=0.5769and w_2=0.5116	170	$F(t-1)=A_{62}\wedge F(t-2)=A_{65}$	w_1=0.4562and w_2=0.5447
81	$F(t-1)=A_{40}\wedge F(t-2)=A_{33}$	w_1=0.6915and w_2=0.4164	171	$F(t-1)=A_{64}\wedge F(t-2)=A_{62}$	w_1=0.5273and w_2=0.5
82	$F(t-1)=A_{53}\wedge F(t-2)=A_{40}$	w_1=0.8753and w_2=0.5	172	$F(t-1)=A_{68}\wedge F(t-2)=A_{64}$	w_1=0.7434and w_2=0.2367
83	$F(t-1)=A_{106}\wedge F(t-2)=A_{53}$	w_1=0.6826and w_2=0.5034	173	$F(t-1)=A_{63}\wedge F(t-2)=A_{68}$	w_1=0.6416and w_2=0.3957
84	$F(t-1)=A_{119}\wedge F(t-2)=A_{106}$	w_1=0.7737and w_2=0.0591	174	$F(t-1)=A_{71}\wedge F(t-2)=A_{63}$	w_1=0.6405and w_2=0.2914
85	$F(t-1)=A_{81}\wedge F(t-2)=A_{119}$	w_1=0.6953and w_2=0.1292	175	$F(t-1)=A_{57}\wedge F(t-2)=A_{71}$	w_1=0.7939and w_2=0.1728
86	$F(t-1)=A_{54}\wedge F(t-2)=A_{81}$	w_1=0.75and w_2=0.136	176	$F(t-1)=A_{54}\wedge F(t-2)=A_{57}$	w_1=0.7108and w_2=0.2695
87	$F(t-1)=A_{39}\wedge F(t-2)=A_{54}$	w_1=0.6137and w_2=0.3156	177	$F(t-1)=A_{52}\wedge F(t-2)=A_{54}$	w_1=0.545and w_2=0.4428
88	$F(t-1)=A_{34}\wedge F(t-2)=A_{39}$	w_1=0.7543and w_2=0.1623	178	$F(t-1)=A_{51}\wedge F(t-2)=A_{52}$	w_1=0.5061and w_2=0.5
89	$F(t-1)=A_{23}\wedge F(t-2)=A_{34}$	w_1=0.536and w_2=0.5			
90	$F(t-1)=A_{33}\wedge F(t-2)=A_{23}$	w_1=0.7522and w_2=0.2271			

Table 6 shows that each fuzzy rule has corresponding weight w_i values resulting from the application of the PSO function. To illustrate, we have the fuzzy rule (1) in Table 6. Accordingly, weights are equal to (w_1 = 0.734, w_2 = 0.1299). The value of the linguistic variable in time F(t-1) is equal to A17 and the value of the linguistic variable in time F(t-2) equals to A_{32}. Because this rule contains only two conditions, the number of

weights corresponding to this rule is only two. However, if the fuzzy rule contains three conditions, as in fuzzy rule 12 in the table, the number of corresponding weights will be three weights.
Throughout the values of the weights that have been obtained by using the PSO function, the formula of the defuzzification coefficient X_i has been applied and used in the defuzzing process to obtain the predictive values time series as shown in Table 7.

Table 7. Predictive values of the TDS series after defuzzification

Linguistic Variable	Month	Year	Linguistic Variable	Month	Year	Linguistic Variable	Month	Year	Linguistic Variable	Month	Year
***	Jan	2004	712.1305	Jan	2008	654.644	Jan	2012	632.0388	Jan	2016
***	Feb		649.8629	Feb		556.5566	Feb		639.9498	Feb	
370.429	Mar		572.022	Mar		552.6428	Mar		622.2322	Mar	
394.6782	Apr		502.1139	Apr		521.5485	Apr		591.8567	Apr	
399.1177	May		658.2267	May		472.8383	May		564.1834	May	
363.6687	June		710.9914	June		519.4693	June		535.0187	June	
373.9654	July		731.6088	July		617.3781	July		557.7008	July	
379.1749	Aug		741.1833	Aug		518.6186	Aug		569.7858	Aug	
387.7769	Sep		773.4207	Sep		630.8485	Sep		585.7851	Sep	
488.6273	Oct		795.3873	Oct		605.61	Oct		596.1049	Oct	
511.591	Nov		761.2142	Nov		596.6727	Nov		626.073	Nov	
499.6896	Dec		766.8576	Dec		620.8294	Dec		614.0409	Dec	
566.9747	Jan	2005	755.4341	Jan	2009	601.6079	Jan	2013	598.9665	Jan	2017
596.7618	Feb		795.9502	Feb		587.3241	Feb		615.0808	Feb	
552.2084	Mar		585.2333	Mar		473.1915	Mar		565.8065	Mar	
507.1153	Apr		492.4093	Apr		516.9208	Apr		579.0081	Apr	
521.0201	May		445.0586	May		547.4635	May		601.2227	May	
436.503	June		430.388	June		544.4603	June		539.8245	June	
447.659	July		514.3901	July		504.0725	July		549.8722	July	
496.5295	Aug		506.2627	Aug		571.7093	Aug		570.0606	Aug	
499.7335	Sep		465.614	Sep		576.8487	Sep		588.7885	Sep	
567.073	Oct		499.0696	Oct		616.6072	Oct		606.806	Oct	
587.239	Nov		517.8678	Nov		657.3602	Nov		619.2767	Nov	
559.3698	Dec		522.5421	Dec		649.9582	Dec		619.9171	Dec	
642.8267	Jan	2006	456.5569	Jan	2010	659.3882	Jan	2014	609.7755	Jan	2018
607.4288	Feb		463.3987	Feb		617.611	Feb		590.8393	Feb	
492.4919	Mar		433.5852	Mar		526.5338	Mar		581.1532	Mar	
514.1895	Apr		490.8415	Apr		503.6562	Apr		586.9667	Apr	
463.9317	May		472.919	May		513.3949	May		600.0026	May	
387.1127	June		428.9556	June		512.018	June		584.9267	June	
450.0782	July		443.6182	July		537.1298	July		585.3918	July	
508.5469	Aug		443.9474	Aug		529.8992	Aug		593.9333	Aug	
602.6637	Sep		477.7888	Sep		551.6953	Sep		573.8334	Sep	
630.7663	Oct		503.9061	Oct		618.2907	Oct		543.7246	Oct	
482.184	Nov		547.5624	Nov		603.9389	Nov		545.4309	Nov	
485.7252	Dec		732.5329	Dec		594.8948	Dec		557.1203	Dec	
593.2829	Jan	2007	777.4091	Jan	2011	611.1763	Jan	2015			
544.3126	Feb		645.3526	Feb		603.2408	Feb				
501.3298	Mar		549.6817	Mar		575.8963	Mar				
500.7565	Apr		500.3671	Apr		539.7923	Apr				
462.5617	May		480.4336	May		565.0178	May				
439.7679	June		443.2208	June		533.7624	June				
510.9367	July		477.4464	July		516.9263	July				
535.2146	Aug		460.177	Aug		563.1071	Aug				
568.8424	Sep		558.3233	Sep		552.8944	Sep				
569.74	Oct		536.5755	Oct		544.7831	Oct				
745.8112	Nov		584.7344	Nov		541.901	Nov				
729.1784	Dec		655.7574	Dec		546.8124	Dec				

It is noted in Table 7 that the predictive values of time series data are close to actual values when using the high order fuzzy time series model and symbolized $Fuzzy_{POS}$

3.3. The proposed procedure in this study

Time series data for the ARIMA(3,1,3) model were taken as a new series as shown in Figure 7.

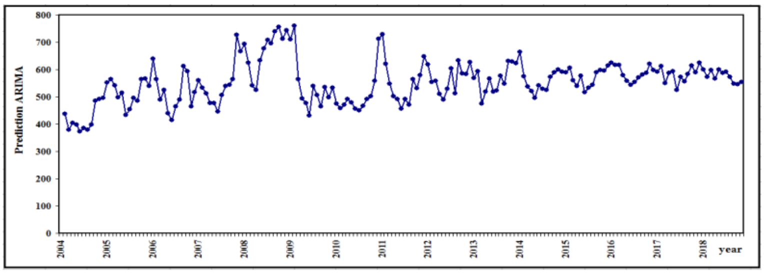

Figure 7. Time series data for the ARIMA model

The high order fuzzy time series model was combined to control the fluctuations of ARIMA series errors and to find a model with high accuracy and quality, by calculating the following: AD = 2.1709 and ADR = 1.2867, U = [372.1043, 761.0917]. The number of fuzzy sets is equal to n = 151. It's being able to define the fuzzy sets for new series (ARIMA) with 151 fuzzy sets. It can be defined by using TMF as shown in Table 8, which can be identified by using the TMF.

Table 8. Fuzzy sets of high order fuzzy model for ARIMA series

Fuzzy Sets	Fuzzy Numbers				Fuzzy Sets	Fuzzy Numbers			
A_{1}	372.1043	373.391	374.6777	375.9644	A_{77}	567.6827	568.9694	570.2561	571.5428
A_{2}	374.6777	375.9644	377.2511	378.5378	A_{78}	570.2561	571.5428	572.8295	574.1162
A_{3}	377.2511	378.5378	379.8245	381.1112	A_{79}	572.8295	574.1162	575.4029	576.6896
A_{4}	379.8245	381.1112	382.3979	383.6846	A_{80}	575.4029	576.6896	577.9763	579.263
A_{5}	382.3979	383.6846	384.9713	386.258	A_{81}	577.9763	579.263	580.5497	581.8364
A_{6}	384.9713	386.258	387.5447	388.8314	A_{82}	580.5497	581.8364	583.1231	584.4098
A_{7}	387.5447	388.8314	390.1181	391.4048	A_{83}	583.1231	584.4098	585.6965	586.9832
A_{8}	390.1181	391.4048	392.6915	393.9782	A_{84}	585.6965	586.9832	588.2699	589.5566
A_{9}	392.6915	393.9782	395.2649	396.5516	A_{85}	588.2699	589.5566	590.8433	592.13
A_{10}	395.2649	396.5516	397.8383	399.125	A_{86}	590.8433	592.13	593.4167	594.7034
A_{11}	397.8383	399.125	400.4117	401.6984	A_{87}	593.4167	594.7034	595.9901	597.2768
A_{12}	400.4117	401.6984	402.9851	404.2718	A_{88}	595.9901	597.2768	598.5635	599.8502
A_{13}	402.9851	404.2718	405.5585	406.8452	A_{89}	598.5635	599.8502	601.1369	602.4236
A_{14}	405.5585	406.8452	408.1319	409.4186	A_{90}	601.1369	602.4236	603.7103	604.997
A_{15}	408.1319	409.4186	410.7053	411.992	A_{91}	603.7103	604.997	606.2837	607.5704
A_{16}	410.7053	411.992	413.2787	414.5654	A_{92}	606.2837	607.5704	608.8571	610.1438
A_{17}	413.2787	414.5654	415.8521	417.1388	A_{93}	608.8571	610.1438	611.4305	612.7172
A_{18}	415.8521	417.1388	418.4255	419.7122	A_{94}	611.4305	612.7172	614.0039	615.2906
A_{19}	418.4255	419.7122	420.9989	422.2856	A_{95}	614.0039	615.2906	616.5773	617.864
A_{20}	420.9989	422.2856	423.5723	424.859	A_{96}	616.5773	617.864	619.1507	620.4374
A_{21}	423.5723	424.859	426.1457	427.4324	A_{97}	619.1507	620.4374	621.7241	623.0108
A_{22}	426.1457	427.4324	428.7191	430.0058	A_{98}	621.7241	623.0108	624.2975	625.5842
A_{23}	428.7191	430.0058	431.2925	432.5792	A_{99}	624.2975	625.5842	626.8709	628.1576
A_{24}	431.2925	432.5792	433.8659	435.1526	A_{100}	626.8709	628.1576	629.4443	630.731
A_{25}	433.8659	435.1526	436.4393	437.726	A_{101}	629.4443	630.731	632.0177	633.3044
A_{26}	436.4393	437.726	439.0127	440.2994	A_{102}	632.0177	633.3044	634.5911	635.8778
A_{27}	439.0127	440.2994	441.5861	442.8728	A_{103}	634.5911	635.8778	637.1645	638.4512
A_{28}	441.5861	442.8728	444.1595	445.4462	A_{104}	637.1645	638.4512	639.7379	641.0246
A_{29}	444.1595	445.4462	446.7329	448.0196	A_{105}	639.7379	641.0246	642.3113	643.598
A_{30}	446.7329	448.0196	449.3063	450.593	A_{106}	642.3113	643.598	644.8847	646.1714
A_{31}	449.3063	450.593	451.8797	453.1664	A_{107}	644.8847	646.1714	647.4581	648.7448
A_{32}	451.8797	453.1664	454.4531	455.7398	A_{108}	647.4581	648.7448	650.0315	651.3182
A_{33}	454.4531	455.7398	457.0265	458.3132	A_{109}	650.0315	651.3182	652.6049	653.8916
A_{34}	457.0265	458.3132	459.5999	460.8866	A_{110}	652.6049	653.8916	655.1783	656.465
A_{35}	459.5999	460.8866	462.1733	463.46	A_{111}	655.1783	656.465	657.7517	659.0384
A_{36}	462.1733	463.46	464.7467	466.0334	A_{112}	657.7517	659.0384	660.3251	661.6118
A_{37}	464.7467	466.0334	467.3201	468.6068	A_{113}	660.3251	661.6118	662.8985	664.1852
A_{38}	467.3201	468.6068	469.8935	471.1802	A_{114}	662.8985	664.1852	665.4719	666.7586
A_{39}	469.8935	471.1802	472.4669	473.7536	A_{115}	665.4719	666.7586	668.0453	669.332
A_{40}	472.4669	473.7536	475.0403	476.327	A_{116}	668.0453	669.332	670.6187	671.9054
A_{41}	475.0403	476.327	477.6137	478.9004	A_{117}	670.6187	671.9054	673.1921	674.4788
A_{42}	477.6137	478.9004	480.1871	481.4738	A_{118}	673.1921	674.4788	675.7655	677.0522
A_{43}	480.1871	481.4738	482.7605	484.0472	A_{119}	675.7655	677.0522	678.3389	679.6256
A_{44}	482.7605	484.0472	485.3339	486.6206	A_{120}	678.3389	679.6256	680.9123	682.199
A_{45}	485.3339	486.6206	487.9073	489.194	A_{121}	680.9123	682.199	683.4857	684.7724
A_{46}	487.9073	489.194	490.4807	491.7674	A_{122}	683.4857	684.7724	686.0591	687.3458
A_{47}	490.4807	491.7674	493.0541	494.3408	A_{123}	686.0591	687.3458	688.6325	689.9192
A_{48}	493.0541	494.3408	495.6275	496.9142	A_{124}	688.6325	689.9192	691.2059	692.4926

A_{49}	495.6275	496.9142	498.2009	499.4876	A_{125}	691.2059	692.4926	693.7793	695.066
A_{50}	498.2009	499.4876	500.7743	502.061	A_{126}	693.7793	695.066	696.3527	697.6394
A_{51}	500.7743	502.061	503.3477	504.6344	A_{127}	696.3527	697.6394	698.9261	700.2128
A_{52}	503.3477	504.6344	505.9211	507.2078	A_{128}	698.9261	700.2128	701.4995	702.7862
A_{53}	505.9211	507.2078	508.4945	509.7812	A_{129}	701.4995	702.7862	704.0729	705.3596
A_{54}	508.4945	509.7812	511.0679	512.3546	A_{130}	704.0729	705.3596	706.6463	707.933
A_{55}	511.0679	512.3546	513.6413	514.928	A_{131}	706.6463	707.933	709.2197	710.5064
A_{56}	513.6413	514.928	516.2147	517.5014	A_{132}	709.2197	710.5064	711.7931	713.0798
A_{57}	516.2147	517.5014	518.7881	520.0748	A_{133}	711.7931	713.0798	714.3665	715.6532
A_{58}	518.7881	520.0748	521.3615	522.6482	A_{134}	714.3665	715.6532	716.9399	718.2266
A_{59}	521.3615	522.6482	523.9349	525.2216	A_{135}	716.9399	718.2266	719.5133	720.8
A_{60}	523.9349	525.2216	526.5083	527.795	A_{136}	719.5133	720.8	722.0867	723.3734
A_{61}	526.5083	527.795	529.0817	530.3684	A_{137}	722.0867	723.3734	724.6601	725.9468
A_{62}	529.0817	530.3684	531.6551	532.9418	A_{138}	724.6601	725.9468	727.2335	728.5202
A_{63}	531.6551	532.9418	534.2285	535.5152	A_{139}	727.2335	728.5202	729.8069	731.0936
A_{64}	534.2285	535.5152	536.8019	538.0886	A_{140}	729.8069	731.0936	732.3803	733.667
A_{65}	536.8019	538.0886	539.3753	540.662	A_{141}	732.3803	733.667	734.9537	736.2404
A_{66}	539.3753	540.662	541.9487	543.2354	A_{142}	734.9537	736.2404	737.5271	738.8138
A_{67}	541.9487	543.2354	544.5221	545.8088	A_{143}	737.5271	738.8138	740.1005	741.3872
A_{68}	544.5221	545.8088	547.0955	548.3822	A_{144}	740.1005	741.3872	742.6739	743.9606
A_{69}	547.0955	548.3822	549.6689	550.9556	A_{145}	742.6739	743.9606	745.2473	746.534
A_{70}	549.6689	550.9556	552.2423	553.529	A_{146}	745.2473	746.534	747.8207	749.1074
A_{71}	552.2423	553.529	554.8157	556.1024	A_{147}	747.8207	749.1074	750.3941	751.6808
A_{72}	554.8157	556.1024	557.3891	558.6758	A_{148}	750.3941	751.6808	752.9675	754.2542
A_{73}	557.3891	558.6758	559.9625	561.2492	A_{149}	752.9675	754.2542	755.5409	756.8276
A_{74}	559.9625	561.2492	562.5359	563.8226	A_{150}	755.5409	756.8276	758.1143	759.401
A_{75}	562.5359	563.8226	565.1093	566.396	A_{151}	758.1143	759.401	760.6877	761.9744
A_{76}	565.1093	566.396	567.6827	568.9694					

Time series data has fuzzified, and Table 9 shows fuzzification of the time series to determine the membership function.

Table 9. Fuzzification ARIMA data according to the high order fuzzy model

Linguistic Variable	Month	Year	Linguistic Variable	Month	Year	Linguistic Variable	Month	Year	Linguistic Variable	Month	Year
***	Jan	2004	A_{115}	Jan	2008	A_{108}	Jan	2012	A_{94}	Jan	2016
A_{39}	Feb		A_{125}	Feb		A_{96}	Feb		A_{98}	Feb	
A_{19}	Mar		A_{98}	Mar		A_{71}	Mar		A_{95}	Mar	
A_{3}	Apr		A_{66}	Apr		A_{72}	Apr		A_{95}	Apr	
A_{12}	May		A_{60}	May		A_{53}	May		A_{81}	May	
A_{10}	June		A_{102}	June		A_{46}	June		A_{73}	June	
A_{1}	July		A_{118}	July		A_{61}	July		A_{67}	July	
A_{5}	Aug		A_{130}	Aug		A_{90}	Aug		A_{70}	Aug	
A_{3}	Sep		A_{126}	Sep		A_{55}	Sep		A_{77}	Sep	
A_{10}	Oct		A_{143}	Oct		A_{101}	Oct		A_{81}	Oct	
A_{44}	Nov		A_{149}	Nov		A_{83}	Nov		A_{83}	Nov	
A_{46}	Dec		A_{132}	Dec		A_{82}	Dec		A_{97}	Dec	
A_{48}	Jan	2005	A_{145}	Jan	2009	A_{99}	Jan	2013	A_{87}	Jan	2017
A_{70}	Feb		A_{132}	Feb		A_{76}	Feb		A_{86}	Feb	
A_{75}	Mar		A_{151}	Mar		A_{86}	Mar		A_{93}	Mar	
A_{66}	Apr		A_{75}	Apr		A_{40}	Apr		A_{69}	Apr	
A_{49}	May		A_{47}	May		A_{57}	May		A_{84}	May	
A_{56}	June		A_{41}	June		A_{76}	June		A_{86}	June	
A_{24}	July		A_{23}	July		A_{57}	July		A_{60}	July	
A_{32}	Aug		A_{65}	Aug		A_{59}	Aug		A_{78}	Aug	
A_{48}	Sep		A_{52}	Sep		A_{80}	Sep		A_{72}	Sep	
A_{44}	Oct		A_{36}	Oct		A_{69}	Oct		A_{82}	Oct	
A_{75}	Nov		A_{63}	Nov		A_{101}	Nov		A_{95}	Nov	
A_{75}	Dec		A_{49}	Dec		A_{100}	Dec		A_{84}	Dec	
A_{65}	Jan	2006	A_{62}	Jan	2010	A_{97}	Jan	2014	A_{98}	Jan	2018
A_{104}	Feb		A_{40}	Feb		A_{114}	Feb		A_{88}	Feb	
A_{75}	Mar		A_{33}	Mar		A_{79}	Mar		A_{78}	Mar	
A_{45}	Apr		A_{38}	Apr		A_{64}	Apr		A_{88}	Apr	
A_{60}	May		A_{47}	May		A_{58}	May		A_{76}	May	
A_{26}	June		A_{42}	June		A_{48}	June		A_{88}	June	
A_{17}	July		A_{33}	July		A_{66}	July		A_{84}	July	
A_{36}	Aug		A_{30}	Aug		A_{61}	Aug		A_{85}	Aug	
A_{46}	Sep		A_{37}	Sep		A_{59}	Sep		A_{78}	Sep	
A_{93}	Oct		A_{46}	Oct		A_{78}	Oct		A_{68}	Oct	
A_{86}	Nov		A_{51}	Nov		A_{85}	Nov		A_{68}	Nov	
A_{36}	Dec		A_{73}	Dec		A_{89}	Dec		A_{71}	Dec	

A_{56}	Jan	2007	A_{132}	Jan	2011	A_{86}	Jan	2015
A_{73}	Feb		A_{138}	Feb		A_{84}	Feb	
A_{63}	Mar		A_{96}	Mar		A_{91}	Mar	
A_{55}	Apr		A_{68}	Apr		A_{73}	Apr	
A_{41}	May		A_{50}	May		A_{65}	May	
A_{41}	June		A_{46}	June		A_{80}	June	
A_{29}	July		A_{33}	July		A_{56}	July	
A_{52}	Aug		A_{46}	Aug		A_{63}	Aug	
A_{65}	Sep		A_{39}	Sep		A_{67}	Sep	
A_{67}	Oct		A_{74}	Oct		A_{85}	Oct	
A_{75}	Nov		A_{62}	Nov		A_{88}	Nov	
A_{138}	Dec		A_{81}	Dec		A_{87}	Dec	

In the stage of defuzzification, FSGS elements have been established as in Table 10. It is clear that there are no frequencies in elements of FSGS.

Table 10. FSGS for the data of ARIMA series

#	FSGS	#	FSGS	#	FSGS	#	FSGS	#	FSGS
1	$\{A_{39},A_{19}\}$	37	$\{A_{73},A_{63}\}$	73	$\{A_{40},A_{33}\}$	109	$\{A_{76},A_{86}\}$	145	$\{A_{98},A_{95}\}$
2	$\{A_{19},A_{3}\}$	38	$\{A_{63},A_{55}\}$	74	$\{A_{33},A_{38}\}$	110	$\{A_{86},A_{40}\}$	146	$\{A_{95},A_{95}\}$
3	$\{A_{3},A_{12}\}$	39	$\{A_{55},A_{41}\}$	75	$\{A_{38},A_{47}\}$	111	$\{A_{40},A_{57}\}$	147	$\{A_{95},A_{81}\}$
4	$\{A_{12},A_{10}\}$	40	$\{A_{41},A_{41}\}$	76	$\{A_{47},A_{42}\}$	112	$\{A_{57},A_{76}\}$	148	$\{A_{81},A_{73}\}$
5	$\{A_{10},A_{1}\}$	41	$\{A_{41},A_{29}\}$	77	$\{A_{42},A_{33}\}$	113	$\{A_{76},A_{57}\}$	149	$\{A_{73},A_{67}\}$
6	$\{A_{1},A_{5}\}$	42	$\{A_{29},A_{52}\}$	78	$\{A_{33},A_{30}\}$	114	$\{A_{57},A_{59}\}$	150	$\{A_{67},A_{70}\}$
7	$\{A_{5},A_{3}\}$	43	$\{A_{52},A_{65}\}$	79	$\{A_{30},A_{37}\}$	115	$\{A_{59},A_{80}\}$	151	$\{A_{70},A_{77}\}$
8	$\{A_{3},A_{10}\}$	44	$\{A_{65},A_{67}\}$	80	$\{A_{37},A_{46}\}$	116	$\{A_{80},A_{69}\}$	152	$\{A_{77},A_{81}\}$
9	$\{A_{10},A_{44}\}$	45	$\{A_{67},A_{75}\}$	81	$\{A_{46},A_{51}\}$	117	$\{A_{69},A_{101}\}$	153	$\{A_{81},A_{83}\}$
10	$\{A_{44},A_{46}\}$	46	$\{A_{75},A_{138}\}$	82	$\{A_{51},A_{73}\}$	118	$\{A_{101},A_{100}\}$	154	$\{A_{83},A_{97}\}$
11	$\{A_{46},A_{48}\}$	47	$\{A_{138},A_{115}\}$	83	$\{A_{73},A_{132}\}$	119	$\{A_{100},A_{97}\}$	155	$\{A_{97},A_{87}\}$
12	$\{A_{48},A_{70}\}$	48	$\{A_{115},A_{125}\}$	84	$\{A_{132},A_{138}\}$	120	$\{A_{97},A_{114}\}$	156	$\{A_{87},A_{86}\}$
13	$\{A_{70},A_{75}\}$	49	$\{A_{125},A_{98}\}$	85	$\{A_{138},A_{96}\}$	121	$\{A_{114},A_{79}\}$	157	$\{A_{86},A_{93}\}$
14	$\{A_{75},A_{66}\}$	50	$\{A_{98},A_{66}\}$	86	$\{A_{96},A_{68}\}$	122	$\{A_{79},A_{64}\}$	158	$\{A_{93},A_{69}\}$
15	$\{A_{66},A_{49}\}$	51	$\{A_{66},A_{60}\}$	87	$\{A_{68},A_{50}\}$	123	$\{A_{64},A_{58}\}$	159	$\{A_{69},A_{84}\}$
16	$\{A_{49},A_{56}\}$	52	$\{A_{60},A_{102}\}$	88	$\{A_{50},A_{46}\}$	124	$\{A_{58},A_{48}\}$	160	$\{A_{84},A_{86}\}$
17	$\{A_{56},A_{24}\}$	53	$\{A_{102},A_{118}\}$	89	$\{A_{46},A_{33}\}$	125	$\{A_{48},A_{66}\}$	161	$\{A_{86},A_{60}\}$
18	$\{A_{24},A_{32}\}$	54	$\{A_{118},A_{130}\}$	90	$\{A_{33},A_{46}\}$	126	$\{A_{66},A_{61}\}$	162	$\{A_{60},A_{78}\}$
19	$\{A_{32},A_{48}\}$	55	$\{A_{130},A_{126}\}$	91	$\{A_{46},A_{39}\}$	127	$\{A_{61},A_{59}\}$	163	$\{A_{78},A_{72}\}$
20	$\{A_{48},A_{44}\}$	56	$\{A_{126},A_{143}\}$	92	$\{A_{39},A_{74}\}$	128	$\{A_{59},A_{78}\}$	164	$\{A_{72},A_{82}\}$
21	$\{A_{44},A_{75}\}$	57	$\{A_{143},A_{149}\}$	93	$\{A_{74},A_{62}\}$	129	$\{A_{78},A_{85}\}$	165	$\{A_{82},A_{95}\}$
22	$\{A_{75},A_{75}\}$	58	$\{A_{149},A_{132}\}$	94	$\{A_{62},A_{81}\}$	130	$\{A_{85},A_{89}\}$	166	$\{A_{95},A_{84}\}$
23	$\{A_{75},A_{65}\}$	59	$\{A_{132},A_{145}\}$	95	$\{A_{81},A_{108}\}$	131	$\{A_{89},A_{86}\}$	167	$\{A_{84},A_{98}\}$
24	$\{A_{65},A_{104}\}$	60	$\{A_{145},A_{132}\}$	96	$\{A_{108},A_{96}\}$	132	$\{A_{86},A_{84}\}$	168	$\{A_{98},A_{88}\}$
25	$\{A_{104},A_{75}\}$	61	$\{A_{132},A_{151}\}$	97	$\{A_{96},A_{71}\}$	133	$\{A_{84},A_{91}\}$	169	$\{A_{88},A_{78}\}$
26	$\{A_{75},A_{45}\}$	62	$\{A_{151},A_{75}\}$	98	$\{A_{71},A_{72}\}$	134	$\{A_{91},A_{73}\}$	170	$\{A_{78},A_{88}\}$
27	$\{A_{45},A_{60}\}$	63	$\{A_{75},A_{47}\}$	99	$\{A_{72},A_{53}\}$	135	$\{A_{73},A_{65}\}$	171	$\{A_{88},A_{76}\}$
28	$\{A_{60},A_{26}\}$	64	$\{A_{47},A_{41}\}$	100	$\{A_{53},A_{46}\}$	136	$\{A_{65},A_{80}\}$	172	$\{A_{76},A_{88}\}$
29	$\{A_{26},A_{17}\}$	65	$\{A_{41},A_{23}\}$	101	$\{A_{46},A_{61}\}$	137	$\{A_{80},A_{56}\}$	173	$\{A_{88},A_{84}\}$
30	$\{A_{17},A_{36}\}$	66	$\{A_{23},A_{65}\}$	102	$\{A_{61},A_{90}\}$	138	$\{A_{56},A_{63}\}$	174	$\{A_{84},A_{85}\}$
31	$\{A_{36},A_{46}\}$	67	$\{A_{65},A_{52}\}$	103	$\{A_{90},A_{55}\}$	139	$\{A_{63},A_{67}\}$	175	$\{A_{85},A_{78}\}$
32	$\{A_{46},A_{93}\}$	68	$\{A_{52},A_{36}\}$	104	$\{A_{55},A_{101}\}$	140	$\{A_{67},A_{85}\}$	176	$\{A_{78},A_{68}\}$
33	$\{A_{93},A_{86}\}$	69	$\{A_{36},A_{63}\}$	105	$\{A_{101},A_{83}\}$	141	$\{A_{85},A_{88}\}$	177	$\{A_{68},A_{68}\}$
34	$\{A_{86},A_{36}\}$	70	$\{A_{63},A_{49}\}$	106	$\{A_{83},A_{82}\}$	142	$\{A_{88},A_{87}\}$	178	$\{A_{68},A_{71}\}$
35	$\{A_{36},A_{56}\}$	71	$\{A_{49},A_{62}\}$	107	$\{A_{82},A_{99}\}$	143	$\{A_{87},A_{94}\}$		
36	$\{A_{56},A_{73}\}$	72	$\{A_{62},A_{40}\}$	108	$\{A_{99},A_{76}\}$	144	$\{A_{94},A_{98}\}$		

The *If-then* rules have evaluated for the sets of Table 10. PSO function algorithm is applied to obtain the w_i weights and calculate the value of the defuzzification coefficient X_i. Table 11 illustrates these steps:

Table 11. Evaluation of the *If-then* rule and w_i results of ARIMA data using PSO function

#	Matching Measure *If - then*	Resultant Weights	#	Matching Measure *If - then*	Resultant Weights
1	$F(t-1)= A_{19}$ ^$F(t-2)= A_{39}$	w_1=0.7487 and w_2=0.143	90	$F(t-1)=A_{46}$^$F(t-2)=A_{33}$	w_1=0.7503and w_2=0.2261
2	$F(t-1)= A_{3}$ ^$F(t-2)= A_{19}$	w_1=0.6097 and w_2=0.4183	91	$F(t-1)=A_{39}$^$F(t-2)=A_{46}$	w_1=0.687and w_2=0.4977
3	$F(t-1)=A_{12}$^$F(t-2)=A_{3}$	w_1=0.704 and w_2=0.3028	92	$F(t-1)=A_{74}$^$F(t-2)=A_{39}$	w_1=0.5328and w_2=0.4947
4	$F(t-1)=A_{10}$^$F(t-2)=A_{12}$	w_1=0.4378 and w_2=0.493	93	$F(t-1)=A_{62}$^$F(t-2)=A_{74}$	w_1=0.5521and w_2=0.5013
5	$F(t-1)=A_{1}$^$F(t-2)=A_{10}$	w_1=0.7465 and w_2=0.2602	94	$F(t-1)=A_{81}$^$F(t-2)=A_{62}$	w_1=0.6606and w_2=0.5044
6	$F(t-1)=A_{5}$^$F(t-2)=A_{1}$	w_1=0.75and w_2=0.2363	95	$F(t-1)=A_{108}$^$F(t-2)=A_{81}$	w_1=0.5127and w_2=0.5
7	$F(t-1)=A_{3}$^$F(t-2)=A_{5}$	w_1=0.7135and w_2=0.3432	96	$F(t-1)=A_{96}$^$F(t-2)=A_{108}$	w_1=0.6475and w_2=0.2382
8	$F(t-1)=A_{10}$^$F(t-2)=A_{3}$	w_1=0.75and w_2=0.5	97	$F(t-1)=A_{71}$^$F(t-2)=A_{96}$	w_1=0.7246and w_2=0.2596

9	F(t-1)=A_{44}^F(t-2)=A_{10}	w_1=0.7351and w_2=0.3326	98	F(t-1)=A_{72}^F(t-2)=A_{71}	w_1=0.5805and w_2=0.3354
10	F(t-1)=A_{46}^F(t-2)=A_{44}	w_1=0.6531and w_2=0.3645	99	F(t-1)=A_{53}^F(t-2)=A_{72}	w_1=0.7055and w_2=0.228
11	F(t-1)=A_{48}^F(t-2)=A_{46}	w_1=0.6438and w_2=0.4851	100	F(t-1)=A_{46}^F(t-2)=A_{53}	w_1=0.7182and w_2=0.3456
12	F(t-1)=A_{70}^F(t-2)=A_{48}	w_1=0.7822and w_2=0.2595	101	F(t-1)=A_{61}^F(t-2)=A_{46}	w_1=0.7513and w_2=0.4257
13	F(t-1)=A_{75}^F(t-2)=A_{70}	w_1=0.4632and w_2=0.5104	102	F(t-1)=A_{90}^F(t-2)=A_{61}	w_1=0.7252and w_2=0.1348
14	F(t-1)=A_{66}^F(t-2)=A_{75}	w_1=0.1828and w_2=0.6967	103	F(t-1)=A_{55}^F(t-2)=A_{90}	w_1=0.7549and w_2=0.4099
15	F(t-1)=A_{49}^F(t-2)=A_{66}	w_1=0.7114and w_2=0.3067	104	F(t-1)=A_{101}^F(t-2)=A_{55}	w_1=0.6263and w_2=0.3618
16	F(t-1)=A_{56}^F(t-2)=A_{49}	w_1=0.6269and w_2=0.2323	105	F(t-1)=A_{83}^F(t-2)=A_{101}	w_1=0.5006and w_2=0.4546
17	F(t-1)=A_{24}^F(t-2)=A_{56}	w_1=0.5328and w_2=0.428	106	F(t-1)=A_{82}^F(t-2)=A_{83}	w_1=0.6719and w_2=0.4063
18	F(t-1)=A_{32}^F(t-2)=A_{24}	w_1=0.6737and w_2=0.4447	107	F(t-1)=A_{99}^F(t-2)=A_{82}	w_1=0.4528and w_2=0.4903
19	F(t-1)=A_{48}^F(t-2)=A_{32}	w_1=0.5091and w_2=0.5056	108	F(t-1)=A_{76}^F(t-2)=A_{99}	w_1=0.7624and w_2=0.262
20	F(t-1)=A_{44}^F(t-2)=A_{48}	w_1=0.7145and w_2=0.4449	109	F(t-1)=A_{86}^F(t-2)=A_{76}	w_1=0.7417and w_2=0.0599
21	F(t-1)=A_{75}^F(t-2)=A_{44}	w_1=0.5656and w_2=0.5052	110	F(t-1)=A_{40}^F(t-2)=A_{86}	w_1=0.7501and w_2=0.2796
22	F(t-1)=A_{75}^F(t-2)=A_{75}	w_1=0.7677and w_2=0.1897	111	F(t-1)=A_{57}^F(t-2)=A_{40}	w_1=0.7146and w_2=0.4236
23	F(t-1)=A_{65}^F(t-2)=A_{75}	w_1=0.7731and w_2=0.4012	112	F(t-1)=A_{76}^F(t-2)=A_{57}	w_1=0.4738and w_2=0.4886
24	F(t-1)=A_{104}^F(t-2)=A_{65}	w_1=0.6309and w_2=0.3002	113	F(t-1)=A_{57}^F(t-2)=A_{76}	w_1=0.8535and w_2=0.1378
25	F(t-1)=A_{75}^F(t-2)=A_{104}	w_1=0.3926and w_2=0.4122	114	F(t-1)=A_{59}^F(t-2)=A_{57}	w_1=0.8854and w_2=0.2278
26	F(t-1)=A_{45}^F(t-2)=A_{75}	w_1=0.6742and w_2=0.351	115	F(t-1)=A_{80}^F(t-2)=A_{59}	w_1=0.4777and w_2=0.5171
27	F(t-1)=A_{60}^F(t-2)=A_{45}	w_1=0.4119and w_2=0.4616	116	F(t-1)=A_{69}^F(t-2)=A_{80}	w_1=0.7474and w_2=0.3818
28	F(t-1)=A_{26}^F(t-2)=A_{60}	w_1=0.6142and w_2=0.2847	117	F(t-1)=A_{101}^F(t-2)=A_{69}	w_1=0.5926and w_2=0.4733
29	F(t-1)=A_{17}^F(t-2)=A_{26}	w_1=0.7736and w_2=0.3248	118	F(t-1)=A_{100}^F(t-2)=A_{101}	w_1=0.75and w_2=0.2348
30	F(t-1)=A_{36}^F(t-2)=A_{17}	w_1=0.697and w_2=0.4019	119	F(t-1)=A_{97}^F(t-2)=A_{100}	w_1=0.7565and w_2=0.3083
31	F(t-1)=A_{46}^F(t-2)=A_{36}	w_1=0.7608and w_2=0.5061	120	F(t-1)=A_{114}^F(t-2)=A_{97}	w_1=0.7268and w_2=0.1549
32	F(t-1)=A_{93}^F(t-2)=A_{46}	w_1=0.767and w_2=0.259	121	F(t-1)=A_{79}^F(t-2)=A_{114}	w_1=0.5511and w_2=0.333
33	F(t-1)=A_{86}^F(t-2)=A_{93}	w_1=0.374and w_2=0.3985	122	F(t-1)=A_{64}^F(t-2)=A_{79}	w_1=0.6368and w_2=0.3169
34	F(t-1)=A_{36}^F(t-2)=A_{86}	w_1=0.7376and w_2=0.2893	123	F(t-1)=A_{58}^F(t-2)=A_{64}	w_1=0.6982and w_2=0.2543
35	F(t-1)=A_{56}^F(t-2)=A_{36}	w_1=0.7157and w_2=0.4127	124	F(t-1)=A_{48}^F(t-2)=A_{58}	w_1=0.6217and w_2=0.4433
36	F(t-1)=A_{73}^F(t-2)=A_{56}	w_1=0.486and w_2=0.5001	125	F(t-1)=A_{66}^F(t-2)=A_{48}	w_1=0.807and w_2=0.1943
37	F(t-1)=A_{63}^F(t-2)=A_{73}	w_1=0.7294and w_2=0.2186	126	F(t-1)=A_{61}^F(t-2)=A_{66}	w_1=0.6308and w_2=0.3491
38	F(t-1)=A_{55}^F(t-2)=A_{63}	w_1=0.4108and w_2=0.5	127	F(t-1)=A_{59}^F(t-2)=A_{61}	w_1=0.7156and w_2=0.3726
39	F(t-1)=A_{41}^F(t-2)=A_{55}	w_1=0.6874and w_2=0.2841	128	F(t-1)=A_{78}^F(t-2)=A_{59}	w_1=0.7132and w_2=0.3494
40	F(t-1)=A_{41}^F(t-2)=A_{41}	w_1=0.4391and w_2=0.4983	129	F(t-1)=A_{85}^F(t-2)=A_{78}	w_1=0.7104and w_2=0.3186
41	F(t-1)=A_{29}^F(t-2)=A_{41}	w_1=0.6953and w_2=0.4059	130	F(t-1)=A_{89}^F(t-2)=A_{85}	w_1=0.4947and w_2=0.5063
42	F(t-1)=A_{52}^F(t-2)=A_{29}	w_1=0.6359and w_2=0.4964	131	F(t-1)=A_{86}^F(t-2)=A_{89}	w_1=0.9188and w_2=0.0821
43	F(t-1)=A_{65}^F(t-2)=A_{52}	w_1=0.5195and w_2=0.5137	132	F(t-1)=A_{84}^F(t-2)=A_{86}	w_1=0.5283and w_2=0.5
44	F(t-1)=A_{67}^F(t-2)=A_{65}	w_1=0.5364and w_2=0.4976	133	F(t-1)=A_{91}^F(t-2)=A_{84}	w_1=0.7147and w_2=0.2121
45	F(t-1)=A_{75}^F(t-2)=A_{67}	w_1=0.751and w_2=0.5481	134	F(t-1)=A_{73}^F(t-2)=A_{91}	w_1=0.468and w_2=0.4595
46	F(t-1)=A_{138}^F(t-2)=A_{75}	w_1=0.5548and w_2=0.4612	135	F(t-1)=A_{65}^F(t-2)=A_{73}	w_1=0.6066and w_2=0.4542
47	F(t-1)=A_{115}^F(t-2)=A_{138}	w_1=0.5589and w_2=0.4447	136	F(t-1)=A_{80}^F(t-2)=A_{65}	w_1=0.7016and w_2=0.2039
48	F(t-1)=A_{125}^F(t-2)=A_{115}	w_1=0.4047and w_2=0.5138	137	F(t-1)=A_{56}^F(t-2)=A_{80}	w_1=0.5363and w_2=0.4423
49	F(t-1)=A_{98}^F(t-2)=A_{125}	w_1=0.7511and w_2=0.1019	138	F(t-1)=A_{63}^F(t-2)=A_{56}	w_1=0.5044and w_2=0.5319
50	F(t-1)=A_{66}^F(t-2)=A_{98}	w_1=0.4912and w_2=0.4106	139	F(t-1)=A_{67}^F(t-2)=A_{63}	w_1=0.6958and w_2=0.4054
51	F(t-1)=A_{60}^F(t-2)=A_{66}	w_1=0.7309and w_2=0.4583	140	F(t-1)=A_{85}^F(t-2)=A_{67}	w_1=0.6043and w_2=0.4482
52	F(t-1)=A_{102}^F(t-2)=A_{60}	w_1=0.7133and w_2=0.429	141	F(t-1)=A_{88}^F(t-2)=A_{85}	w_1=0.7376and w_2=0.2569
53	F(t-1)=A_{118}^F(t-2)=A_{102}	w_1=0.6604and w_2=0.4125	142	F(t-1)=A_{87}^F(t-2)=A_{88}	w_1=0.7427and w_2=0.2808
54	F(t-1)=A_{130}^F(t-2)=A_{118}	w_1=0.5813and w_2=0.4261	143	F(t-1)=A_{94}^F(t-2)=A_{87}	w_1=0.5315and w_2=0.5
55	F(t-1)=A_{126}^F(t-2)=A_{130}	w_1=0.552and w_2=0.5052	144	F(t-1)=A_{98}^F(t-2)=A_{94}	w_1=0.7529and w_2=0.2425
56	F(t-1)=A_{143}^F(t-2)=A_{126}	w_1=0.7678and w_2=0.2758	145	F(t-1)=A_{95}^F(t-2)=A_{98}	w_1=0.7488and w_2=0.249
57	F(t-1)=A_{149}^F(t-2)=A_{143}	w_1=0.5902and w_2=0.3586	146	F(t-1)=A_{95}^F(t-2)=A_{95}	w_1=0.9288and w_2=0.0136
58	F(t-1)=A_{132}^F(t-2)=A_{149}	w_1=0.4768and w_2=0.5332	147	F(t-1)=A_{81}^F(t-2)=A_{95}	w_1=0.4357and w_2=0.4938
59	F(t-1)=A_{145}^F(t-2)=A_{132}	w_1=0.4827and w_2=0.4963	148	F(t-1)=A_{73}^F(t-2)=A_{81}	w_1=0.735and w_2=0.2266
60	F(t-1)=A_{132}^F(t-2)=A_{145}	w_1=0.9724and w_2=0.0927	149	F(t-1)=A_{67}^F(t-2)=A_{73}	w_1=0.516and w_2=0.4878
61	F(t-1)=A_{151}^F(t-2)=A_{132}	w_1=0.6314and w_2=0.1191	150	F(t-1)=A_{70}^F(t-2)=A_{67}	w_1=0.7567and w_2=0.2763
62	F(t-1)=A_{75}^F(t-2)=A_{151}	w_1=0.7653and w_2=0.081	151	F(t-1)=A_{77}^F(t-2)=A_{70}	w_1=0.7894and w_2=0.2407
63	F(t-1)=A_{47}^F(t-2)=A_{75}	w_1=0.7468and w_2=0.196	152	F(t-1)=A_{81}^F(t-2)=A_{77}	w_1=0.4414and w_2=0.5726
64	F(t-1)=A_{41}^F(t-2)=A_{47}	w_1=0.6782and w_2=0.2161	153	F(t-1)=A_{83}^F(t-2)=A_{81}	w_1=0.6822and w_2=0.3866
65	F(t-1)=A_{23}^F(t-2)=A_{41}	w_1=0.7474and w_2=0.4456	154	F(t-1)=A_{97}^F(t-2)=A_{83}	w_1=0.4818and w_2=0.5082
66	F(t-1)=A_{65}^F(t-2)=A_{23}	w_1=0.7203and w_2=0.2811	155	F(t-1)=A_{87}^F(t-2)=A_{97}	w_1=0.4767and w_2=0.4963
67	F(t-1)=A_{52}^F(t-2)=A_{65}	w_1=0.762and w_2=0.1469	156	F(t-1)=A_{86}^F(t-2)=A_{87}	w_1=0.7048and w_2=0.3277
68	F(t-1)=A_{36}^F(t-2)=A_{52}	w_1=0.7671and w_2=0.3626	157	F(t-1)=A_{93}^F(t-2)=A_{86}	w_1=0.4139and w_2=0.4959
69	F(t-1)=A_{63}^F(t-2)=A_{36}	w_1=0.5219and w_2=0.4792	158	F(t-1)=A_{69}^F(t-2)=A_{93}	w_1=0.6327and w_2=0.388
70	F(t-1)=A_{49}^F(t-2)=A_{63}	w_1=0.6871and w_2=0.3492	159	F(t-1)=A_{84}^F(t-2)=A_{69}	w_1=0.7543and w_2=0.2781
71	F(t-1)=A_{62}^F(t-2)=A_{49}	w_1=0.5238and w_2=0.3851	160	F(t-1)=A_{86}^F(t-2)=A_{84}	w_1=0.7583and w_2=0.1238
72	F(t-1)=A_{40}^F(t-2)=A_{62}	w_1=0.6617and w_2=0.2687	161	F(t-1)=A_{60}^F(t-2)=A_{86}	w_1=0.7604and w_2=0.2993
73	F(t-1)=A_{33}^F(t-2)=A_{40}	w_1=0.5447and w_2=0.4658	162	F(t-1)=A_{78}^F(t-2)=A_{60}	w_1=0.6575and w_2=0.3343
74	F(t-1)=A_{38}^F(t-2)=A_{33}	w_1=0.6691and w_2=0.3972	163	F(t-1)=A_{72}^F(t-2)=A_{78}	w_1=0.7224and w_2=0.3211
75	F(t-1)=A_{47}^F(t-2)=A_{38}	w_1=0.5203and w_2=0.4853	164	F(t-1)=A_{82}^F(t-2)=A_{72}	w_1=0.6437and w_2=0.4261
76	F(t-1)=A_{42}^F(t-2)=A_{47}	w_1=0.7543and w_2=0.1968	165	F(t-1)=A_{95}^F(t-2)=A_{82}	w_1=0.7499and w_2=0.2202
77	F(t-1)=A_{33}^F(t-2)=A_{42}	w_1=0.7497and w_2=0.215	166	F(t-1)=A_{84}^F(t-2)=A_{95}	w_1=0.7421and w_2=0.3013
78	F(t-1)=A_{30}^F(t-2)=A_{33}	w_1=0.5357and w_2=0.4958	167	F(t-1)=A_{98}^F(t-2)=A_{84}	w_1=0.4878and w_2=0.502
79	F(t-1)=A_{37}^F(t-2)=A_{30}	w_1=0.5468and w_2=0.5149	168	F(t-1)=A_{88}^F(t-2)=A_{98}	w_1=0.4534and w_2=0.4733

80	$F(t-1)=A_{46}$^$F(t-2)=A_{37}$	w_1=0.5525and w_2=0.5008	169	$F(t-1)=A_{78}$^$F(t-2)=A_{88}$	w_1=0.7079and w_2=0.3161
81	$F(t-1)=A_{51}$^$F(t-2)=A_{46}$	w_1=0.6081and w_2=0.5258	170	$F(t-1)=A_{88}$^$F(t-2)=A_{78}$	w_1=0.5717and w_2=0.3958
82	$F(t-1)=A_{73}$^$F(t-2)=A_{51}$	w_1=0.7559and w_2=0.569	171	$F(t-1)=A_{76}$^$F(t-2)=A_{88}$	w_1=0.7202and w_2=0.3145
83	$F(t-1)=A_{132}$^$F(t-2)=A_{73}$	w_1=0.6315and w_2=0.5015	172	$F(t-1)=A_{88}$^$F(t-2)=A_{76}$	w_1=0.75and w_2=0.2396
84	$F(t-1)=A_{138}$^$F(t-2)=A_{132}$	w_1=0.751and w_2=0.1005	173	$F(t-1)=A_{84}$^$F(t-2)=A_{88}$	w_1=0.8958and w_2=0.113
85	$F(t-1)=A_{96}$^$F(t-2)=A_{138}$	w_1=0.4662and w_2=0.3534	174	$F(t-1)=A_{85}$^$F(t-2)=A_{84}$	w_1=0.749and w_2=0.2235
86	$F(t-1)=A_{68}$^$F(t-2)=A_{96}$	w_1=0.7509and w_2=.1494	175	$F(t-1)=A_{78}$^$F(t-2)=A_{85}$	w_1=0.6512and w_2=0.2885
87	$F(t-1)=A_{50}$^$F(t-2)=A_{68}$	w_1=0.5769and w_2=0.3722	176	$F(t-1)=A_{68}$^$F(t-2)=A_{78}$	w_1=0.5652and w_2=0.4118
88	$F(t-1)=A_{46}$^$F(t-2)=A_{50}$	w_1=0.5427and w_2=0.373	177	$F(t-1)=A_{68}$^$F(t-2)=A_{68}$	w_1=0.5126and w_2=0.5058
89	$F(t-1)=A_{33}$^$F(t-2)=A_{46}$	w_1=0.7436and w_2=0.3146			

Table 11 shows that the number of fuzzy rules for all table results includes only two conditions. To illustrate, we have the fuzzy rule (1) in the table where weights are (w_1=0.7487, w_2=0.143) if the value of the linguistic variable in time F(t-1) is equal to A_{19}. The value of the linguistic variable in time F(t-2) equals A_{39}. Through the values of weights obtained using the PSO function, the formula of the defuzzification coefficient X_i is applied and used in the defuzzing process to obtain the predictive values of the time series as shown in Table 12.

Table 12. Predictive values to model the proposed procedure after defuzzification

Linguistic Variable	Month	Year	Linguistic Variable	Month	Year	Linguistic Variable	Month	Year	Linguistic Variable	Month	Year
***	Jan	2004	663.4145	Jan.	2008	650.8188	Jan.	2012	610.9404	Jan.	2016
***	Feb		695.9739	Feb.		622.6244	Feb.		625.2845	Feb.	
***	Mar		623.7066	March		555.0911	March		619.0692	March	
383.7008	April		539.9798	April		562.1425	April		617.256	April	
407.6307	May		522.3842	May		509.824	May		581.7746	May	
398.8487	June		632.4284	June		486.8357	June		557.2747	June	
373.1327	July		677.6205	July		528.3815	July		542.2177	July	
382.2639	Aug		707.9736	August		606.013	August		553.1081	August	
377.4583	Sep		699.4146	Sep.		508.8152	Sep		568.756	Sep.	
403.2044	Oct		741.5103	October		634.8233	Oct		584.1786	October	
488.7415	Nov		759.8507	Nov.		582.0539	Nov		583.8014	Nov.	
490.0456	Dec		711.4485	Dec.		580.4466	Dec		625.0009	Dec.	
498.0698	Jan	2005	742.5951	Jan.	2009	629.2602	Jan.	2013	597.4777	Jan.	2017
557.1461	Feb		712.5502	Feb.		569.6407	Feb.		593.0387	Feb.	
560.7192	Mar		760.3838	March		597.6857	March		613.1592	March	
543.3961	April		564.3705	April		474.0549	April		547.0503	April	
492.2644	May		493.1036	May		521.8541	May		585.485	May	
519.9787	June		478.9255	June		571.4773	June		595.9629	June	
438.3963	July		430.1039	July		522.1968	July		523.4777	July	
451.5707	Aug.		535.4502	August		521.1665	August		577.347	August	
498.3054	Sep.		509.6862	Sep.		581.1047	Sep.		552.3199	Sep.	
481.3316	Oct.		464.8474	October		546.3205	Oct.		586.2611	Oct.	
567.5838	Nov.		539.7014	Nov.		630.7878	Nov.		612.2019	Nov.	
564.8579	Dec.		501.9928	Dec.		633.9463	Dec.		589.6598	Dec.	
541.8329	Jan.	2006	528.8161	Jan.	2010	620.5329	Jan.	2014	622.4905	Jan.	2018
643.8508	Feb.		470.5099	Feb		664.7681	Feb.		600.2465	Feb.	
565.1427	March		456.8938	Mar		580.1242	March		567.2625	March	
485.0507	April		470.0977	Apr		538.6999	April		594.3109	April	
527.7205	May		496.2688	May		524.1486	May		567.9804	May	
442.1753	June		484.1171	June		500.8726	June		596.4285	June	
419.0493	July		458.414	July		540.134	July		585.3918	July	
464.0198	Aug.		444.8407	Aug		533.1843	Aug.		593.9333	August	
490.8931	Sep.		466.7069	Sep		522.567	Sep.		573.8334	Sep.	
607.3548	Oct.		487.0426	Oct		572.692	Oct.		543.7246	Oct.	
596.0608	Nov.		505.3747	Nov		592.2351	Nov.		545.4309	Nov.	
465.9012	Dec.		563.6063	Dec		601.9455	Dec.		557.1203	Dec.	
514.8782	Jan.	2007	708.6592	Jan	2011	595.6258	Jan.	2015			
561.1036	Feb.		730.0224	Feb		593.3779	Feb.				
530.5382	March		618.4697	Mar		607.066	March				
511.7183	April		546.3573	Apr		558.638	April				
477.4272	May		504.3902	May		541.2054	May				
474.0119	June		493.4597	June		581.3748	June				
447.5377	July		453.703	July		515.3114	July				
504.0109	Aug.		493.9778	Aug		532.2223	Aug.				
543.2326	Sep.		471.8827	Sep		543.46	Sep.				
540.2437	Oct.		568.3954	Oct		594.9789	Oct.				
560.2515	Nov.		533.5177	Nov		600.6681	Nov.				
722.4721	Dec.		575.55	Dec		592.3224	Dec.				

Figure 8 shows the predictive values of the proposed procedure that can be symbolized ARIMA-Fuzzy$_{PSO}$ model.

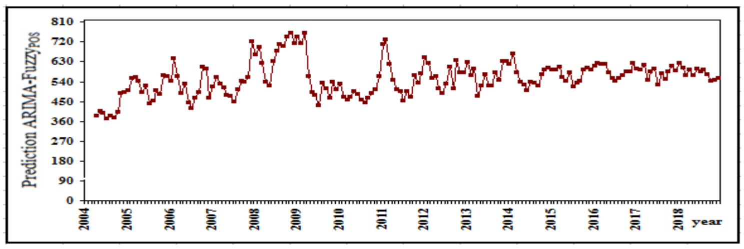

Figure 8. Predictive values of the proposed ARIMA-Fuzzy$_{PSO}$ model

3.4. Comparison of results

The criteria of RMSE, MAE, and MAPE have calculated to compare the three models and find out the best model to predict the TDS in drinking water after calculating the predictive values of each method and comparing them with the actual values. Table 13 shows the results of the comparison.

Table 13. Comparison of prediction for the models

No.	Criteria / Model	RMSE	MAE	MAPE
1	ARIMA(3,1,3)	51.131	39.658	7.910
2	Fuzzy$_{POS}$	48.357	37.250	6.862
3	ARIMA-Fuzzy$_{POS}$	46.016	35.402	6.128

It is clear from the table that the proposed ARIMA-Fuzzy$_{POS}$ model has the lowest value of the criteria in Table 13. This indicates the efficiency of the proposal compared to the two models ARIMA(3,1,3) and Fuzzy$_{POS}$. Therefore, it is considered the best model to predict the data of the studied phenomenon and it has ability to more predict with better performance.

4. Conclusions

1- When studying the B-J method for testing the dissolved solids for drinking water in Baghdad city, it was found that the ARIMA(3,1,3) model is appropriate for the time series data which has the lowest value in the statistical criteria of RMSE, MAE, MAPE with its significant parameters.

2- There are no frequencies in FSGs elements in the proposal when calculating the high order fuzzy time series of the ARIMA series. In addition, all fuzzy rules included only two conditions and number of weights per rule is two. This did not appear when studying actual time series in high order fuzzy time series.

3- The use of the trapezoidal membership function in the fuzzification of time series data with the application of the PSO algorithm influenced the results of the predictive values. The fuzzy method has a mechanical ability to find solutions to different field's, which in turn affected the results of the proposal model by owning the lowest values in the comparison criteria.

4- Use the adaptation procedure between the high order fuzzy time series method and the B-J method as an alternative to classical statistical methods. It has given better predictive results if the B-J method was used only on the data. Thus, the best criteria of the proposed model appeared was ARIMA-Fuzzy$_{POS}$ which improved the quality of the model with increased predictability, followed by the Fuzzy$_{POS}$ model, while the latter is the ARIMA model.

5. Recommendations

1- Using the high order fuzzy time series to the dissolved solids data for drinking water in Baghdad city, due to its high accuracy in data processing and obtaining the best data predicate.

2 - Application of the ARIMA-Fuzzy$_{POS}$ of proposed model for data dissolved solids for drinking water in the city of Baghdad, where it proved its accuracy and superiority.

3 - Use of high order fuzzy time series method for multivariate to be compared with the vector autoregressive moving average (VARMA) method by taking several chemical examinations of the drinking water pollution.

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