Periodicals of Engineering and Natural Sciences ISSN 2303-4521
Vol. 8, No. 1, March 2020, pp.313-320

Fully modified least absolute devotions and fully modified M in estimating regression model with non-stationary explanatory variable and auto correlated random errors

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ABSTRACT

The research is concerned with the adoption of two robust estimation methods: The fully modified least absolute devotions method (FM-LAD) and the fully modified M method (FM-M), in estimating the parameters of regression model with non-stationary explanatory variable and autocorrelated random errors which can be modeled according to one of the mixed models, autoregressive and moving average (ARMA). The research aims to make a comparison between these two methods based on the results of their estimation using simulation experiments prepared for this purpose. The results of the simulation experiment showed the advantage of the fully modified M method over the second method depending on the trade-off criterion mean squared error (MSE).

Keywords: Robust estimating methods, fully modified least Absolute devotions method, fully modified M method, Stationary and nonstationary time series, Autoregressive-Moving average models, autocorrelated.

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1. Introduction

Statistically, the linear regression model has been based on some basic assumptions which facilitate the estimation process and the significance tests of the estimated model. However, some or all of these assumptions may not be realized and in particular, when the model variables data is in a time series format and the explanatory variable is unstable and the random error terms are autocorrelated. This leads to the fact that the least-squares estimators are inefficient, which calls for the search for alternative estimation methods, some of which are Fully Modified Least Absolute deviation method (FM-LAD) and fully modified M method, which is robust estimation methods. The author of [1], suggested these two methods by making a modification to both of the robust estimation methods, least absolute deviation (LAD) and M method. This modification was based on semiparametric correction of the variance-covariance matrix of the random errors to address the effects of unstable explanatory variables of the regression model and serial correlation of random error terms. The authors of [2], suggested a parametric fully modified M method for estimating the regression model with integrated regressors and autoregressive moving average (ARMA) errors, by this method the parameters of ARMA model were estimated together with the parameters of the regression model, they compare their method with the semi-parametric fully modified M method suggested by Philips. The research aims to adopt fully modified least absolute deviation method (FM-LAD) and fully modified M method in estimating the regression model with non-stationary explanatory variables (I(1)) and autocorrelated random error terms, and compared between the results of these two methods using a simulation study. The paper is arranged as follows: Section 2 presents in detail the regression model under consideration. Section 3 shows the assumptions of the random error terms. Sections 4, 5 and 6 explain why the robust estimation methods are used, as well as the two estimation methods under consideration. Section 7 deals with the significance test of the estimated parameter vector. Sections 8 and 9 summarize the simulation experiments and the results which have been determined from these experiments. Section 10 summarizes the conclusions which have been obtained from the results of simulation experiments. Finally, section 11 offers an expansion of what our research paper covered.

2. A regression model with integrated regressors and ARMA error

The regression model can be written as:

$$Y_t = X_t^T \beta + u_{0t} \qquad \text{(a)}$$
$$X_t = X_{t-1} + u_{1t} \qquad \text{(b)} \qquad (1)$$
$$\emptyset(B) u_{0t} = \theta(B) e_t \qquad \text{(c)}$$

Where X_t the matrix of the explanatory variables which are nonstationary, integrated, and correlated with the regression random error terms u_{0t} . the explanatory variable matrix was a full rank matrix, and that means that they are endogenous variables and can be modeled according to the random walk model as in (b), [3]. The time series of regression random error terms are autocorrelated and can be modeled according to the mixed autoregressive-moving average model ARMA (p,q) as in (c). The random error terms of the explanatory variables model (u_{1t}) are stationary time series with zero mean and may be correlated with the random error series (e_t) of the ARMA (p,q), which are also stationary time series with zero mean,[4]. The coefficients of the regression model in (1) can be detailed as follows:

$$\varphi = (\beta^T, \delta^T)^T \qquad (2)$$

Where φ is the regression model coefficient vector of size (1*p+q+k+1), which consists of β the vector of the explanatory variables coefficient of size (1* k+1), and $\delta^T = (\emptyset^T, \theta^T)^T$ for the vector of the ARMA (p,q) coefficient of size (1*p+q), where :

$$\emptyset = \left(\emptyset_1, \emptyset_2, \dots\dots\dots, \emptyset_p\right) \qquad (3)$$

$$\theta = \left(\theta_1, \theta_2, \dots\dots\dots.., \theta_q\right) \qquad (4)$$

3. The assumptions of the random error terms

Assume that $\underline{\cup}_t$ represents the vector of random errors which included (u_{0t}) for random errors of the regression model (1-a) and the random errors of the random walk model (u_{1t}), as shown below [5]:

$$\underline{\cup}_t = (u_{0t}, u_{1t}) \qquad (5)$$

This vector assumes to satisfy the following assumptions:

a- It is a completely stable and mixed sequence in mixed numbers α_m, which satisfied:

$$\sum_1^\infty \alpha_m^{(l-\beta)/l\beta} < \infty \quad , \quad l > \beta > 2 \qquad (6)$$

b- The moment condition which states that:

$$\left\|\underline{\cup}_t\right\|_l < \infty \qquad (7)$$

c- The probability density function of the random errors vector $\underline{\cup}_t$ is symmetric, positive, and continuous over the interval ($-b,b$) for $b > 0$, [1].

Under the three assumptions outlined above, the long-term variance and covariance matrix of the vector $\underline{\cup}_t$ exist and can be expressed as:

$$\Omega_{uu} = \sum_{j=-\infty}^{\infty} E(U_t U_{t-j}^T) = \begin{bmatrix} \Omega_{00} & \Omega_{o1} \\ \Omega_{1o} & \Omega_{11} \end{bmatrix} \qquad (8)$$

Using the sine transformation for the random errors series u_{ot} as following:

$$v_t = sign(u_{ot})\begin{cases} 1 & for \quad u_{ot} \geq 0 \\ -1 & for \quad u_{ot} < 0 \end{cases} \tag{9}$$

Defining the error vector $Z_t = (v_t,\ u_{01}^T)$, then the long-term variance and covariance matrix of that vector also exists under the three error assumptions, and because the error vector v_t is a finite function of the error vector u_{ot}. The variance and covariance matrix of the vector Z_t can be as:

$$\Omega_{zz} = \sum_{j=-\infty}^{\infty} E(z_t z_{t-j}^T) = \begin{bmatrix} \Omega_{vv} & \Omega_{v1} \\ \Omega_{1v} & \Omega_{11} \end{bmatrix} \tag{10}$$

In the same way, we can define the one direction long-term variance and covariance matrix for each of the two vectors U_t and Z_t respectively [6]:

$$\Delta_{uu} = \sum_{j=0}^{\infty} E(u_t u_{t-j}^T) = \begin{bmatrix} \Delta_{oo} & \Delta_{o1} \\ \Delta_{1o} & \Delta_{11} \end{bmatrix} \tag{11}$$

$$\Delta_{ZZ} = \sum_{j=0}^{\infty} E(z_t z_{t-j}^T) = \begin{bmatrix} \Delta_{vv} & \Delta_{v1} \\ \Delta_{1v} & \Delta_{11} \end{bmatrix} \tag{12}$$

4. Robust methods for estimating model parameters

The adoption of traditional estimation methods for estimating the regression model with nonstationary explanatory variable and autocorrelated random errors leads to inefficient estimates. This affects the statistical inference which calls for the adoption of robust estimation methods leading to efficient estimates of model parameters. Among these robust methods are fully modified least absolute deviations and fully modified M.

5. Fully modified least absolute deviation (FM-LAD)

The vector parameters (φ) of the regression model shown in formula (1), can be estimated using least absolute deviation method (LAD). The estimators of this method is the solution to the set of equations resulting from the minimization of the following objective function [2] :

$$\sum|e_t(\varphi)| = 0 \tag{13}$$

Where $e_t(\varphi)$ is the white noise vector of the mixed model autoregressive and moving average (ARMA) which represents the random error term u_{0t}.

The resulting estimators are consistent but biased of order two because of the correlation between the explanatory variable X and the robust function (v_t) shown in formula (9), which can be measured by the variance-covariance matrix (Δ_{1V}) which is an element of the variance-covariance matrix (Δ_{ZZ}) shown in formula (12). Since the limiting distribution of these estimators affected by the variance-covariance matrix (Δ_{1V}), because this matrix exists in the probability formula of that limiting distribution [7].

To address these failures, Philips proposed a modification to the LAD method to obtain a modified estimate known as Fully Modified Least Absolute Deviations (FM-LAD), this method treats second-order bias and leads to estimates with an approximate normal distribution under which standardized tests such as t and Wald can be used. These estimates are robust and resistant to outlier values when applied to heavy tail data, [3]. The vector parameters of the regression model are estimated according to the FM-LAD method by correcting the correlation between the explanatory variable and the robust function (v_t). The procedures are as follows:

a- Make a modification to the random error terms of the model (1-a) to get the robust errors according to the transformation formula shown in the formula (9).

b- Estimate the vector parameters by Least Absolute Deviations (LAD) method.

c- Estimate random error terms based on the vector parameters estimated in step (2).

d- Estimate the errors $({v_t}^+)$ according to the following formula:

$${v_t}^+ = v_t - \Omega_{v1}\Omega_{11}^{-1}\Delta x_t \tag{14}$$

e- Estimate the variance-covariance matrix (Δ_{1V}) between (v_t) and (u_{1t}) depending on the kernel estimate of Ω_{11} and Ω_{V1} , as follows [9]:

$$\Delta_{1v}^+ = \sum_{t=0}^{\infty}(u_{10}\, v_t^+) = \Delta_{xy} - \Delta_{11}\Omega_{11}^{-1}\Omega_{1v} \tag{15}$$

f- Depending on Δ_{1v}^+ , the estimation of parameters according to the Fully Modified Least Absolute Deviations (FM-LAD) method is according to the following formula, [8]:

$$B^{+}_{LAD} = B_{LAD} - [2\hat{h}(0)X^T X]^{-1}\left[X^T \Delta X \hat{\Omega}^{-1}_{11}\hat{\Omega}_{1v} + T\hat{\Delta}_{1v}{}^{+}\right] \qquad (16)$$

Where:

$\hat{h}(0)$ is the consistent nonparametric estimator for the probability density function of the random error terms of the regression model (1-a) at the point of origin, which has been estimated according to the following formula:

$$h(0) = \frac{1}{TK}\sum_{t=1}^{T} w\left(\frac{0-u_{0t}}{M}\right) \qquad (17)$$

where $w(\frac{0-u_{0t}}{M})$ represents the parameter of the kernel function.

The probability distribution of the estimated parameters vector B^{+}_{LAD} can be approximated to the normal distribution with a mean equal to β and variance-covariance matrix equal to $[1/(2h(0))]^2 q(X^T X)^{-1}$, that is mean, $\beta^{+}_{FM-LAD} \sim N(\beta\ , [1/(2h(0))]^2 q(X^T X)^{-1}\ )$, the consistent estimator of the variance-covariance matrix (q) is as follows, [5];

$$\hat{q} = \hat{\Omega}_{vv} - \hat{\Omega}_{v1}\hat{\Omega}^{-1}_{11}\hat{\Omega}_{1v} \qquad (18)$$

6. Fully modified M method (*FM-M*)

The M estimators are a general set of robust estimation methods used in estimating the vector parameters (φ) of the regression model shown in formula (1). It is the solution to the set of equations resulting from the minimization of the following objective function, [2] :

$$\varphi_M = argmin[\textstyle\sum_1^T \rho(Y_t - x_t^T\beta)] \qquad (19)$$

Where ρ is a weight function that can take the form ($\rho(u) = |u|^{\delta}$, for $\delta \in [1,2]$), and it may take the loss function for Huber according to the following formula:

$$\rho_c(u) = \begin{cases} \left(\frac{1}{2}\right)u^2 & for|u| \le c \\ c|u| - \left(\frac{1}{2}\right)c^2 & for\ |u| > c \end{cases} \qquad (20)$$

The estimates $\hat{\varphi}_M$ are the solution to the following set of equations after being equal to zero:

$$\textstyle\sum_{t=1}^{T} \psi\big(e_t(\varphi)\big)e_{\varphi t} = 0 \qquad (21)$$

Where $e_t(\varphi)$ is the white noise vector of the mixed model autoregressive and moving average (*ARMA*) which represents the random error term u_{0t}, $e_{\varphi t}$ is the vector of the white noise derivatives with respect to all model parameters as follows:

$$e_{\varphi t} = \partial e_t(\varphi)/\partial\varphi \qquad (22)$$

That is:

$$e_{\varphi t}(\varphi) = (e^T_{\beta t}(\varphi), e^T_{\emptyset t}(\varphi), e^T_{\theta t}(\varphi))^T \qquad (23)$$

Where:

$$e_{\beta t}(\varphi) = -\theta^{-1}(B)\emptyset(B)X_t \qquad (24)$$

$$e_{\emptyset t}(\varphi) = -\theta^{-1}(B)\emptyset\big(B_p\big)(Y_t - X_t^T\beta) \qquad (25)$$

$$e_{\theta t}(\varphi) = \theta^{-2}(B)\emptyset\big(B_q\big)(Y_t - X_t^T\beta) \qquad (26)$$

When ρ is a differentiable and concave function, and ($\psi = \acute{\rho}$), then the two relations (17 and 19) are equivalent. In this case, there is a unique solution to Eq. (19) [5]. The robust M estimators are consistent but biased of second-order, because in spite of removing the autocorrelation between the random error terms u_{0t} using *ARMA* model, and therefore, $\psi(e_t)$ will be uncorrelated, the white noise error e_t of the *ARMA* model is still correlated with the weighted error $\psi(e_t)$ which lead to the bias in the estimation of the vector parameter $\underline{\beta}$. To treat this drop, Philips (1995), and Dong Wan Shin and Oesook Lee (2004), suggested to make a modification on the robust *M* estimators to get fully modified *M* estimators (*FM-M*), in this way we can treat the second-order bias and lead to estimators with approximately normal distribution so we can use the standard tests of significant like t-test or wald test,[7].

The formula of the estimated regression parameters vector can be represented according to the *FM-M* method as follows:

$$\beta_M^+ = \hat{\beta}_M - [(T^{-1} \sum_1^T \acute{\psi}\,(\hat{u}_{0t}))X^T X)^T (X^T \Delta X \hat{\Omega}_{xx}^{-1} \hat{\Omega}_{x\psi} + T\hat{\Delta}_{x\psi}^+)] \qquad (27)$$

Where,

$\hat{\Omega}_{xx}$ is the long-run estimated variance-covariance matrix between u_{xt-j} and u_{xt} .

$\hat{\Omega}_{x\psi}$ is the long run consistent estimated variance-covariance matrix between u_{xt} and $\psi(u_{ot)}$,where :

$$\Omega_{x\psi} = \sum_{j=-\infty}^{\infty} E\big(u_{xt}\psi(u_{ot+j})\big) \qquad (28)$$

$\hat{\Delta}_{x\psi}^+$ is the estimate of the one direction variance-covariance matrix between u_{xt} and $\psi(u_{ot})$, where :

$$\Delta_{x\psi}^+ = \Delta_{x\psi} - \Delta_{xx}\Omega_{xx}^{-1}\Omega_{x\psi} \qquad (29)$$

And

$$\Delta_{x\psi} = \sum_{j=0}^{\infty} E\big(u_{xt}\psi(u_{ot+j})\big) \qquad (30)$$

Variance-covariance matrices are estimated using kernel function, which means that $\hat{\Omega}_{x\psi}$ and $\hat{\Delta}_{x\psi}^+$ is the consistent kernel estimator which is one of the nonparametric estimators [6].

The approximated distribution of the estimated parameter vector according to FM-M method is the normal distribution with mean equal to β and variance-covariance matrix ($q(X^T X)^{-1}$), the consistent estimator of the variance-covariance matrix q is as follows [9]:

$$\hat{q} = \hat{\Omega}_{\psi\psi} - \hat{\Omega}_{\psi x}\hat{\Omega}_{xx}^{-1}\hat{\Omega}_{x\psi} \qquad (31)$$

7. A significant test of the estimated parameter vector

As stated above, the robust fully modified M method gives an estimator with approximately normal distribution, so we can depend on the significant tests like t-test or Wald test to test the following hypothesis:

$$H_0 = R\underline{\beta} - r = 0 \qquad V.S \qquad H_1 = R\underline{\beta} - r \neq 0$$

The formula of the t-statistics for testing the above hypothesis can be expressed as:

$$t_i = \frac{(\beta_{FM-OLS}^+ - \beta_i)}{s_i} \qquad (32)$$

Where s_i computed using the following formula for all i= 1,2,… , k :

$$s_i = \sqrt{[\hat{q}(X^T X)^{-1}]_{ii}} \qquad i = 1,2, \dots \dots k \qquad (33)$$

The null hypothesis can be rejected if the computed value of the t-statistics is greater than the table value of the t-distribution, [5].

We can also use wald statistics (W^+) which are computed as follows:

$$W^+ = (R\beta_{FM}^+ - r)^T \{R\hat{q}(X^T X)^{-1} R^T\}^{-1} (R\beta_{FM}^+ - r) \,/\, \hat{q} \qquad (34)$$

Under the null hypothesis wald statistics (W^+) are distributed as qui square distribution (χ_r^2) with *r* degree of freedom where r represents the number of leaner constraints on the model parameters,[1].

8. Simulation experiments

Simulation experiments were carried out based on three sample sizes (n1 = 30 , n2 = 70 , n3 = 100), and two levels of standard deviation (σ1 = 0.1 , σ2 = 0.25), as well as three different parameter values (**β**0 = 3 , 8 , 12) . (**β**1 = 1 , 1.5 , 2), which were determined based on the model parameters estimated in previous studies on the same research topic. For the purpose of generating random errors, three mathematical models were adopted which are AR(1), MA(1), and ARMA(1,1). Carrying out the simulation requires writing a program in MATLAB language. Based on the results of the simulation experiments, the two robust estimation methods understudy will be compared using the trade-off criterion mean squared error (MSE).

9. Results and discussion

We will discuss the results of the simulation experiments to show which of the two robust estimation methods under study are better at estimating the regression model parameters. Table 1 shows the results of the first simulation experiment in which we assumed the random errors are generated according to *AR(1)* model, these results indicated *FM-M* preference over *FM-LAD*, based on the criterion of goodness of fit (*MMSE), FM-M* method achieved the lowest value of the rate of mean squared errors which is (0.017) at the sample size ($n_1=30$), the level of standard deviation ($\sigma_1 = 0.1$) and the value of autoregressive parameter ($\emptyset_1=0.4$). The second simulation experiment assumed that the random errors are generated according to MA(1) model, the results of this experiment are shown in Table 2, which indicated also the preference of FM-M method over *FM-LAD* method according to the lowest value of *MMSE* which is (0.037) at the two sample size ($n_1=30$ *and* $n_2=70$), the level of standard deviation ($\sigma_1 = 0.1$) and the two values of moving average parameters ($\theta_1= 0.2, 0.6$).

Table 3 shows the results of the third experiment which assumed that the random errors generated are according to the mixed model *ARMA(1,1)*, as in the previous two experiments, the *FM_M* method yielded better results than the *FM-LAD* method, since it achieved the lowest value of *MMSE* (0.044) at the two sample size ($n_1=30$ *and* $n_2=70$), the level of standard deviation ($\sigma_1 = 0.1$), the values of *AR* parameter ($\emptyset_1=0.4$) and MA parameter ($\theta_1= 0.2$).

Table 1. The simulation results of the first experiment which assumed that the random errors generated according to AR(1) model

levels of standard deviation	n	AR parameter	$\emptyset_1$			$\emptyset_1$			$\emptyset_1$		
			0.4			-0.4			0.9		
		Reg. parameter	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1
			3 1	8 1.5	12 2	3 1	8 1.5	12 2	3 1	8 1.5	12 2
σ_1 (0.1)	30	FM-LAD	0.031036	0.035183	0.032813	0.087403	0.093116	0.089246	0.141315	0.140594	0.137314
		FM-M	0.01750	0.01752	0.01757	0.06974	0.06969	0.06943	0.11558	0.11743	0.11444
	70	FM-LAD	0.026298	0.02949	0.027287	0.084222	0.084034	0.084838	0.129731	0.138348	0.137814
		FM-M	0.01858	0.01774	0.01764	0.06940	0.06987	0.06921	0.11378	0.11502	0.11546
	100	FM-LAD	0.021268	0.020703	0.020165	0.076061	0.075408	0.078404	0.123993	0.12386	0.12275
		FM-M	0.02094	0.02088	0.02078	0.07385	0.06937	0.07334	0.11512	0.11273	0.11281
σ_2 (0.25)	30	FM-LAD	0.098284	0.101964	0.092143	0.156801	0.163922	0.17288	0.222265	0.201835	0.202447
		FM-M	0.052	0.053	0.053	0.105	0.106	0.104	0.154	0.151	0.152
	70	FM-LAD	0.093738	0.081308	0.091472	0.146719	0.141059	0.156713	0.189658	0.202222	0.191372
		FM-M	0.054	0.052	0.053	0.108	0.105	0.107	0.153	0.151	0.149
	100	FM-LAD	0.06381	0.066258	0.063005	0.119967	0.12408	0.116239	0.175744	0.158669	0.176639
		FM-M	0.056	0.055	0.054	0.103	0.106	0.101	0.152	0.144	0.150

Table 2. The simulation results of the second experiment which assumed that the random errors generated according to MA(1) model

levels of standard deviation	n	AR parameter	θ_1			θ_1			θ_1		
			0.2			0.6			-0.8		
		Reg. parameter	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1
			3 1	8 1.5	12 2	3 1	8 1.5	12 2	3 1	8 1.5	12 2
σ_1 (0.1)	30	FM-LAD	0.054	0.054	0.056	0.055	0.052	0.055	0.170	0.173	0.170
		FM-M	0.037	0.037	0.037	0.037	0.037	0.037	0.143	0.144	0.141
	70	FM-LAD	0.048	0.050	0.048	0.052	0.048	0.048	0.169	0.165	0.168
		FM-M	0.037	0.037	0.037	0.037	0.038	0.037	0.141	0.139	0.145
	100	FM-LAD	0.041	0.040	0.042	0.041	0.041	0.041	0.159	0.156	0.154
		FM-M	0.039	0.040	0.040	0.039	0.039	0.039	0.143	0.145	0.142
σ_2 (0.25)	30	FM-LAD	0.129	0.128	0.124	0.111	0.139	0.118	0.243	0.248	0.243
		FM-M	0.073	0.073	0.072	0.071	0.073	0.071	0.179	0.179	0.182
	70	FM-LAD	0.110	0.106	0.113	0.109	0.101	0.108	0.230	0.228	0.215
		FM-M	0.073	0.073	0.073	0.071	0.071	0.071	0.181	0.177	0.177
	100	FM-LAD	0.088	0.086	0.085	0.087	0.086	0.083	0.196	0.202	0.193
		FM-M	0.073	0.074	0.073	0.074	0.076	0.073	0.173	0.177	0.174

Table 3. The simulation results of the thrid experiment which assumed that the random errors generated according to mixed model ARMA(1,1)

levels of standard deviation	n	AR parameter	$\varnothing_1$ θ_1			$\varnothing_1$ θ_1			$\varnothing_1$ θ_1		
			0.4 0.2			-0.4 0.6			0.9 -0.8		
		Reg. parameter	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1	β_0 β_1
			3 1	8 1.5	12 2	3 1	8 1.5	12 2	3 1	8 1.5	12 2
σ_1 (0.1)	30	FM-LAD	0.592	0.140	0.139	0.348	0.179	0.205	0.299	0.252	0.188
		FM-M	0.045	0.044	0.045	0.199	0.206	0.202	0.516	0.517	0.515
	70	*FM-LAD*	0.203	0.860	0.102	0.176	0.147	0.137	0.257	0.174	0.278
		FM-M	0.044	0.046	0.045	0.202	0.200	0.200	0.512	0.511	0.518
	100	*FM-LAD*	0.359	0.356	0.279	0.711	0.619	0.679	0.160	0.177	0.155
		FM-M	0.048	0.047	0.047	0.202	0.196	0.196	0.477	0.491	0.482
σ_2	30	*FM-LAD*	0.092	0.086	0.087	0.259	0.275	0.259	0.583	0.589	0.598

(0.25)		*FM-M*	0.046	0.045	0.046	0.200	0.201	0.201	0.504	0.505	0.505
	70	*FM-LAD*	0.057	0.062	0.059	0.230	0.231	0.233	0.533	0.562	0.578
		FM-M	0.048	0.046	0.048	0.194	0.200	0.198	0.481	0.504	0.502
	100	*FM-LAD*	0.863	0.756	0.907	0.116	0.106	0.143	0.169	0.157	0.600
		FM-M	0.182	0.183	0.179	0.328	0.330	0.339	0.621	0.649	0.507

10. Conclusions

The simulation results of FM-M estimates indicated that they were better than FM-LAD estimates, they achieved the lowest value of the average mean squared errors for all the three experiments. It was found through simulation experiments that there is an effect of the value of the standard deviation of the random error terms since the lowest value of the mean error squares average was achieved at ($\sigma1 = 0.1$) Among the two default standard deviation values ($\sigma_1 = 0.1$, $\sigma_2 = 0.25$), these results also showed that the best model for representing the time series of random errors in the autoregressive model AR (1) for all the three sample sizes and for the two standard deviation levels.

11. Recommendations

As an expansion of our research paper, other robust estimation methods can be used and compared with the estimates obtained for the two estimation methods under consideration. Non-parametric estimation methods can also be used as penalized Spline regression and wavelet regression.

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