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Constructing a new mixed probability distribution with fuzzy reliability estimation

Anam Abdul Rahman Noaman[1], Suhad Ahmed Ahmed[2], Mohammed Kadhim Hawash[3]
[1] College of Administration & Economic, University of Diyala
[2] College of Education Ibn Al Haitham, University of Baghdad
[3] Dijlah University College

ABSTRACT:

This paper deals with constructing mixed probability distribution from mixing exponential (β) and Rayleigh along with β. Accordingly, the mixing proportions are $\left(\frac{\alpha}{\alpha+1}\right)$ and $\left(\frac{1}{\alpha+1}\right)$.At that point, the mixed PDF and CDF were investigated in this study. The mixed reliability has determined based on estimating its two parameters (α,β) by three different methods, which are maximum likelihood, moments and percentiles method. The fuzzy reliability estimators are compared and the results of comparison are explained based on simulation procedure with detailed tables.

Keywords: Maximum likelihood estimator, Moments method, Percentiles method, One scale parameter (β)Exp(β), One scale parameter(β)Rayleigh

Corresponding Author:

Anam Abdul Rahman Noaman
College of Administration & Economic
University of Diyala, Iraq
E-mail: inamabd66@gmail.com

1. Introduction

Rayleigh distribution was introduced by Lord Rayleigh in 1880 in connection with problems in the field of acoustics. Rayleigh distribution has been applied in many different areas of science and technology. The two-parameter Rayleigh distribution is a special case of the three-parameter for Weibull distribution [1, 2]. The researchers of [3], estimated the parameters of Rayleigh distribution numerical iterative methods or routines. In this work, an exact method on the constant minimization of the goal function has proposed. Reliability analysis is a tool kit of statistical procedures for analyzing time-to-failure data. Usually, reliability analysis is carried out using the classical or Bayesian statistical analysis of parametric reliability models. Reliability analysis datasets are represented by a single univariate or multivariate statistical distributions such as Exponential, Rayleigh, inverse Weibull, Pareto and Burr distributions [4, 5].

The mixture distributions have vital role in practical applications for researches that deal with economics, medicine, agriculture, life testing, and reliability for classical reliability theory. There are several methods and models, in which the parameters are assumed to be precise. Nevertheless, in real world application due to vague, randomness with the life time's distribution, and when the parameters of life time distribution are fuzzy, there is difficulty for handling reliability and hazard functions. Many researches have worked on fuzzy reliability and introduced development for this field. In [6], the researchers discussed about fuzzy exponential distribution and how to compute reliability in case of stress- strength model and ranked set sampling. The Rayleigh distribution has a number of applications in settings where magnitudes of normal variables are important [7]. An application for the Rayleigh distribution is the analysis of wind velocity [8].

In this study, we continue the work in the field of fuzzy Reliability and work on comparing three different estimators of fuzzy Reliability function, and explain the results of comparison of simulation by comprehensive tables.

2. Material and methods

Let the below function be exponential distribution with scale parameter(β):

$$f_1(x) = \beta \;\; e^{-\beta x} \qquad x > 0, \beta > 0 \tag{1}$$

And the corresponding cumulative distribution function (CDF) is

$$F_1(x) = 1 - \; e^{-\beta x} \qquad x > 0, \beta > 0 \tag{2}$$

The second PDF function to be mixed is Rayleigh with scale parameter (β) as follows:

$$f_2(x) = \beta x \;\; e^{-\frac{\beta}{2}x^2} \qquad x > 0, \beta > 0 \tag{3}$$

So the mixed PDF function for Exponential-Rayleigh is:

$$f(x,\alpha,\beta) = \frac{\alpha}{(\alpha+1)}\,\beta \;\; e^{-\beta x} + \frac{1}{(\alpha+1)}\,\beta \;\; e^{-\frac{\beta}{2}x^2} \qquad \alpha > 0, \beta > 0, x > 0 \tag{4}$$

The mixed distribution from Exponential(β) and Rayleigh $(\frac{\beta}{2})$ with mixing proportions of $(\frac{\alpha}{\alpha+1})$ and $(\frac{1}{\alpha+1})$ are obtained from:

$$f(x,\alpha,\beta) = \frac{\alpha}{(\alpha+1)}\,\beta \;\; \mathrm{e}^{-\beta x} + \frac{1}{(\alpha+1)}\,\beta \;\; \mathrm{e}^{-\,-\frac{\beta}{2}x^2} \qquad \alpha > -1, \beta > 0, x > 0 \tag{5}$$

And the corresponding CDF formula is :

$$F(x,\alpha,\beta) = \frac{\alpha}{(\alpha+1)}\left(1 - \; e^{-\beta x}\right) + \frac{1}{(\alpha+1)}\left(1 - e^{-\,-\frac{\beta}{2}x^2}\right) \quad \alpha > -1, \beta > 0, x > 0 \tag{6}$$

The Reliability function will be:

$$F(x,\alpha,\beta) = \frac{\alpha}{(\alpha+1)}\,\beta \;\; \mathrm{e}^{-\beta x} + \frac{1}{(\alpha+1)}\,\beta \;\; \mathrm{e}^{-\frac{\beta}{2}x^2} \qquad \alpha > -1, \beta > 0, x > 0 \tag{7}$$

Accordingly, the corresponding mixed Reliability is given in equation (8) as below:

$$R_{Mixed}(xi,\alpha,\beta) = \frac{\alpha}{(\alpha+1)} \;\; \mathrm{e}^{-\beta xi} + \frac{1}{(\alpha+1)} \;\; \mathrm{e}^{-\,-\frac{\beta}{2}xi^2} \tag{8}$$

The fuzzy Reliability function is defined in equation (9):

$$\hat{R}(xi,ki,\alpha,\beta) = \frac{\alpha}{(\alpha+1)} \;\; \mathrm{e}^{-\beta kixi} + \frac{1}{(\alpha+1)} \;\; \mathrm{e}^{-\frac{\beta}{2}kixi^2} \tag{9}$$

Mean and Variance of mixed distribution from exponential(β), and Rayleigh with (β) are:

$$E(T) = \left(\frac{\alpha}{\alpha+1}\right)\left(\frac{1}{\beta}\right) + \left(\frac{1}{\alpha+1}\right)\sqrt{\frac{\pi}{2\beta}}$$

$$V(T) = \left(\frac{\alpha}{\alpha+1}\right)^2\left(\frac{1}{\beta^2}\right) + \left(\frac{1}{\alpha+1}\right)^2\left(\frac{2}{\beta}\right)(1 - \frac{\pi}{4})$$

3. Estimation methods

The two parameters (α,β) are estimated by maximum likelihood (MLE), moment method (MoM), and percentiles (PEC) method.

3.1. Estimation by maximum likelihood

Assume $t_1, t_2, t_3, \dots\dots\dots, t_n$ are related to PDF in equation (10-11), then

$$L = \prod_{i=1}^{n} g_T(xi,\alpha,\beta)$$

$$= \alpha^n(\alpha+1)^{-n}\;\beta^n \;\; \mathrm{e}^{-\beta\sum xi} + \;(\alpha+1)^{-n}\;\beta^n \prod_{i=1}^{n} xi \;\; \mathrm{e}^{-\,-\frac{\beta}{2}\sum xi^2} \tag{10}$$

$\log L =$ $\quad \mathrm{n}\log\alpha - 2n\log(\alpha+1) + 2n\log\beta - \beta\sum_{i=1}^{n} xi - \frac{\beta}{2}\sum_{i=1}^{n} xi^2$

Then,

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{\alpha} - \frac{2n}{\alpha+1}$$

$$\frac{\partial \log L}{\partial \alpha} = 0 \quad \rightarrow \quad \frac{n}{\alpha} = \frac{2n}{\alpha+1}$$
$$n\,(\alpha+1) = \;2n\alpha \qquad , n = n\alpha \;\; , \alpha = 1$$
And based on above, we get the following:
$$\frac{\partial \log L}{\partial \beta} = \frac{2n}{\beta} - \sum_{i=1}^{n} xi - \frac{1}{2}\sum_{i=1}^{n} xi^2 = 0$$
$$\frac{2n}{\hat{\beta}} = \sum_{i=1}^{n} xi - \frac{1}{2}\sum_{i=1}^{n} xi^2$$
$$\hat{\beta}_{MLE} = \frac{2n}{\sum_{i=1}^{n} xi + \frac{1}{2}\sum_{i=1}^{n} xi^2} \tag{11}$$

3.2. Estimation by moment method

The Moments estimators of two parameters (α, β) of mixed (Exponential- Rayleigh) are obtained from solving the following equations:

$\acute{\mu}_r = E(t^r)$ with

$E(t^r) = \frac{\sum_{i=1}^{n} ti^r}{n} \quad , for\; r = 1{,}2$

Since we have two parameters (α, β) and solving the resulted equation, we get:
$$\frac{\sum_{i=1}^{n} ti^2}{n} = \frac{1}{(\hat{\alpha}+1)^2}\left[\frac{\alpha^2}{\beta^2} + \frac{2}{\beta}(1-\frac{\pi}{4})\right] \tag{12}$$
$$\frac{\sum_{i=1}^{n} ti}{n} = \frac{1}{(\hat{\alpha}+1)}\left[\frac{\hat{\alpha}}{\hat{\beta}} + \sqrt{\frac{\pi}{2\,\hat{\beta}}}\right]$$
These two equations are solved numerically to find($\hat{\alpha}_{MOM\,,}\, \hat{\beta}_{MOM}$) in the case of $\hat{\alpha} > -1$
$$\bar{t}\,(\hat{\alpha}+1) = \frac{\hat{\alpha}}{\hat{\beta}} + \sqrt{\frac{\pi}{2\,\hat{\beta}}}$$
$$(\hat{\alpha}+1)^2\,\frac{\sum_{i=1}^{n} ti^2}{n} = \frac{\alpha^2}{\beta^2} + \frac{2}{\beta}\left(1-\frac{\pi}{4}\right)$$
$$E(T) = \left(\frac{\alpha}{\alpha+1}\right)\left(\frac{1}{\beta}\right) + \left(\frac{1}{\alpha+1}\right)\sqrt{\frac{\pi}{2\beta}}$$
And,
$$V(T) = \left(\frac{\alpha}{\alpha+1}\right)^2\left(\frac{1}{\beta^2}\right) + \left(\frac{1}{\alpha+1}\right)^2\left(\frac{2}{\beta}\right)(1-\frac{\pi}{4})$$
Then, by solving $\quad E(T^2) = V(T) + (ET)^2$
$$\frac{\sum_{i=1}^{n} ti}{n} = \frac{\hat{\alpha}}{(\hat{\alpha}+1)}\,\frac{1}{\hat{\beta}} + \frac{1}{(\hat{\alpha}+1)}\sqrt{\frac{\pi}{2\hat{\beta}}}$$

3.3. Estimation by percentiles method

The estimators($\hat{\alpha}_{PEC\,,}\, \hat{\beta}_{PEC}$) by this method are obtained from minimizing total sum square of the difference between $F_{(xi,\alpha,\beta)}$ and $\hat{F}_{(xi,\alpha,\beta)}$ that may be equal to ($\frac{i}{n+1}$, or $\frac{\frac{3}{8}i}{(n+2)}$, ….).

Here, we choose $\hat{F}_{(xi,\alpha,\beta)} = \frac{\frac{3}{8}i}{(n+2)}$

Total sum square of $F_{(xi,\alpha,\beta)}$ and $\hat{F}_{(xi,\alpha,\beta)}$ can be minimize by (T) as in equation (13). Then, by deriving $\frac{\partial T}{\partial \alpha}$ and $\frac{\partial T}{\partial \beta}$ and solving ($\frac{\partial T}{\partial \alpha} = , \frac{\partial T}{\partial \beta} = 0$) by iterative procedure, we can obtain ($\hat{\alpha}_{PEC\,,}\, \hat{\beta}_{PEC}$) as follows:
$$T = \sum_{i=}^{n}\left[\frac{\alpha\,\beta\;\; \mathrm{e}^{-\beta xi} + \beta xi\;\; \mathrm{e}^{-\frac{\beta}{2}xi^2}}{(\,\alpha+1)} - \frac{\frac{3}{8}\,i}{(n+2)}\right]^2 \tag{13}$$

$$\frac{\partial T}{\partial \alpha}=2\sum_{i=1}^{n}\left[\frac{\alpha\,\beta\ \ \mathrm{e}^{-\beta xi}+\beta xi\ \mathrm{e}^{-\frac{\beta}{2}xi^{2}}}{(\alpha+1)}-\frac{\frac{3}{8}\,i}{(n+2)}\right]\left[\sum_{i=1}^{n}\frac{\beta\ \ \mathrm{e}^{-\beta xi}+\beta xi\ \mathrm{e}^{-\frac{\beta}{2}xi^{2}}}{(\alpha+1)^{2}}\right] \tag{14}$$

$$\frac{\partial T}{\partial \beta}=2\sum_{i=1}^{n}\left[\frac{\alpha\,\beta\ \ \mathrm{e}^{-\beta xi}+\beta xi\ \mathrm{e}^{-\frac{\beta}{2}xi^{2}}}{(\alpha+1)}-\frac{\frac{3}{8}\,i}{(n+2)}\right]$$
$$\times 2\sum_{i=1}^{n}\frac{\alpha\,\beta\ (-xi)\mathrm{e}^{-\beta xi}+\alpha\ \mathrm{e}^{-\frac{\beta}{2}xi}}{(\alpha+1)}+\frac{\beta xi\ \mathrm{e}^{-\frac{\beta}{2}xi^{2}}}{(\alpha+1)}\left(-\frac{1}{2}xi^{2}\right)+\mathrm{e}^{-\frac{\beta}{2}xi^{2}}(xi) \tag{15}$$

Then from below, we can find $\hat{\alpha}_{Pec}$,$\hat{\beta}_{Pec}$.

$$\frac{\partial T}{\partial \alpha}=0$$
$$\frac{\partial T}{\partial \beta}=0$$

4. Results

The fuzzy Reliability function of mixed Exponential-Rayleigh is compared by three different methods which are MLE, MOM and PEC methods

Where,

$$\hat{R}_{Mixed}(ki,\hat{\alpha},\hat{\beta})=\frac{\hat{\alpha}}{(\hat{\alpha}+1)}\ \mathrm{e}^{-\hat{\beta}kixi}+\frac{1}{(\hat{\alpha}+1)}\ \mathrm{e}^{-\frac{\hat{\beta}}{2}kixi^{2}}\quad \alpha>-1\ ,\beta>0\ ,ki>0$$

$$F(xi,\alpha,\beta)=\frac{\alpha}{(\alpha+1)}\left(1-\ \mathrm{e}^{-\beta xi}\right)+\frac{1}{(\alpha+1)}\left(1-\ \mathrm{e}^{-\,-\frac{\beta}{2}xi^{2}}\right)$$
$$=\frac{1}{(\alpha+1)}\,(\alpha+1)+\frac{1}{(\alpha+1)}\,(\alpha\,\mathrm{e}^{-\beta xi}-\alpha\,\mathrm{e}^{-\,-\frac{\beta}{2}xi^{2}})$$
$$=1-\frac{1}{(\alpha+1)}\left(\alpha\,\mathrm{e}^{-\beta xi}-\alpha\,\mathrm{e}^{-\,-\frac{\beta}{2}xi^{2}}\right)$$

$$G(ti)=Ui$$

$$ui=1-\frac{1}{(\alpha+1)}\left(\alpha\,\mathrm{e}^{-\beta xi}-\ \mathrm{e}^{-\,-\frac{\beta}{2}xi^{2}}\right)$$

Let $Zi=1-ui$, then:

$$Zi\ =1-\frac{1}{(\alpha+1)}\left(\alpha\,\mathrm{e}^{-\beta xi}-\ \mathrm{e}^{-\,-\frac{\beta}{2}xi^{2}}\right)$$
$$=\frac{1}{(\alpha+1)}\left(\mathrm{e}^{-\beta xi}-\alpha\,\mathrm{e}^{-\,-\frac{\beta}{2}xi^{2}}\right)$$

$$Zi=\frac{1}{(\alpha+1)}(\alpha\,Z_1-\ Z_2)$$

$$Z_1=\mathrm{e}^{-\beta xi1}$$

$$Z_2=\mathrm{e}^{-\,-\frac{\beta}{2}{xi_2}^{2}}$$

At that moment, we get

$$lnZ_1=-\beta xi\qquad\rightarrow\quad xi_1=-\frac{1}{\beta}\,lnZ_1$$

and $lnZ_2=-\ -\frac{\beta}{2}{xi_2}^{2}\qquad\rightarrow\quad {xi_2}^{2}=-\frac{1}{\beta}\,lnZ_2$

The methods of estimation are firstly by Maximum Likelihood, secondly by Moments method and thirdly by Percentiles method.

Here, we have two parameters (α,β) as well as (Ki) fuzzy factor.

The sample size is taken with n = (25, 50, 75), and the results are compared by mean square error.

There are different values for $(\alpha=0.5\ ,1\ )$, $(\beta=0.5\ ,1.2\ )$and $(\ \widehat{ki}=0.3,0.6\ )$. Each experiment is repeated with $R=300$

All the results of comparisons for fuzzy reliability functions are explained in different tables as stated by Tables 1-8, according to different sets of initial values, different combination parameters (ki, α, β) ,and the best fuzzy reliability estimation is highest one, and all the results in tables.

Table 1. Estimator of fuzzy Reliability when $\beta = 0.5$, $\alpha = 0.5$, $\tilde{k} = 0.3$

n	t_i	Rreal Ri	$\hat{R}_{mom}$	$\hat{R}_{Mle}$	$\hat{R}_{pec}$	Best
25	1.55	0.3102	0.3068	0.3106	0.3187	Pec
	1.60	0.3753	0.3688	0.3742	0.3632	Mle
	1.65	0.4165	0.4091	0.4147	0.4234	Pec
	1.70	0.4452	0.4371	0.4437	0.4415	Mle
	1.75	0.4653	0.4611	0.4632	0.4523	Mle
	1.80	0.4817	0.4652	0.4962	0.4616	Mle
	1.85	0.4942	0.5068	0.5031	0.4718	Mom
	1.90	0.5066	0.5165	0.5232	0.4872	Mle
	1.95	0.5137	0.5242	0.5307	0.4605	Mle
	2.00	0.5198	0.5308	0.5334	0.4152	Mle
50	1.55	0.3102	0.3116	0.3704	0.3185	Mle
	1.60	0.3753	0.3624	0.4169	0.3832	Mle
	1.65	0.4165	0.3762	0.4452	0.4246	Mle
	1.70	0.4452	0.4168	0.4656	0.4522	Mle
	1.75	0.4653	0.4452	0.4815	0.4629	Mle
	1.80	0.4817	0.4656	0.4843	0.4734	Pec
	1.85	0.4942	0.4823	0.5042	0.4885	Mle
	1.90	0.5066	0.4917	0.5123	0.5221	Pec
	1.95	0.5137	0.5023	0.5233	0.5413	Pec
	2.00	0.5198	0.5233	0.5292	0.5296	Pec
75	1.55	0.3102	0.3224	0.3114	0.3685	Pec
	1.60	0.3753	0.3742	0.3764	0.3769	Pec
	1.65	0.4165	0.4137	0.4172	0.4189	Pec
	1.70	0.4452	0.4561	0.4454	0.4323	Mom
	1.75	0.4653	0.4713	0.4662	0.4632	Mom
	1.80	0.4817	0.4866	0.4720	0.4072	Mom
	1.85	0.4942	0.4988	0.4822	0.4047	Mom
	1.90	0.5066	0.5172	0.5047	0.5132	Mom
	1.95	0.5137	0.5238	0.5132	0.5266	Mom
	2.00	0.5198	0.5246	0.5200	0.5427	Pec

Table 2. Estimator of fuzzy Reliability when $\beta = 0.5$, $\alpha = 0.5$, $\tilde{k} = 0.6$

n	t_i	Rreal Ri	$\hat{R}_{mom}$	$\hat{R}_{Mle}$	$\hat{R}_{pec}$	Best
25	1.55	0.3202	0.3125	0.2956	0.2966	Mom
	1.60	0.3764	0.3507	0.3376	0.3397	Mom
	1.65	0.4166	0.3662	0.3768	0.3846	Mom
	1.70	0.4352	0.3872	0.4003	0.3864	Mle
	1.75	0.4656	0.4030	0.4025	0.4125	Pec

n	t_i	Rreal Ri	$\widehat{R}_{mom}$	$\widehat{R}_{Mle}$	$\widehat{R}_{pec}$	Best
	1.80	0.4817	0.4155	0.4166	0.4224	Pec
	1.85	0.4942	0.4265	0.4255	0.4334	Pec
	1.90	0.5043	0.4338	0.4338	0.4854	Pec
	1.95	0.5127	0.4406	0.4409	0.3645	Mle
	2.00	0.5197	0.4437	0.4465	0.3532	Mle
50	1.55	0.3202	0.2868	0.2982	0.2826	Mle
	1.60	0.3764	0.3284	0.3368	0.3225	Mle
	1.65	0.4166	0.3564	0.3634	0.3512	Mle
	1.70	0.4352	0.3768	0.3979	0.3619	Mle
	1.75	0.4656	0.4052	0.4294	0.3847	Mle
	1.80	0.4817	0.4235	0.4272	0.4002	Mle
	1.85	0.4942	0.3305	0.3338	0.4186	Pec
	1.90	0.5043	0.4305	0.4338	0.4654	Pec
	1.95	0.5127	0.4237	0.4372	0.4318	Mle
	2.00	0.5197	0.4246	0.4352	0.3768	Mle
75	1.55	0.3202	0.4342	0.4406	0.3668	Mle
	1.60	0.3764	0.4428	0.4546	0.3945	Mle
	1.65	0.4166	0.4472	0.4438	0.4106	Mom
	1.70	0.4352	0.4482	0.4439	0.4128	Mom
	1.75	0.4656	0.4492	0.4516	0.4238	Mle
	1.80	0.4817	0.3264	0.3206	0.3387	Pec
	1.85	0.4942	0.4152	0.4247	0.3632	Mle
	1.90	0.5043	0.4267	0.4537	0.4234	Mle
	1.95	0.5127	0.4382	0.4638	0.4415	Mle
	2.00	0.5197	0.4982	0.4962	0.4523	Mom

Table 3. Estimator of fuzzy Reliability when $\beta = 0.5$, $\alpha = 1, \tilde{k} = 0.3$

n	t_i	Rreal Ri	$\widehat{R}_{mom}$	$\widehat{R}_{Mle}$	$\widehat{R}_{pec}$	Best
25	1.55	0.5530	0.6042	0.5727	0.5726	MOM
	1.60	0.6032	0.6519	0.6216	0.6216	MOM
	1.65	0.6333	0.6824	0.6543	0.6514	MOM
	1.70	0.6571	0.7032	0.6772	0.6772	MOM
	1.75	0.6752	0.7192	0.6846	0.6844	MOM
	1.80	0.6888	0.7324	0.7082	0.7082	MOM
	1.85	0.7005	0.7424	0.7188	0.7189	MOM
	1.90	0.7092	0.7504	0.7268	0.7278	MOM
	1.95	0.7234	0.7634	0.7352	0.7352	MOM
	2.00	0.7415	0.7724	0.7415	0.7415	MOM
50	1.55	0.5530	0.5802	0.5366	0.5903	PEC
	1.60	0.6032	0.6336	0.6033	0.6487	PEC
	1.65	0.6333	0.6655	0.6443	0.6825	PEC
	1.70	0.6571	0.6644	0.6725	0.6748	PEC
	1.75	0.6752	0.6882	0.6932	0.6936	PEC
	1.80	0.6888	0.7049	0.7088	0.7072	MLE

n	t_i	Rreal Ri	$\widehat{R}_{mom}$	$\widehat{R}_{Mle}$	$\widehat{R}_{pec}$	Best
	1.85	0.7005	0.7181	0.7211	0.7182	MLE
	1.90	0.7092	0.7284	0.7311	0.7271	MLE
	1.95	0.7234	0.7374	0.7393	0.7345	MLE
	2.00	0.7415	0.7446	0.7483	0.7409	MLE
75	1.55	0.5530	0.5418	0.5625	0.5442	MLE
	1.60	0.6032	0.6029	0.6133	0.6026	MOM
	1.65	0.6333	0.6299	0.6469	0.6362	MLE
	1.70	0.6571	0.6344	0.6709	0.6512	MLE
	1.75	0.6752	0.6662	0.6889	0.6612	MLE
	1.80	0.6888	0.7009	0.7028	0.6798	MLE
	1.85	0.7005	0.7124	0.7142	0.6789	MLE
	1.90	0.7092	0.7219	0.7232	0.7057	MLE
	1.95	0.7234	0.7294	0.7371	0.7152	MLE
	2.00	0.7415	0.7492	0.7571	0.7236	MLE

Table 4. Estimator of fuzzy Reliability when $\beta = 0.5$, $\alpha = 1, \tilde{k} = 0.6$

n	t_i	Rreal Ri	$\widehat{R}_{mom}$	$\widehat{R}_{Mle}$	$\widehat{R}_{pec}$	Best
25	1.55	0.6000	0.5822	0.5727	5826	PEC
	1.60	0.6333	0.6336	0.6216	0.6362	PEC
	1.65	0.6572	0.6676	0.6541	0.6687	PEC
	1.70	0.6760	0.6927	0.6772	0.6844	PEC
	1.75	0.6884	0.7097	0.6946	0.7098	PEC
	1.80	0.7002	0.7236	0.7081	0.7128	MLE
	1.85	0.7092	0.7348	0.7278	0.7272	MOM
	1.90	0.7167	0.7438	0.7352	0.7374	MOM
	1.95	0.7232	0.7525	0.7415	0.7447	MOM
	2.00	0.7578	0.7574	0.7422	0.7520	MOM
50	1.55	0.6000	0.5728	0.5703	0.5726	MOM
	1.60	0.6333	0.6234	0.6199	0.6232	MOM
	1.65	0.6572	0.6569	0.6526	0.6563	MOM
	1.70	0.6760	0.6818	0.6758	0.6816	MLE
	1.75	0.6884	0.6992	0.6936	0.6498	PEC
	1.80	0.7002	0.7132	0.7072	0.7139	PEC
	1.85	0.7092	0.7242	0.7182	0.7252	PEC
	1.90	0.7167	0.7333	0.7273	0.7343	PEC
	1.95	0.7232	0.7409	0.7342	0.7420	PEC
	2.00	0.7578	0.7473	0.7408	0.7466	PEC
75	1.55	0.6000	0.5466	0.5043	0.7043	PEC
	1.60	0.6333	0.5699	0.5594	0.7485	PEC
	1.65	0.6572	0.6188	0.6083	0.7566	PEC
	1.70	0.6760	0.6233	0.6412	0.7141	PEC
	1.75	0.6884	0.6414	0.7644	0.7233	MLE
	1.80	0.7002	0.6523	0.7825	0.7308	MLE
	1.85	0.7092	0.6887	0.7957	0.7341	MLE

n	t_i	Rreal Ri	$\hat{R}_{mom}$	$\hat{R}_{Mle}$	$\hat{R}_{pec}$	Best
	1.90	0.7167	0.7028	0.7466	0.7355	MLE
	1.95	0.7232	0.7140	0.7458	0.7402	MLE
	2.00	0.7578	0.7307	0.7531	0.7342	MLE

Table 5. Estimator of fuzzy Reliability when $\beta = 1.2, \alpha = 0.5, \tilde{k} = 0.3$

n	t_i	Rreal Ri	$\hat{R}_{mom}$	$\hat{R}_{Mle}$	$\hat{R}_{pec}$	Best
25	1.55	0.3776	0.2968	0.2864	0.2955	MLE
	1.60	0.3182	0.3376	0.3329	0.3362	MLE
	1.65	0.3482	0.3552	0.3642	0.3542	PEC
	1.70	0.3667	0.3671	0.3862	0.3556	PEC
	1.75	0.3824	0.4032	0.4043	0.4006	PEC
	1.80	0.3906	0.4155	0.4268	0.4135	PEC
	1.85	0.4924	0.4256	0.4354	0.4226	MLE
	1.90	0.4482	0.4338	0.4429	0.4375	MLE
	1.95	0.4563	0.4468	0.4456	0.4434	MOM
	2.00	0.4528	0.4520	0.4532	0.4736	PEC
50	1.55	0.3776	0.2868	0.2854	0.2866	MLE
	1.60	0.3182	0.3284	0.3362	0.3234	MLE
	1.65	0.3482	0.3563	0.3642	0.3512	MLE
	1.70	0.3667	0.3747	0.3846	0.3718	MLE
	1.75	0.3824	0.3906	0.4002	0.3817	MLE
	1.80	0.3906	0.4052	0.4125	0.3876	MLE
	1.85	0.4924	0.4152	0.4224	0.4102	MLE
	1.90	0.4482	0.4235	0.4306	0.4281	MLE
	1.95	0.4563	0.4236	0.4305	0.4354	PEC
	2.00	0.4528	0.5302	0.4434	0.4468	MOM
75	1.55	0.3776	0.2824	0.2846	0.4148	PEC
	1.60	0.3182	0.3232	0.3252	0.4458	PEC
	1.65	0.3482	0.3513	0.3546	0.4341	PEC
	1.70	0.3667	0.4719	0.3748	0.4492	MOM
	1.75	0.3824	0.4877	0.3727	0.4488	MOM
	1.80	0.3906	0.4601	0.4151	0.4695	PEC
	1.85	0.4924	0.5399	0.4237	0.5062	MOM
	1.90	0.4482	0.4306	0.4257	0.5132	PEC
	1.95	0.4563	0.4309	0.4266	0.5224	PEC
	2.00	0.4528	0.4317	0.4426	0.5338	PEC

Table 6. Estimator of fuzzy Reliability when $\beta = 1.2, \alpha = 0.5, \tilde{k} = 0.6$

n	t_i	Rreal Ri	$\hat{R}_{mom}$	$\hat{R}_{Mle}$	$\hat{R}_{pec}$	Best
25	1.55	0.2778	0.3126	0.2864	0.2965	MOM
	1.60	0.3184	0.3508	0.3329	0.3352	MOM
	1.65	0.3462	0.3768	0.3642	0.3542	MOM
	1.70	0.3667	0.3962	0.3865	0.3844	MOM
	1.75	0.3824	0.4109	0.4031	0.4002	MOM
	1.80	0.3926	0.4226	0.4166	0.4125	MOM

n	t_i	Rreal Ri	$\widehat{R}_{mom}$	$\widehat{R}_{Mle}$	$\widehat{R}_{pec}$	Best
	1.85	0.4058	0.4319	0.4265	0.4224	MOM
	1.90	0.4133	0.4397	0.4354	0.4305	MOM
	1.95	0.4262	0.4497	0.4426	0.4375	MOM
	2.00	0.4295	0.4483	0.4488	0.4487	MLE
50	1.55	0.2778	0.2952	0.3761	0.2754	MLE
	1.60	0.3184	0.3268	0.3224	0.3255	MOM
	1.65	0.3462	0.3564	0.3538	0.3532	PEC
	1.70	0.3667	0.3769	0.3746	0.3731	PEC
	1.75	0.3824	0.3868	0.3816	0.3886	PEC
	1.80	0.3926	0.4015	0.3916	0.4066	PEC
	1.85	0.4058	0.4116	0.4151	0.4167	PEC
	1.90	0.4133	0.4195	0.4267	0.4188	PEC
	1.95	0.4262	0.4266	0.4308	0.4257	MOM
	2.00	0.4295	0.4327	0.4357	0.4289	MLE
75	1.55	0.2778	0.2912	0.2817	0.3336	PEC
	1.60	0.3184	0.3216	0.3172	0.3827	PEC
	1.65	0.3462	0.3492	0.3473	0.3945	PEC
	1.70	0.3667	0.3687	0.3665	0.3956	PEC
	1.75	0.3824	0.3751	0.3856	0.4055	PEC
	1.80	0.3926	0.3972	0.3987	0.4142	PEC
	1.85	0.4058	0.4071	0.4082	0.4212	PEC
	1.90	0.4133	0.4155	0.4178	0.4315	PEC
	1.95	0.4262	0.4223	0.4250	0.4462	PEC
	2.00	0.4295	0.4282	0.4311	0.4548	PEC

Table 7. Estimator of fuzzy Reliability when $\beta = 1.2\,, \alpha = 1, \tilde{k} = 0.6$

n	t_i	Rreal Ri	$\widehat{R}_{mom}$	$\widehat{R}_{Mle}$	$\widehat{R}_{pec}$	Best
25	1.55	0.3202	0.3367	0.3066	0.3396	PEC
	1.60	0.3753	0.3873	0.3658	0.3977	PEC
	1.65	0.4165	0.4346	0.4098	0.4352	PEC
	1.70	0.4452	0.4522	0.4372	0.4820	PEC
	1.75	0.4558	0.4703	0.4574	0.4952	PEC
	1.80	0.4717	0.4851	0.4728	0.4962	PEC
	1.85	0.4926	0.5066	0.4852	0.4506	MOM
	1.90	0.5042	0.5262	0.5030	0.5196	MOM
	1.95	0.5127	0.5223	0.5132	0.5308	PEC
	2.00	0.5188	0.5234	0.5334	0.5107	MLE
50	1.55	0.3202	0.3302	0.3217	0.4177	PEC
	1.60	0.3753	0.3917	0.3447	0.4168	PEC
	1.65	0.4165	0.4312	0.4248	0.4299	PEC
	1.70	0.4452	0.4582	0.4728	0.5040	PEC
	1.75	0.4558	0.4566	0.4525	0.5123	PEC
	1.80	0.4717	0.4834	0.4772	0.5192	PEC
	1.85	0.4926	0.5054	0.4884	0.5056	PEC

n	t_i	Rreal Ri	$\hat{R}_{mom}$	$\hat{R}_{Mle}$	$\hat{R}_{pec}$	Best
	1.90	0.5042	0.5152	0.5104	0.5135	PEC
	1.95	0.5127	0.5133	0.5102	0.5144	PEC
	2.00	0.5188	0.5255	0.5233	0.5226	MLE
75	1.55	0.3202	0.3215	0.5163	0.5066	MLE
	1.60	0.3753	0.3847	0.5806	0.5136	MLE
	1.65	0.4165	0.4248	0.4249	0.3085	MLE
	1.70	0.4452	0.4526	0.4463	0.3729	MOM
	1.75	0.4558	0.4728	0.4688	0.4139	MOM
	1.80	0.4717	0.4864	0.4980	0.4246	MLE
	1.85	0.4926	0.5104	0.5080	0.4786	MOM
	1.90	0.5042	0.5255	0.5163	0.4976	MOM
	1.95	0.5127	0.4563	0.4626	0.4535	MLE
	2.00	0.5188	0.4632	0.5206	0.4668	MLE

Table 8. Estimator of fuzzy Reliability when $\beta = 1.2\,, \alpha = 1, \tilde{k} = 0.3$

n	t_i	Rreal Ri	$\hat{R}_{mom}$	$\hat{R}_{Mle}$	$\hat{R}_{pec}$	Best
25	1.55	0.3699	0.3872	0.4166	0.3995	MLE
	1.60	0.4127	0.4566	0.4682	0.4722	PEC
	1.65	0.5062	0.5029	0.5306	0.5168	MLE
	1.70	0.5392	0.5329	0.5592	0.5472	MLE
	1.75	0.5614	0.5536	0.5806	0.5692	MLE
	1.80	0.5782	0.5702	0.5964	0.5855	MLE
	1.85	0.5927	0.5821	0.6019	0.6089	PEC
	1.90	0.6026	0.5934	0.6182	0.6074	MLE
	1.95	0.6108	0.6018	0.6277	0.6175	MLE
	2.00	0.6192	0.6092	0.6345	0.6246	MLE
50	1.55	0.3699	0.4006	0.3944	0.3915	MOM
	1.60	0.4127	0.4724	0.4665	0.4648	MOM
	1.65	0.5062	0.5162	0.5116	0.5152	MOM
	1.70	0.5392	0.5427	0.5423	0.5407	MOM
	1.75	0.5614	0.5468	0.5466	0.5628	PEC
	1.80	0.5782	0.5668	0.5643	0.5796	PEC
	1.85	0.5927	0.5679	0.5811	0.5927	PEC
	1.90	0.6026	0.5792	0.5942	0.6032	PEC
	1.95	0.6108	0.5826	0.6047	0.6118	PEC
	2.00	0.6192	0.5116	0.6133	0.6171	PEC
75	1.55	0.3699	0.3929	0.3888	0.3897	PEC
	1.60	0.4127	0.4652	0.4523	0.4684	PEC
	1.65	0.5062	0.5108	0.5043	0.5085	PEC
	1.70	0.5392	0.5420	0.5362	0.5492	PEC
	1.75	0.5614	0.5628	0.5369	0.5862	PEC
	1.80	0.5782	0.5742	0.6021	0.5732	PEC
	1.85	0.5927	0.5871	0.6244	0.5972	PEC

n	t_i	Rreal Ri	$\widehat{R}_{mom}$	$\widehat{R}_{Mle}$	$\widehat{R}_{pec}$	Best
	1.90	**0.6026**	**0.6027**	**0.6151**	**0.6463**	**PEC**
	1.95	**0.6108**	**0.6114**	**0.6224**	**0.6427**	**PEC**
	2.00	**0.6192**	**0.6321**	**0.7042**	**0.7532**	**PEC**

5. Discussion

This section explores the significance of the work, and it is explained by Table 9. From the summary of results in this table, we concluded that percentile is the best with PEC in the case of $\frac{96}{240} = 0.4\%$. Then, MLE is best in the case of $\frac{82}{240} = 0.3416\%$. Lastly, MOM has been the finest in the case of $\frac{62}{240} = 0.2583\%$.

Table 9. Results of preference of fuzzy Reliability estimators

Tables	$\widehat{R}_{Mom}$	$\widehat{R}_{Mle}$	$\widehat{R}_{Pec}$
Table 1	$\frac{8}{30} = 0.266$	$\frac{12}{30} = 0.4$	$\frac{10}{30} = 0.333$
Table 2	$\frac{7}{30} = 0.233$	$\frac{17}{30} = 0.466$	$\frac{6}{30} = 0.2$
Table 3	$\frac{11}{30} = 0.366$	$\frac{14}{30} = 0.466$	$\frac{5}{30} = 0.166$
Table 4	$\frac{7}{30} = 0.233$	$\frac{8}{30} = 0.266$	$\frac{15}{30} = 0.5$
Table 5	$\frac{6}{30} = 0.233$	$\frac{11}{30} = 0.333$	$\frac{13}{30} = 0.666$
Table 6	$\frac{12}{30} = 0.4$	$\frac{4}{30} = 0.133$	$\frac{14}{30} = 0.466$
Table 7	$\frac{7}{30} = 0.233$	$\frac{8}{30} = 0.266$	$\frac{15}{30} = 0.5$
Table 8	$\frac{4}{30} = 0.133$	$\frac{8}{30} = 0.266$	$\frac{18}{30} = 0.6$

6. Conclusion

From the results of simulation for comparing the fuzzy reliability function of mixed Exponential-Rayleigh proportions, we find that the first best one is the PEC estimator, the second one is MLE, and the third one is MOM. Also, we applied the Exponential-Rayleigh and Maxwell for comparing fuzzy Reliability as the exponential is necessary for this distribution. As the values are small, the powering to certain exponent make the data flexible and feasible for estimation of parameters, especially for reliability or risk function.

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