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Robust Bayesian estimators for binomial distribution under prior data conflict

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ABSTRACT

In this paper, the regular Bayes method and robust Bayes method were used to estimate the parameter (p) and the survival function of the Binomial distribution in the case of prior data conflict for two simulation experiments. The first experiment was in the case of unconflicted prior data. The simulation results of the first experiment showed that the robust Bayes method is best by using the comparative criterion (IMSE). The second experiment was in case of prior data conflict. The simulation results showed that the robust Bayes method is best by using the comparative criterion (IMSE). Thus, the robust Bayes method is best in both cases to estimate the parameter (p) and the survival function of the Binomial distribution.

Keywords: Binomial distribution, Robust Bayesian, Regular Bayesian, Prior data conflict, Survival function, iLuck Model.

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1. Introduction

In statistical inference there are several methods of estimation, As the Bayes method, as is known, depends mainly on the prior distribution or prior information, the prior distribution is combined with the distribution of observations to obtain the posterior distribution according to the rule of Bayes. When we combine the prior distribution with the distribution of observation, we may have a problem which is prior data conflict [1]. This means that the prior information does not necessarily correspond to the sample information under study. Therefore, the existence of this problem should be verified by using a method for modeling the prior distribution parameters [2]. After the prior distribution is modeled, the standard deviation of the prior distribution is extracted and compared to the standard deviation of the posterior distribution [3]. If the standard deviation of the posterior distribution is less than the standard deviation of the prior distribution, then there is a prior data conflict [4]. From here, the main objective of the research is to obtain the best estimate in the case of prior data conflict from the model through which we get a set of prior distributions and thus get a set of posterior distributions [5, 1]. Therefore, we obtain more accurate and efficient estimators so that this method is called the robust Bayes method. The regular Bayes method and the robust Bayes method will be used to estimate the parameter (P) and the survival function of the Binomial distribution and compare the methods by using (IMSE).

2. The estimation of methods

The parameter (P) and the survival function of the Binomial distribution will be estimated using the regular Bayes method and the robust Bayes method.

2.1. Regular Bayes

The Bayes estimator, which relies primarily on the prior information, considers that the parameter to be estimated is a random variable. Thus, the random variable has a distribution called the prior distribution and then the prior distribution is combined with the distribution of the sample under study in accordance with the rule Bayes to get the posterior distribution. After obtaining the posterior distribution, we get the Bayes estimator using one of the loss functions. In this paper the quadratic loss function is used to obtain the Bayes estimator [6]**:**

$$\mathrm{f(x)} = \binom{\mathrm{n}}{\mathrm{x}}\mathrm{p^x}(1-\mathrm{p})^{\mathrm{n-x}}\,, \mathrm{x} = 0,1,\dots,\mathrm{n} \qquad (1)$$

The appropriate prior distribution is the beta distribution as follows:

$$\mathrm{f}(p/a,b) = \frac{1}{\beta(\alpha,\beta)}p^{\alpha-1}(1-p)^{\beta-1} \qquad (2)$$

Using the Bayes rule, we get the posterior distribution as follows:

$$= \frac{1}{\beta(\alpha+s,n+\beta-s)}p^{\alpha+s-1}(1-p)^{n+\beta-s-1} \qquad (3)$$

2.1.1. Bayesian estimation for the parameter (P)

From equation (4) which represents the posterior distribution, we get the Bayes estimator for parameter (P) as follows [7]**:**

$$\hat{p} = \frac{\alpha+s}{n+\alpha+\beta} \qquad (4)$$

2.1.2. Bayesian estimation for the survival function

From equation (4), we get the Bayes estimator for the survival function as shown below [8]**:**

$$\hat{\mathrm{S}}(\mathrm{t}) = \frac{1}{\beta(\alpha+s,n+\beta-s)}\sum_{j=t}^{n}\frac{n!}{j!(n-j)!}\beta(\alpha+s+j,2n+\beta-s-j) \qquad (5)$$

2.2. Robust Bayesian method

2.2.1. Checking for prior data conflict

The problem of the prior data conflict can be tested by modeling the prior distribution parameters as described in the following steps [9,10]**:**

$$f(x) = \binom{n}{x}p^x(1-p)^{n-x} \qquad (6)$$

The prior distribution of parameter (P) is the beta distribution as follows:

$$f(p/a,b) = \frac{1}{\beta(\alpha,\beta)}p^{\alpha-1}(1-p)^{\beta-1} \qquad (7)$$

Next we need to modeling the parameters of the prior distribution in two ways:

The first method: It is the method of conditional expectation as described below:

$$E(p/\alpha,\beta) =$$

$$y^0 = \frac{\alpha}{n^0}$$

$$y^0 = \frac{\alpha}{n^0} \Rightarrow$$

Then, we replace the parameters $\alpha = n^0 y^0, \beta = n^0(1-y^0)$ with the prior distribution to get the prior distribution with the updated parameters:

$$f(p/n^0, y^0) = \frac{1}{\beta(n^0y^0, n^0(1-y^0))} p^{n^0y^0-1}(1-p)^{n^0(1-y^0)-1} \qquad \textbf{(8)}$$

Method 2: In this method, the prior distribution of the updated parameters can be determined in two steps, as follows [4]**:**

Step 1: If the model can be written in the form of (canonical exponential family) as shown below:

$\mathrm{f}(x/p) = a($

$f(s/p) = ($

$$= \binom{n}{s} exp\left\{log\left(\frac{p}{1-p}\right)s - n(-log(1-p))\right\} \qquad (9)$$

Step 2: The prior distribution can be constructed through the following model:

$\mathrm{f}(\psi/n^0, y^0)$

$f(\psi/n^0, y^0$

$\left|\frac{d\psi}{dp}\right| = \frac{}{p(1}$

$f(p/n^0, y^0)$

The prior distribution with the updated parameters is then:

$$\mathrm{f}(p/n^0, y^0) = \frac{1}{\beta(n^0y^0, n^0(1-y^0))} p^{n^0y^0-1}(1-p)^{n^0(1-y^0)-1} \qquad (10)$$

From the equation above we extract the standard deviation as follows:

$$\mathrm{s.d\ prior} = \sqrt{\frac{y^0(1-y^0)}{n^0+1}} \qquad (11)$$

From equation (1) that represents the distribution of Binomial and equation (10) that represents the prior distribution and using the Bayes rule, we get the posterior distribution as follows:

$$= \frac{1}{\beta(n^0y^0+s, n^0(1-y^0)+n-s)}\ p^{n^0y^0+s-1}(1-p)^{n^0(1-y^0)+n-s-1} \qquad (12)$$

From the above equation, we extract the standard deviation for the posterior distribution as follows:

$$\mathrm{s.d\ posterior} = \sqrt{\frac{y^n(1-y^n)}{n^n+1}} \qquad (13)$$

Then, the standard deviation of the prior distribution is compared with the standard deviation of the posterior distribution. If the value of the standard deviation of the prior distribution is greater than the value of the standard deviation of the posterior distribution. This means that there is a problem of prior data conflict and to solve this problem from the following steps.

2.2.2 Address the problem of prior data conflict

The problem of prior data conflict is addressed by the form $\prod{}^{0} = n^0 x[\underline{y}^0, \overline{y}^0]$ which is presented by [1]. There is also another model presented by [5]. The model is $\prod{}^{0} = [\underline{n}^0, \overline{n}^0] x [\underline{y}^0, \overline{y}^0]$. Through this model, we get a set of posterior distributions, which is called (generalized iLuck-model) and as shown below [4]:

$$f_1(p/s) = \frac{1}{\beta(\underline{n}^0\underline{y}^0+s,\underline{n}^0\left(1-\underline{y}^0\right)+n-s)}\ p^{\underline{n}^0\underline{y}^0+s-1}(1-p)^{\underline{n}^0\left(1-\underline{y}^0\right)+n-s-1} \quad (15)$$

$$f_2(p/s) = \frac{1}{\beta(\underline{n}^0\overline{y}^0+s,\underline{n}^0(1-\overline{y}^0)+n-s)}\ p^{\underline{n}^0\overline{y}^0+s-1}(1-p)^{\underline{n}^0(1-\overline{y}^0)+n-s-1} \quad (16)$$

$$f_3(p/s) = \frac{1}{\beta(\overline{n}^0\underline{y}^0+s,\overline{n}^0\left(1-\underline{y}^0\right)+n-s)}\ p^{\overline{n}^0\underline{y}^0+s-1}(1-p)^{\overline{n}^0\left(1-\underline{y}^0\right)+n-s-1} \quad (17)$$

$$f_4(p/s) = \frac{1}{\beta(\overline{n}^0\overline{y}^0+s,\overline{n}^0\left(1-\overline{y}^0\right)+n-s)}\ p^{\overline{n}^0\overline{y}^0+s-1}(1-p)^{\overline{n}^0\left(1-\overline{y}^0\right)+n-s-1} \quad (18)$$

Equations (15-18) represent a set of prior distributions and thus we get a set of posterior distributions as follows:

$$f_1(p/s) = \frac{1}{\beta(\underline{n}^0\underline{y}^0+s,\underline{n}^0\left(1-\underline{y}^0\right)+n-s)}\ p^{\underline{n}^0\underline{y}^0+s-1}(1-p)^{\underline{n}^0\left(1-\underline{y}^0\right)+n-s-1} \quad (19)$$

$$f_2(p/s) = \frac{1}{\beta(\underline{n}^0\overline{y}^0+s,\underline{n}^0(1-\overline{y}^0)+n-s)}\ p^{\underline{n}^0\overline{y}^0+s-1}(1-p)^{\underline{n}^0(1-\overline{y}^0)+n-s-1} \quad (20)$$

$$f_3(p/s) = \frac{1}{\beta(\overline{n}^0\underline{y}^0+s,\overline{n}^0\left(1-\underline{y}^0\right)+n-s)}\ p^{\overline{n}^0\underline{y}^0+s-1}(1-p)^{\overline{n}^0\left(1-\underline{y}^0\right)+n-s-1} \quad (21)$$

$$f_4(p/s) = \frac{1}{\beta(\overline{n}^0\overline{y}^0+s,\overline{n}^0\left(1-\overline{y}^0\right)+n-s)}\ p^{\overline{n}^0\overline{y}^0+s-1}(1-p)^{\overline{n}^0\left(1-\overline{y}^0\right)+n-s-1} \quad (22)$$

Equations (19-22) represent a set of posterior distributions. By using the quadratic loss function, we get the following equation which represents the iLuck-model:

$$\underline{y}^n = \text{lower}(y^n) = \begin{cases} \frac{\overline{n}^0\underline{y}^0+\tau(x)}{\overline{n}^0+n} \text{ if } \overline{\tau}(x) \geq \underline{y}^0 \\ \frac{\underline{n}^0\underline{y}^0+\tau(x)}{\underline{n}^0+n} \text{ if } \overline{\tau}(x) < \underline{y}^0 \end{cases} \quad (23)$$

$$\overline{y}^n = \text{upper}(y^n) = \begin{cases} \frac{\overline{n}^0\overline{y}^0+\tau(x)}{\overline{n}^0+n} \text{ if } \overline{\tau}(x) \leq \overline{y}^0 \\ \frac{\underline{n}^0\overline{y}^0+\tau(x)}{\underline{n}^0+n} \text{ if } \overline{\tau}(x) > \overline{y}^0 \end{cases} \quad (24)$$

The posterior distribution can therefore be written in its final form as follows:

$$f(p/n^m, y^m) = \frac{1}{\beta(n^m y^m, n^m(1-y^m))} p^{n^m y^m-1}(1-p)^{n^m(1-y^m)-1} \quad (25)$$

$n^m = \frac{\text{lower}}{}$

2.2.3. Robust Bayesian to estimate the parameter (P)

From equation (25), we get the robust Bayes estimator of parameter (P) using the quadratic loss function as follows:

$$\hat{p}_{Rob} = \frac{n^m y^m}{n^m y^m + n^m(1-y^m)} \quad (26)$$

2.2.4. Robust Bayesian to estimate the survival function

From equation (25), we get the robust Bayes estimator of the survival function using the quadratic loss function as follows:

$$\hat{S}_{Rob}(x) = \frac{1}{\beta(n^m y^m, n^m(1-y^m))} \sum_{j=t}^{n} \frac{n!}{j!(n-j)!} \beta(n^m y^m + j, n^m(1-y^m) + n - j) \quad (27)$$

The program was written using R and according to the following steps:

The first step

This stage is one of the basic stages in which the default values are selected so that they depend on them mainly the subsequent stages and the default values are selected as follows:

Different default values for parameter (p) and prior distribution parameters (n^0, y^0) were selected as shown in the following tables:

Table 1. Table of default values in case of prior data unconflicted

Model	P	y^0		n^0	
		Lower	Upper	Lower	Upper
1	0.1	0.01	0.1	10	12
2	0.2	0.02	0.2	10	12
3	0.3	0.03	0.3	10	12
4	0.4	0.04	0.4	10	12
5	0.5	0.05	0.5	10	12
6	0.4	0.05	0.4	10	12

Table 2. Table of default values in case of prior data conflict

Model	P	y^0		n^0	
		Lower	Upper	Lower	Upper
1	0.5	0.3	0.5	2	4
2	0.6	0.4	0.6	2	4
3	0.5	0.3	0.5	5	7
4	0.6	0.4	0.6	5	7
5	0.5	0.3	0.5	10	12
6	0.6	0.4	0.6	10	12

The replicate of the experiment was equal to 1000.

The second step

At this stage, the data is generated by the instruction within the R program for the distribution of Binomial.

Third Step

At this stage the problem of prior data conflict is tested whether or not by comparing the standard deviation and the standard error of the mean for the prior distribution with the standard deviation and the standard error of the average for the posterior distribution. If the standard deviation and standard error of the mean for the prior distribution is greater than the value of the standard deviation and error of the posterior distribution, there will be prior data conflict using formula Eqs. 11 and 13.

The fourth step

At this stage, the parameter (p) and the survival function are estimated according to the regular Bayes method and the robust Bayes method.

The fifth step

At this stage, the estimation methods are compared using by (IMSE) as follows [11]:

$$\mathrm{IMSE}\left[\widehat{\mathrm{pmf}}\right] = \frac{1}{r}\sum_{i=1}^{r}\left\{\frac{1}{n_t}\sum_{j=1}^{n_t}\left[\widehat{\mathrm{pmf}} - \mathrm{pmf}\right]^2\right\} \tag{28}$$

$$\mathrm{IMSE}\left[\hat{S}(t)\right] = \frac{1}{r}\sum_{i=1}^{r}\left\{\frac{1}{n_t}\sum_{j=1}^{n_t}\left[\hat{S}_i(t_j) - S(t_j)\right]^2\right\} \tag{29}$$

Where,

r: The frequency of experiment.

n_t : The sample size for each experiment (t_i)

The simulation results will then be analyzed to estimate the parameter (P) and the survival function of the Binomial distribution according to subsequent tables as follows:

Table 3. Integrated mean square error (IMSE) for the parameter (P) in case of prior data unconflicted

Model							
	Lower					Upper	
	10					12	
			P	K	n		
	Lower	upper					
1	0.01	0.1	0.1	1	10	0.006105	0.002362
				2	20	0.001623	0.001209
				4	40	0.001138	0.000894
Best						Robust Bayesian Estimator	
2	0.02	0.2	0.2	1	10	0.005990	0.003090
				2	20	0.003512	0.002229
				4	40	0.001055	0.000731
Best						Robust Bayesian Estimator	
3	0.03	0.3	0.3	1	10	0.009127	0.004608
				2	20	0.003920	0.002091
				4	40	0.001173	0.000793
Best						Robust Bayesian Estimator	
4	0.04	0.4	0.4	1	10	0.013262	0.006026
				2	20	0.004645	0.002449
				4	40	0.001240	0.000786
Best						Robust Bayesian Estimator	
5	0.05	0.5	0.5	1	10	0.016659	0.006938
				2	20	0.005068	0.002478
				4	40	0.001282	0.000718
Best						Robust Bayesian Estimator	
6	0.05	0.4	0.4	1	10	0.012577	0.005444
				2	20	0.004400	0.002319
				4	40	0.000409	0.000244
Best						Robust Bayesian Estimator	

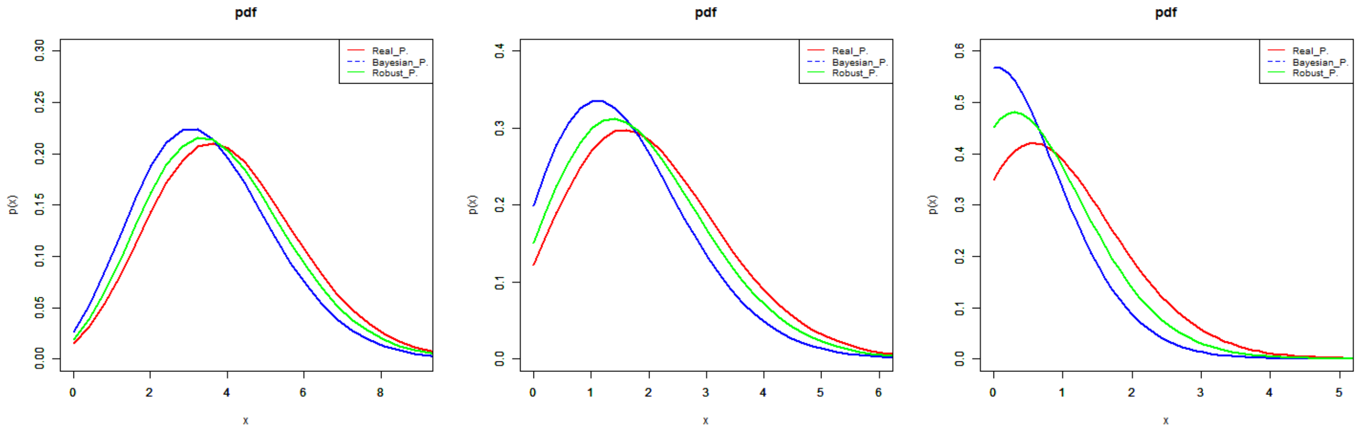


Figure1. Shows model (1) for the (pmf) in the case of prior data unconflicted

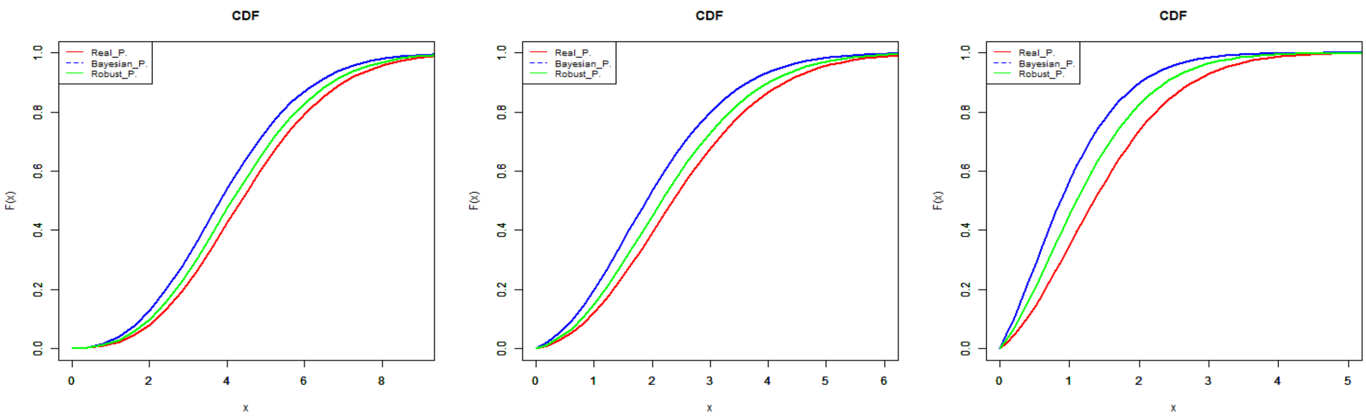


Figure 2. Shows model (1) for the (cdf) in the case of prior data unconflicted

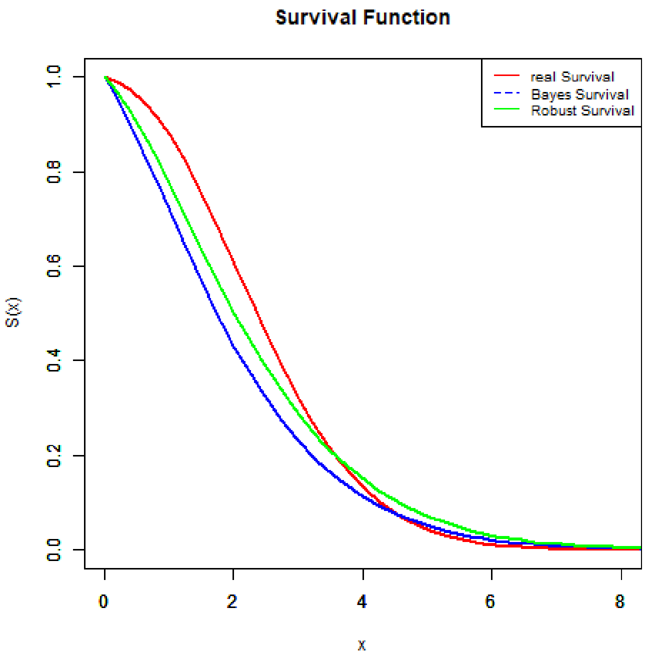
From Table 3 which shows (IMSE) to compare the estimation methods for parameter (p) in the case of unconflicted prior data, the simulation results showed the following:

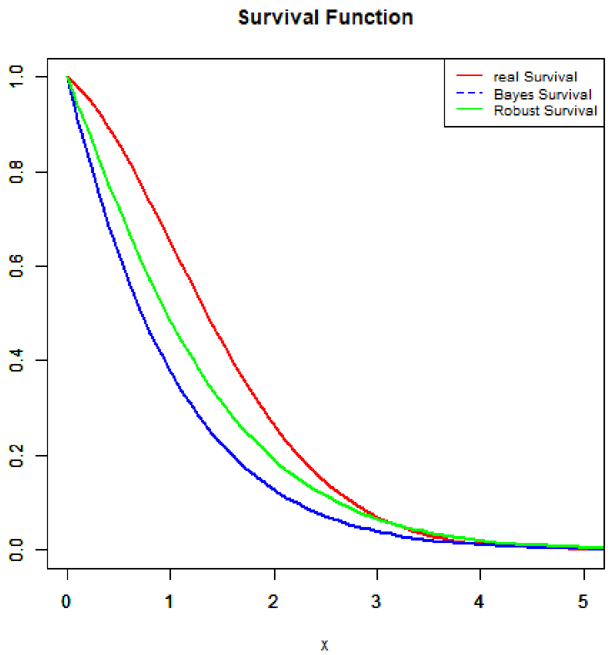
1. The simulation results showed that the robust Bayes estimator for parameter (P) is better than the regular Bayes estimator by using IMSE as a criterion for comparing.
2. The simulation results showed that the best model is model (1).
3. 3. The simulation results showed that IMSE decreases in case of increasing the sample size and this corresponds to statistical theory.
4. Figure (1) illustrates the behaviour of a function (pmf) for model (1) and Figure (2) shows the behaviour of (cdf) for model (1).

Table 4. Integrated mean square error (IMSE) for the survival function in case of prior data unconflicted

Model							
	Lower					upper	
	10					12	
				k	n		
	Lower	Upper					
1	0.01	0.1	0.1	5	10	0.019722	0.010686
				10	20	0.010539	0.007429
				20	40	0.008633	0.007207
Best				Robust Bayesian Estimator			

Model	Lower					upper	
	10					12	
	Lower	Upper		k	n		
2	0.02	0.2	0.2	5	10	0.036682	0.017008
				10	20	0.023583	0.014465
				20	40	0.012673	0.009412
Best				Robust Bayesian Estimator			
3	0.03	0.3	0.3	5	10	0.051084	0.021409
				10	20	0.031874	0.017438
				20	40	0.019501	0.012958
Best				Robust Bayesian Estimator			
4	0.04	0.4	0.4	2	10	0.070779	0.028195
				4	20	0.047031	0.023959
				8	40	0.026544	0.016323
Best				Robust Bayesian Estimator			
5	0.05	0.5	0.5	2	10	0.097546	0.037579
				4	20	0.057222	0.026510
				8	40	0.031539	0.016851
Best				Robust Bayesian Estimator			
6	0.05	0.4	0.4	2	10	0.068898	0.028235
				4	20	0.044183	0.022412
				8	40	0.008372	0.006161
Best				Robust Bayesian Estimator			





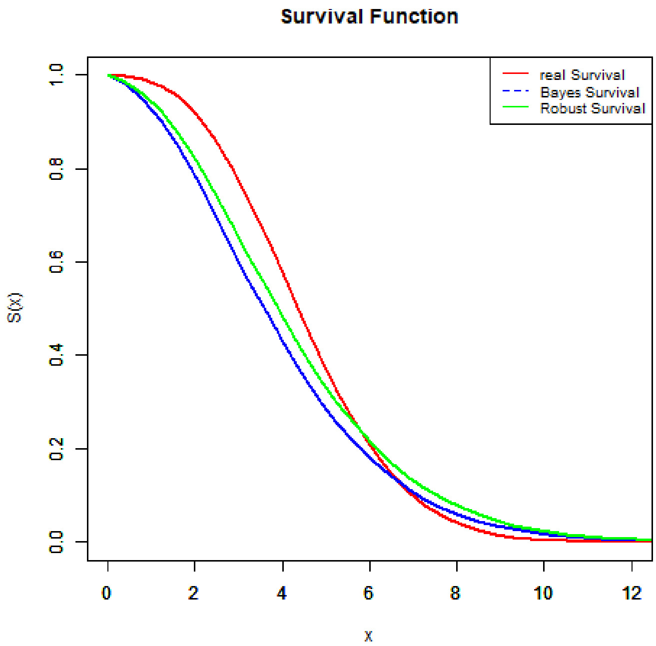


Figure 3. Model (1) for the survival function in the case of prior data unconflicted

From Table 4, which shows (IMSE) to compare the estimation methods for the survival function in the case of unconflicted prior data, the simulation results showed the following:

1. The simulation results showed that the robust Bayes estimator for the survival function is better than the regular Bayes estimator by using (IMSE) as a criterion for comparing.
2. The simulation results showed that the best model is model (1).
3. The simulation results showed that (IMSE) decreases in case of increasing the sample size and this corresponds to statistical theory.
4. Figure 3 illustrates the behavior of a survival function for model (1).

Table 5. Integrated mean square error (IMSE) for the parameter (P)in case of prior data conflict

Model								
	lower					upper		
	2					4		
	Lower	upper	P	K	n			Best
1	0.3	0.5	0.5	5	10	0.006121	0.005225	Robust Bayesian Estimator
				10	20	0.002207	0.002041	
				20	40	0.000832	0.000802	
2	0.4	0.6	0.6	5	10	0.005728	0.004898	
				10	20	0.002369	0.002206	
				20	40	0.000770	0.000743	
Model								
	5					7		
	Lower	upper	P	k	n			Best
3	0.3	0.5	0.5	2	10	0.005229	0.003931	Robust Bayesian
				4	20	0.002113	0.001837	

Model								
	lower					upper		
	2					4		
			P	K	n			Best Estimator
	Lower	upper						
				8	40	0.000777	0.000725	
4	0.4	0.6	0.6	2	10	0.005108	0.004025	
				4	20	0.002009	0.001758	
				8	40	0.000744	0.000699	
Model								
	10					12		
			P	k	n			Best
	Lower	upper						
5	0.3	0.5	0.5	1	10	0.005558	0.003024	Robust Bayesian Estimator
				2	20	0.002085	0.001445	
				4	40	0.000795	0.000649	
6	0.4	0.6	0.6	1	10	0.005402	0.003077	
				2	20	0.002062	0.001445	
				4	40	0.000791	0.000651	

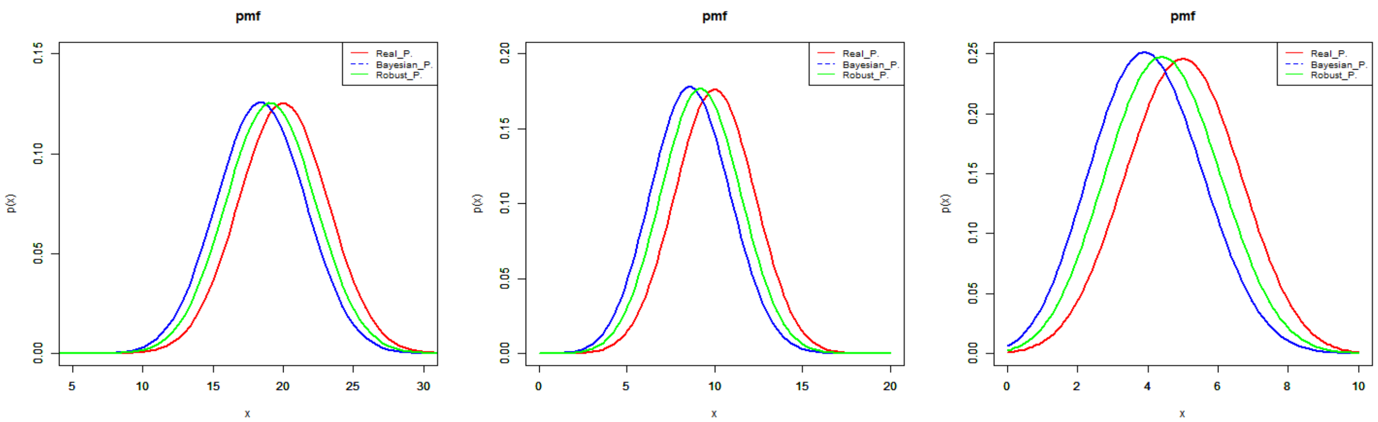


Figure 4. Model (5) for the (pmf) in the case of prior data conflict

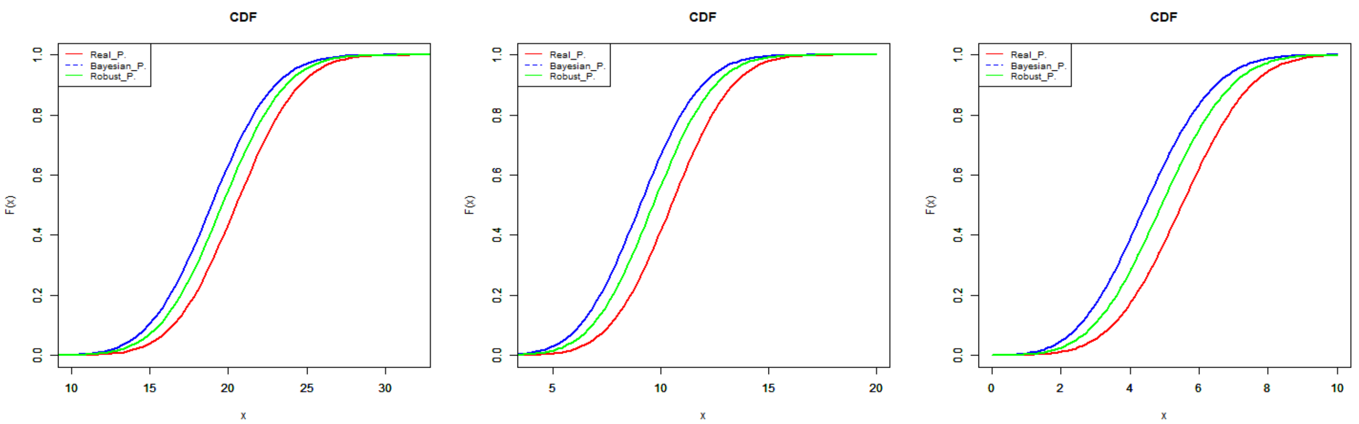


Figure 5. Model (5) for the (cdf) in the case of prior data conflict

From Table (5), which shows (IMSE) to compare the estimation methods for parameter (p) in the case of prior data conflict, the simulation results showed the following:

1. The simulation results showed that the robust Bayes estimator for parameter (P) is better than the regular Bayes estimator by using IMSE as a criterion for comparing.
2. The simulation results showed that the best model is model (5).
3. 3. The simulation results showed that IMSE decreases in case of increasing the sample size and this corresponds to statistical theory.
4. Figure 4 illustrates the behavior of a function (pmf) for model (5) and Figure 5 shows the behavior of (cdf) for model (5).

Table 6. Integrated mean square error (IMSE) for the survival function in case of prior data conflict

Model								
	Lower					upper		
	2					4		
	Lower	Upper		k	n			Best
1	0.3	0.5	0.5	5	10	0.034540	0.028961	Robust Bayesian Estimator
				10	20	0.024535	0.022408	
				20	40	0.016549	0.015884	
2	0.4	0.6	0.6	5	10	0.032064	0.026952	
				10	20	0.022817	0.021020	
				20	40	0.016745	0.016075	
Model								
	Lower					upper		
	5					7		
	Lower	Upper		k	n			Best
3	0.3	0.5	0.5	2	10	0.027907	0.020847	Robust Bayesian Estimator
				4	20	0.020612	0.017634	
				8	40	0.017146	0.015815	
4	0.4	0.6	0.6	2	10	0.026458	0.019954	
				4	20	0.020877	0.017867	
				8	40	0.016666	0.015492	
Model								
	Lower					upper		
	10					12		
	Lower	Upper		k	n			Best
5	0.3	0.5	0.5	1	10	0.028392	0.015736	Robust Bayesian Estimator
				2	20	0.023755	0.014419	
				4	40	0.016926	0.013868	
6	0.4	0.6	0.6	1	10	0.027082	0.014751	
				2	20	0.021034	0.014121	
				4	40	0.016913	0.013560	

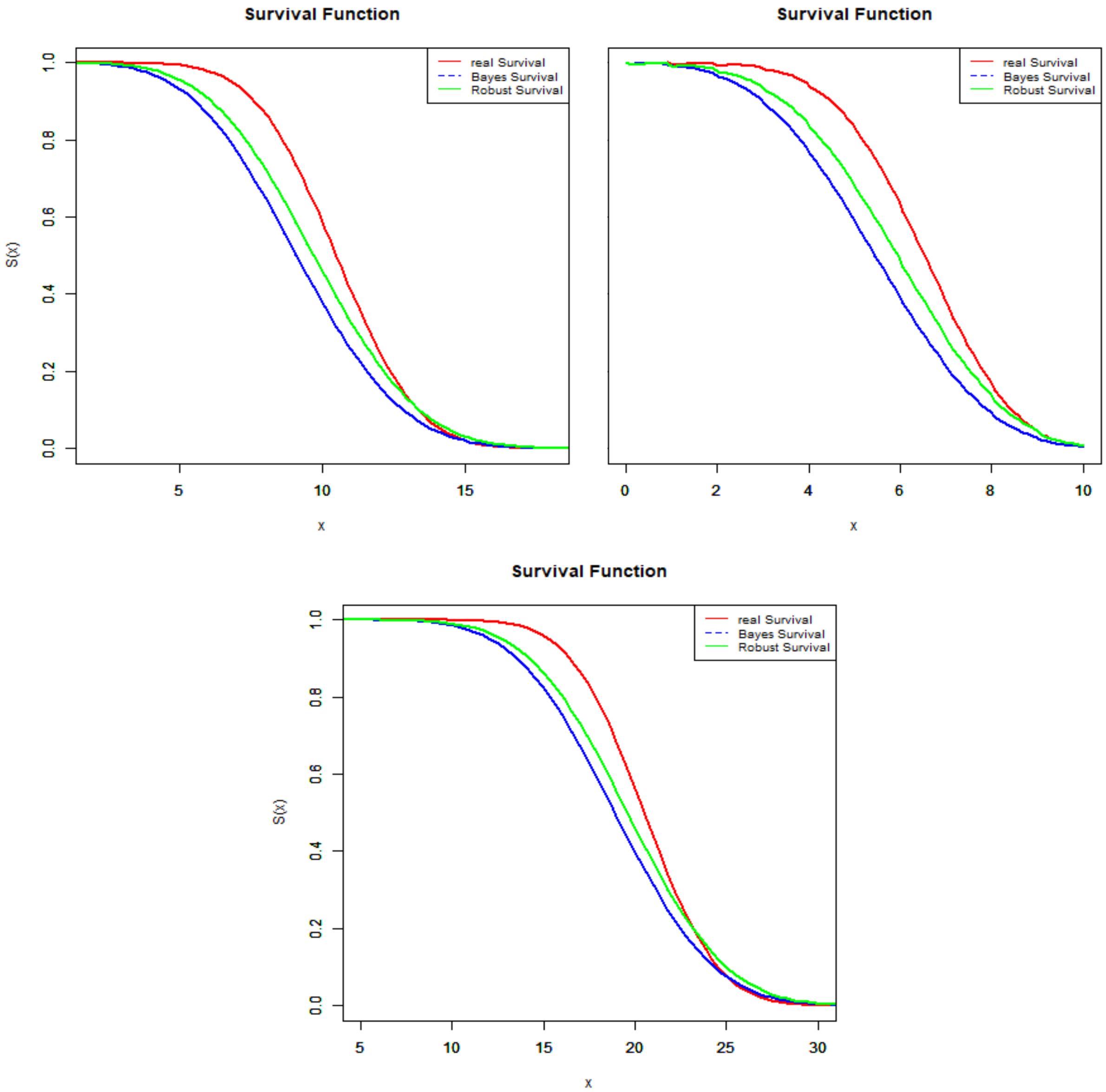


Figure 6. Model (6) for the survival function in the case of prior data conflict

From Table 6, which shows (IMSE) to compare the estimation methods for the survival function in the case of prior data conflict, the simulation results showed the following:

1. The simulation results showed that the robust Bayes estimator for the survival function is better than the regular Bayes estimator by using (IMSE) as a criterion for comparing.
2. The simulation results showed that the best model is model (6).
3. The simulation results showed that (IMSE) decreases in case of increasing the sample size and this corresponds to statistical theory.
4. Figure 6 illustrates the behaviour of a survival function for model (6).

3. Application side

From the experimental side, the results showed that in the case of prior data conflict, the robust Bayes method is best by using the Integrated mean square error (IMSE) as a criterion for comparing.

4. Describing the real data

Mortality data for patients with breast cancer were collected from Yarmouk Teaching Hospital for the period from 2010 to 2017. The data collected are as follows:

Table7. Real data

Year	2010	2011	2012	2013	2014	2015	2016	2017
X	3	4	2	3	2	4	1	0

5. Goodness of fit

Easy fit program was used for goodness of fit based on real data and we found that it is distributed Binomial distribution as shown below:

Table 8. Kolmogorov-Smirnov

Sample Size	8
Statistic	0.37884
P-Value	0.1536
Rank	1

Table 9. Estimation of parameter (P) in case of prior data conflict

		Lower	Upper	
		4	6	
Lower	Upper		s.d prior	s.d posterior
0.3	0.6	0.603595	0.042	0.006646

Table 10. Estimation of the survival function in case of prior data conflict

Lower	Upper	x	$\hat{S}_{rob}(x)$
0.3	0.6	0	1.0000000000
		1	0.9999999805
		2	0.9999996926
		3	0.9999974868
		4	0.9999858386

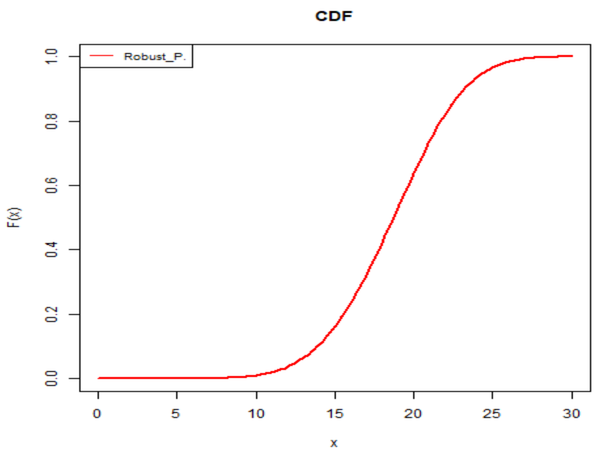


Figure 7. the (cdf) for real data in the case of prior data conflict

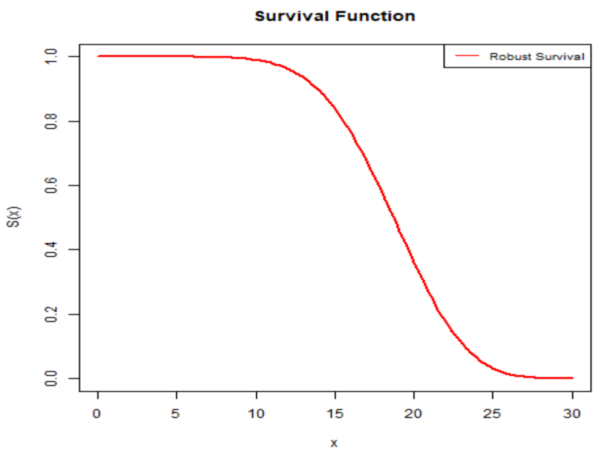


Figure 8. the survival function for real data in the case of prior data conflict

From Table 8, the real data follow the distribution of Binomial, and from the experimental side the simulation results showed that the method of the robust Bayes is better in the case of prior data conflict. So, this method was used to estimate the parameter (P) and survival function as shown in Table 9 and 10. Figure 8 shows the behavior of the survival function as decreasing as the value of (X) increases.

6. Conclusions

1. The simulation results showed that the robust Bayes method is best for estimating parameter (P) in the case of unconflicted prior data and in the case of prior data conflict using the IMSE comparison criterion.
2. The simulation results showed that the robust Bayes method is best for estimating the survival function in the case of unconflicted prior data and in the case of prior data conflict using the IMSE.
3. The simulation results showed that if the sample size increases, the integrated mean square error (IMSE) decreases and this corresponds to the statistical theory.
4. The applied side has shown that the data collected from Yarmouk Teaching Hospital follow the Binomial distribution.
5. The applied side has shown that the survival function is decreasing and this is consistent with the statistical theory for the survival function analysis.

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