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The robust estimators of reliability function for some compound distribution

Entsar Arebe Al-Doori[1], Worood Basim Noor-Abihea[2]
[1] College of Administration and Economics, University of Baghdad
[2]Dijlah University College

ABSTRACT

This paper investigates the use of the POT technique for natural phenomena weather. Their data contain extreme values (high or rare values) by applying two methods. The first method is based on the estimate using the order statistical, and the second method is based on likelihood ratio. The comparative model is based on MSE of the estimated parameters, the probability density function for each distribution of the mixed distributions, and their reliability function. The POT sample has shown significant improvement in data and its superiority, especially in simple exponential distribution.

Keywords: Mixed Pareto, Mixed exponential, Sample peaks above threshold, Cross-entropy algorithm

Corresponding Author:

Entsar arebe Al-Doori
College of Administration and Economics
University of Baghdad
E-mail: entsar.arebe@gmail.com

1. Introduction

The single probability of continuous or discrete distributions are important statistical tools for studying the phenomenon and forecasting its future behavior. But, we may face some problems such as the inability of these distributions to apply to some phenomena such as the natural phenomena (rain, earthquakes, etc.). We resort to the use of merging distributions to obtain mixed distributions more suitable for these phenomena, especially for nonhomogeneous societies. There is a possibility to mix two or more of the same or different kinds. There is some data of the phenomenon of natural need to have a certain technique to deal with the cases of extremism in which a sampling technique was used above the threshold POT to treat the extremism of these data and be applied to the distributions. The most suitable distributions are mixed Pareto and mixed exponential distributions. These two mixed distributions are more suitable for weather phenomena. The cross-entropy algorithm was used as a method for estimating the parameters of the mixed distributions using two methods; the first one is the cross-entropy using the statistical order and the second is the cross-entropy using the statistical order and the likelihood ratio. The third method is based on previous two methods that take a certain path of view values while the suggested method takes the full path with the use of the POT sample.
In 2002, Robert studied the subject of mixed distributions related to demand modeling on the communication network and its simple exponential distribution model with one parameter. This work extends to the modeling technique using mixed exponential distributions [1]. Two methods of estimation have used including nonlinear programming and EM algorithm for more robust estimators based on distribution of data for large samples. In 2007, the authors of [2], introduced the applications of modern innovations to analyze the frequency of the floods for series of the regional earthquakes. The Gumble distribution was used, which distributed the general extreme values of the AM series, Pareto and exponential distribution POT, and compared them with the L-Moment. In 2010, the auhors of [3], studied the atmosphere through atmospheric monitoring of the seasonal rainfall or extreme precipitations using the annual maximum (AM) and peak over

threshold (POT) samples.Then, it has compared the appropriate distributions for the samples through the boundaries of confidence. In 2011, the auhors of [4], examined the comparison of the robustness and the reliability of six probability distributions and also used the annual maximum (AM) and POT peak over threshold which are suitable for the distributions in the maximum likelihood method for observing extreme or major rainfall in eight different climatic conditions. In 2014, , the auhors of [5] conducted an analytical study of the annual maximum (AM) and peaks-over-threshold (POT) studies and comparisons by estimating three different methods (maximum likelihood, logarithm likelihood, and moment methods) for six common probability distributions.

This research work adopts POT technique for weather. It uses dual methods; the first one is the estimate using the order statistical and the second method is likelihood ratio. The comparative model is based on MSE of the estimated parameters and the probability density function for each distribution of the mixed distributions as well as their reliability function. The POT sample has given substantial improvement in terms of data and its superiority, particularly in simple exponential distribution.

2. Theoretical Side

This section includes the following subsections:

2.1. Model POT

The POT model depends on certain probability distributions (ESI, Pareto, Poisson, Binomial, etc.). Distributions for modeling the annual number of events are above the threshold or distributions of values beyond the threshold. The first condition of this method is the events must be independent. The second condition is to be a standard for successive summits of three times. The smallest value between successive peaks must be two to three times the value of the first peaks [5].

2.2. Threshold selection

The threshold value defined by (Langbein, 1949) is a selection equal to the lowest event of the AM. At least, one event a year is within available data. Madsen in 1993, used the k-frequency and data properties (mean arithmetic value and standard deviation) based on $x_0 = \sigma_x k + \mu_x$, where x_0 is the threshold value, and k is the lowest value of the three times when we choose the POT3. The threshold was also defined as the average peaks chosen from one year (POT_1, POT_3, POT_5 ...) [5].

$$x_0 = x_1 + x$$
$$y = x - x_0$$

2.3. Mixture probability distribution

The distribution of the mixture is called "mixture distribution" or "compound distribution" based on multiplication or compilation of heterogeneous components of statistical distributions. This distribution occurs when the withdrawal of a sample of heterogeneous communities with different or same function probability with different parameters for each community. It requires statistical tests to determine if the extent of the mixed distribution returns to the same family or not, rather than a single distribution. Mixed distributions occur when the society is not homogeneous. For instance, a group contains partial societies (sp_1, sp_2, ..., sp_k) mixed with (p_1, p_2, ..., p_k) groups. The function of failure time in each partial society has a probability density function $f(t, \lambda_i)$ i=1,2,..,k [6] .

$$G(t, \lambda_i) = \sum_{i=1}^{k} p_i \, f(t, \lambda_i). \qquad (2)$$

Here, p_i represents the proportion of the micro-society. In this research, we will highlight two mixtures in which a POT sample is used:

2.3.1 Mixture Exponential Distribution

It is a mixed probabilistic model applied to specific regions, called mixed exponential distribution (MEXP). The probability density and the cumulative function are represented by [6]:

Where,

λ_j is the measurement parameter for the exponential distribution of j

p_j represents the parameter of the mixing ratio of j

$0 < p_j < 1 \qquad \sum_{j=1}^{k} p_j = 1 \qquad \int f(x)\, d(x) = 1$

2.3.2 Mixture Pareto distribution

It is as well a mixed probabilistic model applied to specific regions. It can be called the general mixed Pareto distribution (MGP). The probability density and cumulative, function is [4]:

$$R(t) = \sum_{j=1}^{k} p_j\, R_j(t) \qquad (6)$$

3. Reliability criteria

The reliability criterion was determined to assess the probability of extreme events by probability model and high-frequency observations. We need to extract the FF, which is the basis for the division and presentation of the sample by Garcon (1995). D is a regional data set with length N, and D_i, is the time series in location i. By calculating the FF standard, we can classify it into tracking steps [4].

1. Each D_i is divided into two successful subsamples equal to N / 2 length$(x_1, \dots, x_{\frac{N}{2}})$, $\left(X_N, \dots, X_2, X_{\frac{N+2}{2}} \right)$.
2. Suppose that $m_1 = \left[x_1, \dots, x_{\frac{N}{2}}\right]$, $m_2 = \left[x_{\frac{N+2}{2}}, \dots, x_N\right]$.
3. Adopt ing two cumulative probability density functions cdf F_2, F_1 for the same probability model using each sub-sample under the hypothesis of probability random variables (m_2, m_1).

The authors suggests using the following method to calculate the value of the probabilistic probability function:

$$FF = max(maxF_1\,, maxF_2)$$
$$R = 1 - FF. \qquad (7)$$

In addition, using the criterion of reliability without dividing the sample calculated by:

$$R\_all = 1 - maxF(t) \qquad (8)$$

Where,

F is cumulative density function, R is reliability of the divided sample and R_all stands for reliability for the complete sample.

4. Methods of Estimation(M.E)

There are several ways to estimate the parameters for each distribution of the two distributions (i.e., the Exponential mixture and the Pareto mixture). The aim is to get the best estimators, and we will detail some of them in next sections.

5. Algorithm of the cross-entropy

It is the algorithm used by the Entropy method and developed by the Monte-Carlo approach to multi-continuity, multi-radical improvement and the importance of sampling. This method can used in the simulation of rare events requiring probabilities as in reliability analysis, queue models, performance analysis of communication systems, and improvement problems for the sales representative and maximum values. It is also called minimal cross-entropy or kullback-leibler. The aim of this algorithm is to find the best solution to the problem by using simulation, and it has two steps [7]:
1. Generating a random sample based on specific distributions or mechanisms.
2. Updating the parameters based on random data through the best sample for duplicates.

6. The C.E method benefits

Their benefits are:
1. Optimal estimates with both continuous and discrete variables.
2. Initially, a set of candidate solutions was generated, and the algorithm was optimized.
3. Using a range of candidate solutions, treatment can be performed.
4. Mixed problems can be solved by extremism, multi-goal, and multivariate without the use of derivative information.
5. Nonlinear functions can be used.
6. It does not require the help of a physical expert.

The algorithm also referred to the associated stochastic problem (ASP), which is as follows [8]:

$$l(\gamma) = p(s($$

Where H (x) is a real function called Shannon entropy where it can be found in case of a discrete distribution

$$H(x) = -\sum$$

Or in the case of a continuous distribution,
$H(x) = -\int f(x) lnf(x)dx.$
In this research, the equivalent function was used,
$H(x) = I[s(x) \geq \gamma] = 1(s(x) \geq \gamma),$
where $I[s(x) \geq \gamma]$ is the indicator function
Which takes a path of x_i values when greater or equal to γ.
There are several methods of estimating the studied models and two methods were chosen:

6.1. Cross-Entropy Algorithm using order statistic (C.E1)

S (x) is the special case values of x that approximate the value of γ^t [7] :

$$S(x) = \max$$

$\gamma^t = S_{(1-\rho)n}$.

The sample is called quantile, which is similar to the method of order statistic
$\gamma^* = max[S(x)]$,
and that $\gamma^t = \gamma^*$is followed by a stop, i.e. the best results were obtained

$$\hat{v}_t = \max \frac{1}{N}$$

And taking the first derivative on estimates of parameters. There is an optional step to update the parameters called (Smooth update)

$$\hat{v}^* = \alpha \hat{v}_t + (1-\alpha)\hat{v}_{t-1} \qquad 0.3 < \alpha < 0.9 \qquad (11)$$

$$\hat{v}_t = \max \frac{1}{N}$$

In this context, the following formulas for Mixture Exponential Distribution and Mixture Pareto Distribution(MGP) can be adopted:

1. Mixture Exponential Distribution(MEXP)

$$\hat{v}_t = \max \frac{1}{N_s} \sum_{i=1}^{N_s} I[s(x) \geq \gamma_t] ln \left[\sum_{j=1}^{k} \pi_j \frac{1}{\lambda_j} exp\left(-\frac{x}{\lambda_j}\right) \right]$$

$$\hat{v}_t = \begin{cases} \hat{\lambda} = \dfrac{\sum_{i=1}^{N_s} I[s(x) \geq \gamma_t] x_{ij}}{\sum_{i=1}^{N_s} I[s(x) \geq \gamma_t]} & (12) \\ \hat{\pi} = \dfrac{\sum_{i=1}^{N_s} I[s(x) \geq \gamma_t]}{N_S} & (13) \end{cases}$$

$$R\_all = P_1(1 - maxF)_1 + P_2(1 - maxF)_2 + (1 - P_1 - P_2)(1 - maxF)_3 \qquad (14)$$

$$R = P_1(1 - FF_1) + P_2(1 - FF_2) + (1 - P_1 - P_2)(1 - FF_3) \quad \dots \; (15).$$

2. Mixture Pareto Distribution(MGP)

$$\hat{v}_t = \max \frac{1}{N_s} \sum_{i=1}^{N_s} I[s(x) \geq \gamma_t] \ln f(x_i\,; v_{t-1})$$

$$\hat{v}_t = \max \frac{1}{N_s} \left[\sum_{i=1}^{N_s} \sum_{j=1}^{k} I[s(x) \geq \gamma_t] \, ln\pi_j - \sum_{i=1}^{N_s} \sum_{j=1}^{k} I[s(x) \geq \gamma_t] \, ln\lambda_j - \left(\frac{1+\varepsilon_j}{\varepsilon_j}\right) ln \left[1 + \varepsilon_j \left(\frac{x_i}{\lambda_j}\right)\right] \right] \dots (16)$$

[1]

$$\hat{v}_t = \begin{cases} \hat{\lambda} = \dfrac{\sum_{i=1}^{N_S} I[s(x) \geq \gamma_t](1+\varepsilon_j) x_i}{\sum_{i=1}^{N_S} I[s(x) \geq \gamma_t]\left[1+\varepsilon_j\left(\frac{x_i}{\lambda_j}\right)\right]} & (17) \\ \hat{\varepsilon} = \dfrac{\sum_{i=1}^{N_S} I[s(x) \geq \gamma_t] \frac{1}{\varepsilon_j} ln\left[1+\varepsilon_j\left(\frac{x_i}{\lambda_j}\right)\right] + \sum_{i=1}^{N_S} I[s(x) \geq \gamma_t] \frac{\left(\frac{x_i}{\lambda_j}\right)}{\left[1+\varepsilon_j\left(\frac{x_i}{\lambda_j}\right)\right]}}{\sum_{i=1}^{N_S} I[s(x) \geq \gamma_t] \frac{\left(\frac{x_i}{\lambda_j}\right)}{\left[1+\varepsilon_j\left(\frac{x_i}{\lambda_j}\right)\right]}} & (18) \\ \hat{\pi} = \dfrac{\sum_{i=1}^{N_S} I[s(x) \geq \gamma_t]}{N_S} & (19) \end{cases}$$

$R\_all = P_1(1 - maxF)_1 + P_2(1 - maxF)_2 + (1 - P_1 - P_2)(1 - maxF)_2$ (20)

$R = P_1(1 -$

6.2. Cross-entropy algorithm using order statistic and likelihood ratio (C.E2)

This method is similar to the previous method, but the estimations here are multiplied by a function called probability ratio and a function consisting of dividing the probability density function by another probability function for the same distribution, but having another sample different from the numerator that can be found through the following steps [8]:

1. $l(\gamma) = p(s(x) \geq \gamma) = \int_{\gamma}^{\infty} f(x,u)dx.$ (22)

Where u is a vector of the original distribution parameters, and if the probability is too small less than 〖10〗^ (- 5)

2. In order to estimate $l(\gamma)$ the best method is to use a change of a measure of the density function in the following manner:

$$g^*(x) = \frac{I[s(x) \geq \gamma]f(x,u)}{l(\gamma)} \ . \quad (23)$$

Using the "change of measure", we get the

$\frac{I[s(x)\geq\gamma]f(x,u)}{g^*(x)} = l,$

and also appreciation $\hat{l}$ proves to be unbiased

$$\hat{l} = \frac{1}{N}\sum_{i=1}^{N} \frac{I[$$

3. The likelihood ratio that we need to estimate the parameters

$w(x,u,\hat{v}_t) = \frac{f(x;u)}{f(x;\hat{v}_t)}.$

$f(x;u)$: The probability density function of the original distribution

$f(x;\hat{v}_t)$: The probability density function $g^*(x)$or any other source of parameter change that can be extracted in several ways for the N_s . The sample is termed as an important sample and its symbol is (IS)

As for S_i, the statistical order is called for each i , $i = 1,2,3, \dots, n$.

$S(x) = \max$

S (x) represents some of the values of variable x for all i values after ranking them from smallest to largest and then calculating:

$\gamma^t = S_{(1-\rho)n}.$

The sample is called quantile, which is similar to the method of order statistic $\gamma^* = max[S(x)].$

And that $\gamma^t = \gamma^*$is followed by a stop where the best results were obtained.

$\rho = 0.1 or\ 0.01\ all\ \widehat{\gamma_1}, \widehat{\gamma_2}, \dots, \widehat{\gamma^*}\ for\ function\ p.d.f[f(.,v_0), f(.,\hat{v}_1), \dots, f(.,\widehat{v^*})],$

$f(.,v^*)$The best probability density function is v_t which represents the vector of parameters.

We find the best estimation of the parameters using the likelihood ratio and after finding a formula (8)

$$\hat{v}_t = \max \frac{1}{N}$$

In the case of the above formula, we need the estimates extracted in the first method of each distribution with the estimated multiplication in $w(x,u,\hat{v}_t)$

or

$$\hat{v}_t = \frac{\sum_{i=1}^{N} I[}{\sum_{i=1}^{N}}$$

In the case of compounds, we use the formula:

$$\hat{v}_{t,j} = \frac{\sum_{i=1}^{N} I[s(x) \geq \gamma_t] w(x_i, u, \hat{v}_{t-1}) x_{ij}}{\sum_{i=1}^{N} I[s(x) \geq \gamma_t] w(x_i, u, \hat{v}_{t-1})}. \qquad (27)$$

To update the vector, the parameter has called "smoothed updating",

$$\hat{v}^* = \alpha \hat{v}_t + (1-\alpha)\hat{v}_{t-1} \qquad 0.3 < \alpha < 0.9 \qquad (28)$$

When applying the algorithm above to the mixed distributions, we get the follwing:

1. Mixture exponential distribution(MEXP)

$$l(\gamma) = \int_{\gamma}^{\infty} \sum_{j=1}^{k} \pi_j \frac{1}{\lambda_j} exp\left(-\frac{x}{\lambda_j}\right) dx$$

$$= \sum_{j=1}^{k} \pi_j \, exp\left(-\frac{\gamma}{\lambda_j}\right), \qquad (29)$$

$$g^*(x) = I[s(x) \geq \gamma] \frac{\sum_{j=1}^{k} \pi_j \frac{1}{\lambda_j} exp\left(-\frac{x}{\lambda_j}\right)}{\sum_{j=1}^{k} \pi_j \, exp\left(-\frac{\gamma}{\lambda_j}\right)},$$

$$w(x; u, v) = \frac{\sum_{j=1}^{k} \pi_j \frac{1}{\lambda_j} exp\left(-\frac{x}{\lambda_j}\right)}{\frac{\sum_{j=1}^{k} \pi_j \frac{1}{\lambda_j} exp\left(-\frac{x}{v_j}\right)}{\sum_{j=1}^{k} \pi_j \, exp\left(-\frac{\gamma}{v_j}\right)}},$$

$$\hat{v}_t = \max \frac{1}{N}$$

$$R\_all = P_1($$

$$R = P_1(1 - FF_1) + P_2(1 - FF_2) + (1 - P_1 - P_2)(1 - FF_3). \qquad \ldots(32)$$

2. Mixture Pareto distribution (MGP)

$$l(\gamma) = \int_{\gamma}^{\infty}$$

$$= \sum_{j=1}^{k} \pi_j$$

$$g^*(x) = I[s(x) \geq \gamma] \frac{\sum_{j=1}^{k} \pi_j \frac{1}{\lambda_j}\left[1 + \varepsilon_j\left(\frac{x}{\lambda_j}\right)\right]^{-\left(\frac{1+\varepsilon_j}{\varepsilon_j}\right)}}{\sum_{j=1}^{k} \pi_j \left[1 + \varepsilon_j\left(\frac{\gamma}{\lambda_j}\right)\right]^{-\left(\frac{1}{\varepsilon_j}\right)}}.$$

$$w(x; u, v) = \frac{\sum_{j=1}^{k} \pi_j \frac{1}{\lambda_j}\left[1 + \varepsilon_j\left(\frac{x}{\lambda_j}\right)\right]^{-\left(\frac{1+\varepsilon_j}{\varepsilon_j}\right)}}{\frac{\sum_{j=1}^{k} \pi_j \frac{1}{v_{1,j}}\left[1 + v_{2,j}\left(\frac{x}{v_{1,j}}\right)\right]^{-\left(\frac{1+v_{2,j}}{v_{2,j}}\right)}}{\sum_{j=1}^{k} \pi_j \left[1 + v_{2,j}\left(\frac{\gamma}{v_{1,j}}\right)\right]^{-\left(\frac{1}{v_{2,j}}\right)}}}$$

$\hat{v}_t = \max\frac{1}{N}$

$$R_{all} = P_1(1-maxF)_1 + P_2(1-maxF)_2 + (1-P_1-P_2)(1-maxF)_3 \ , \qquad (35)$$

$R = P_1(1 -$

6.2.3. Proposed cross-entropy (PC.E2)

This suggestion uses the same steps as the seconed method of algorithm Cross-Entropy(C.E2) , but without the function, I [s (x) γ]. So, the entropy does not take a given path but takes all the values and the variable with a likelihood ratio.

7. Simulation concept

Simulation is known as the best way to solve many problems that are difficult to solve in real life, including complex mathematical processes, or the difficulty of providing real data when studying a particular phenomenon [9]. There are also experiments that cannot be conducted in real life. It is difficult to observe and draw the different changes and interactions in the case. It is best to describe these cases in a way similar to the real-life by building a model of the problem in question and implementing different experiences of that model to bring us a convergence of real life, which helps in the connection to the goal of the research. The simulation experience has been carried out according to MATLAB program included as mentioned for distribution (Mixed Exponential, Mixed Pareto). This experiment was implemented for distribution by the following steps:

1. Selecting initial default values for parameters by distribution.

Table (1). The number of models and default values of the parameters.

Model									
MEXP	**I**	**0.5**	**1**	**0.8**	———	———	———	**0.35**	**0.1**
	II	**1.5**	**2**	**2.5**	———	———	———	**0.25**	**0.1**
	III	**2**	**4**	**5.5**	———	———	———	**0.25**	**0.1**
	V	**0.7**	**1.5**	**2.5**	———	———	———	**0.35**	**0.1**
MGP	**I**	**0.4**	**0.5**	**0.8**	**0.5**	**0.3**	**0.05**	**0.15**	**0.1**
	II	**1.5**	**2**	**2.5**	**0.6**	**0.7**	**0.2**	**0.25**	**0.1**
	III	**2**	**4**	**5.5**	**3**	**5**	**7**	**0.25**	**0.1**
	V	**0.7**	**1.5**	**2.5**	**0.8**	**1.4**	**2**	**0.35**	**0.1**

2. Selection of four different sizes of small, medium and large samples (n = 25, n = 50, n = 75, n = 100).
3. In order to obtain high accuracy and homogeneity of parameter capabilities, each experiment (Replications = 1000) was repeated once.
4. The parameters are estimated according to the distribution based on the methods studied in the theoretical side, which are defined in the tables according to the following symbols. The adopted method is according to the cross-entropy algorithm (C.E).
5. Comparison of the estimation methods according to the sizes of the samples and the different models of the default parameters mentioned. The estimations that have the lowest value for the comparison scale used in this search are the best. Statistical Mean square Error (MSE) is adopted to compare the parameters according to the following:

$$\text{MSE}\left(\hat{\lambda}\right) = \frac{1}{n\text{T}}\sum_{\text{i}=1}^{\text{T}}\left(\hat{\lambda}_{\text{i}} - \lambda\right)^2 \ , \qquad (37)$$

$$\text{MSE}\left(\widehat{\text{pdf}}\right) = \frac{1}{\text{T}}\sum_{\text{j}=1}^{\text{T}}\left[\frac{1}{\text{n}}\sum_{\text{i}=1}^{\text{n}}(\widehat{\text{pdf}}_{\text{ij}} - \text{pdf}_{\text{ij}})^2\ \right] , \qquad (38)$$

$$\mathrm{MSE}\left(\widehat{\mathrm{R}}\right) = \frac{1}{\mathrm{T}} \sum_{j=1}^{\mathrm{T}} \sum_{j=1}^{\mathrm{T}} \left[\frac{1}{\mathrm{n}} \sum_{i=1}^{\mathrm{n}} \left(\widehat{\mathrm{R}}_{\mathrm{ij}} - \mathrm{R}_{\mathrm{ij}}\right)^2 \right]. \qquad (39)$$

8. Results of the simulation experiment

Table 1 contains the initial parameters for estimating the parameters using distributions (MEXP, MGP). Table (2) explains MSE estimation of the distribution parameters (MEXP, MGP) and their probability density function for the two models (I, II) shown in Table (1).
Table (3) depicts MSE estimation of the distribution parameters (MEXP, MGP) and their probability density function for the models (V, III) given in Table (1). Table (4) illustrates MSE estimation of the reliability function for tow distribution (MEXP, MGP) (III) in Table (1).

Table (2). MSE estimation of the distribution parameters (MEXP, MGP) and their probability density function for the two models (I, II) shown in Table (1)

model	n	M.E	Dist.	MSE(λ_1)	MSE(λ_2)	MSE(λ_3)	MSE(ε_1)	MSE(ε_2)	MSE(ε_3)	MSE(pdf)
I	25	C.E1	MGP	1.02860e-05	6.68753e-05	3.13101e-04	6.36626e-05	3.85807e-05	2.4240e-05	9.1792e-06
			MEXP	7. 8952e-04	7. 8950e-04	7. 8956e-04	—	—	—	9. 5397e-06
		C.E2	MGP	6.68418e-06	2.02946e-05	3.13101e-04	7.05389e-05	5.31645e-05	2.42409e-05	9.292083e-06
			MEXP	7. 8958e-04	7. 8953e-04	7. 8965e-04	—	—	—	9. 53968e-06
		PC.E2	MGP	5.33619e-05	1.57294e-04	0.00277145	1.06252863	0.83214967	0.35734916	1.189134e-04
			MEXP	5. 730333	5. 730332	5. 730329	—	—	—	1.193312e-04
	50	C.E1	MGP	3. 9087e-06	1. 87773e-05	1. 9938e-04	3.57856e-05	2. 2427e-05	1. 1906e-05	5.18137e-04
			MEXP	4. 4275e-04	4. 4269e-04	4. 4267e-04	—	—	—	5.173341e-04
		C.E2	MGP	3. 9085e-06	1. 87772e-05	1. 9937e-04	3. 7855e-05	2. 2425e-05	1. 9055e-05	5.180065e-04
			MEXP	4. 4272e-04	4. 4268e-04	4. 4266e-04	—	—	—	5.173340e-04
		PC.E2	MGP	2. 2038e-05	7. 1261e-05	0. 15263956	2.56617534	2.05356830	0.94170706	0.006222413
			MEXP	14.517950	14. 7951752	14. 7904665	—	—	—	0.006221781
	75	C.E1	MGP	2. 9258e-06	6. 9728e-06	1. 6552e-04	2.65811e-05	2.79835e-05	9.30911e-06	7.592646e-04
			MEXP	4. 9554e-04	4. 2964e-04	4. 2956e-04	—	—	—	7.587349e-04
		C.E2	MGP	2. 9257e-06	6. 79727e-06	1. 6550e-04	2. 5801e-05	2. 9834e-05	9. 910e-06	7.591993e-04
			MEXP	4. 886e-04	4. 889e-04	4. 891e-04	—	—	—	7.587347e-04
		PC.E2	MGP	1.78644e-05	5.36670e-05	0.00115376	4.97388032	4.09073525	2.03119103	0.009114241
			MEXP	28.440238	28. 4402376	28. 4402359	—	—	—	0.009113841
	100	C.E1	MGP	1. 1949e-06	5. 5538e-06	9. 8464e-05	1. 4613e-05	1. 8561e-05	6. 2344e-06	1.425620e-04
			MEXP	2. 4498e-04	2. 64478e-04	2. 4466e-04	—	—	—	1.422150e-04
		C.E2	MGP	1. 2194e-06	5. 25536e-06	9. 8462e-05	1. 4612e-05	1. 8560e-05	6. 2343e-06	1.4252e-04
			MEXP	2. 4496e-04	2. 4495e-04	2. 4497e-04	—	—	—	1.4220e-04
		PC.E2	MGP	1.33035e-05	3.97355e-05	8.11056e-04	6.52126959	5.35487139	2.68363225	0.00171351
			MEXP	33.5806951	33. 51959	33. 9593549	—	—	—	0.00171341
II	25	C.E1	MGP	3. 2600e-05	8. 16875e-05	3. 3887e-04	4. 5605e-05	3. 2557e-05	2.44625e-05	9.173079e-06
			MEXP	7. 8960e-04	7. 78966e-04	7. 8976e-04	—	—	—	9.3192351e-06
		C.E2	MGP	6. 1720e-05	8. 16874e-05	3. 3886e-04	4. 8332e-05	3. 2556e-05	2.44612e-05	9.293168e-06
			MEXP	7. 8965e-04	7. 78953e-04	7. 8955e-04	—	—	—	9.3192350e-06
		PC.E2	MGP	3.3328e-04	6.82311e-04	0.61117458	0. 14987578	0. 56511718	0.27372429	1.188304e-04
			MEXP	5. 0730353	5. 073036	5. 0730332	—	—	—	1.193313e-04
	50	C.E1	MGP	1. 7899e-05	4. 40827e-05	1. 9051e-04	2. 9764e-05	1. 9192e-05	1. 8581e-05	5.180895979e-04
			MEXP	4. 4272e-04	4. 4288e-04	4. 4266e-04	—	—	—	5.17335495e-04
		C.E2	MGP	3. 7133e-05	4. 40826e-05	1. 9049e-04	1. 9257e-05	1. 9190e-05	1. 8580e-05	5.180078e-04
			MEXP	4. 4264e-04	4. 4266e-04	4. 4267e-04	—	—	—	5.1733400e-04
		PC.E2	MGP	2. 2120e-04	3. 49784e-04	0. 15209982	1.63738106	1.51741818	0.96408662	0.006222415
			MEXP	14.5204796	14.52047953	14.5204795	—	—	—	0.006221790
	75	C.E1	MGP	1. 1420e-05	3. 80095e-05	1. 5745e-04	2. 0865e-05	1. 7999e-05	9. 3834e-06	7.592443318e-04
			MEXP	4. 3883e-04	4. 3886e-04	4. 3889e-04	—	—	—	7.587348013e-04
		C.E2	MGP	2. 1608e-05	3. 80094e-05	1. 5743e-04	1. 9317e-05	1. 7998e-05	9. 3833e-06	7.59201660e-04
			MEXP	4. 3890e-04	4. 3881e-04	4. 3886e-04	—	—	—	7.58734801e-04
		PC.E2	MGP	1. 0153e-04	2. 54904e-04	0.00113057	3. 88351523	3. 73558412	2. 8850462	0.00911423215
			MEXP	28. 4402344	28. 4402341	28. 4402376	—	—	—	0.00911395283
	100	C.E1	MGP	7. 9333e-06	2. 84435e-05	9. 2899e-05	1. 2296e-05	1. 3279e-05	6. 7163e-06	1.425494024e-04
			MEXP	2. 4490e-04	2. 4494e-04	2. 498e-04	—	—	—	1.422132208e-04
		C.E2	MGP	1. 0877e-05	2. 84432e-05	9. 2898e-05	1. 3279e-05	1. 3277e-05	6. 7161e-06	1.42527815e-04
			MEXP	2. 4497e-04	2. 4503e-04	2. 496e-04	—	—	—	1.42215020e-04
		PC.E2	MGP	1. 3932e-04	1. 51434e-04	8. 7104e-04	4. 85807902	4. 3578902	2. 8725043	0.00171352000
			MEXP	33. 9593559	33. 5935483	33. 5935505	—	—	—	0.00171341578

Table (3). MSE estimation of the distribution parameters (MEXP, MGP) and their probability density function for the models (V, III) shown in Table (1)

model	n	M.E	Dist.	MSE(λ_1)	MSE(λ_2)	MSE(λ_3)	MSE(ε_1)	MSE(ε_2)	MSE(ε_3)	MSE(pdf)
III	25	C.E1	MGP	3. 6289e-05	9. 46643e-05	1. 4641e-04	5. 0075e-05	4. 4884e-05	4. 0289e-05	1.5563616934e-05
			MEXP	0. 27545734	0. 27545745	0. 27545739	—	—	—	1.5734924231e-05
		C.E2	MGP	3. 6287e-05	9. 46641e-05	1. 4639e-04	5. 0074e-05	4. 4883e-05	4. 0287e-05	1.5592344061e-05
			MEXP	0. 27545734	0. 27545737	0. 27545728	—	—	—	1.5734924230e-05
		PC.E2	MGP	2. 6405e-04	8. 33480e-04	0. 01186292	0.77572969	0.45612706	0.84845825	1.9614779971e-04
			MEXP	9. 42516935	9. 25169221	9.25169342	—	—	—	1.9637170016e-04
	50	C.E1	MGP	1. 4749e-05	5.03473e-05	8.50561e-05	2.67489e-05	2.57636e-05	1.51751e-05	8.5191669806e-04
			MEXP	6. 2669e-04	6. 2665e-04	6. 2661e-04	—	—	—	8.5110102350e-04
		C.E2	MGP	1. 4747e-05	5. 03471e-05	8. 0560e-05	2. 7488e-05	2.57635e-05	1.51750e-05	8.5187914019e-04
			MEXP	6. 2674e-04	6. 2671e-04	6. 2669e-04	—	—	—	8.5110102350e-04
		PC.E2	MGP	1. 0897e-04	4. 84928e-04	7. 8556e-04	2. 03904007	1. 06086765	1. 38649293	0.01022766463467
			MEXP	23. 267998	23. 268008	23. 268017	—	—	—	0.01022648729073
	75	C.E1	MGP	1.9396e-05	3. 94474e-05	5. 9011e-05	2.50285e-05	1.37839e-05	1.40784e-05	0.00124816321053
			MEXP	7. 4104e-04	7. 4107e-04	7. 4101e-04	—	—	—	0.0012476077040
		C.E2	MGP	1.9395e-05	3.94473e-05	5. 9001e-05	2.50284e-05	1.37838e-05	1.40782e-05	0.0012481465277
			MEXP	7. 4113e-04	7. 4106e-04	7. 4097e-04	—	—	—	0.00124760671406
		PC.E2	MGP	9.75748e-05	2.65196e-04	4.55184e-04	4.02517521	3.14215602	3.70812126	0.0149807315007
			MEXP	47.2785006	47. 2785009	47. 2784978	—	—	—	0.01498031743209
	100	C.E1	MGP	8. 0391e-06	2.76443e-05	4.35281e-05	1. 2439e-05	1.21492e-05	1.54513e-05	2.3449980164e-04
			MEXP	4. 5259e-04	4. 55265e-04	4. 5255e-04	—	—	—	2.3412652026e-04
		C.E2	MGP	8. 0390e-06	2. 76442e-05	4.35280e-05	1.52438e-05	1.21491e-05	1.54506e-05	2.3448993182e-04
			MEXP	4. 5263e-04	4. 55266e-04	4. 5259e-04	—	—	—	2.3412652025e-04
		PC.E2	MGP	6.80449e-05	2.67907e-04	3.99441e-04	5.44009078	4.87361334	4.90691629	0.0028164690859
			MEXP	55. 0413364	55. 0413367	55.0413449	—	—	—	0.0028163565684
V	25	C.E1	MGP	1.71012e-05	2.23127e-05	5.82569e-05	5.33407e-05	4.78870e-05	4.52369e-05	7.1933500267e-06
			MEXP	6.40961e-04	6.40967e-04	6.09670e-04	—	—	—	7.3808270342e-06
		C.E2	MGP	1.71002e-05	2.52312e-05	5.82568e-05	5.33406e-05	4.78869e-05	4.52368e-05	7.2047147380e-06
			MEXP	6. 0764e-04	6. 0770e-04	6. 0772e-04	—	—	—	7.3808260241e-06
		PC.E2	MGP	8.80443e-05	2.08168e-04	4.40574e-04	0.55837982	0.8526405	0.31071215	9.5809844573e-05
			MEXP	4.83329826	4.83329830	483329833	—	—	—	9.6038166320e-05
	50	C.E1	MGP	5.37339e-06	1.69777e-05	2.38999e-05	2.90447e-05	2.05186e-05	1.89540e-05	4.1642004474e-04
			MEXP	3. 8970e-04	3. 8979e-04	3. 8977e-04	—	—	—	4.1642004473e-04
		C.E2	MGP	5.37334e-06	1.69769e-05	2.38992e-05	2.0446e-05	2.05196e-05	1.9539e-05	4.1721302411e-04
			MEXP	3. 2202e-04	3. 2206e-04	3. 2205e-04	—	—	—	4.1642004471e-04
		PC.E2	MGP	4.28731e-05	1.03632e-04	2.67327e-04	2.63510726	1.80870506	1.53711348	0.00501148678944
			MEXP	11. 391313	11. 391310	11. 391320	—	—	—	0.00501081014554
	75	C.E1	MGP	3.416732-06	9.72843e-06	1.54923e-05	2.8246e-05	2.36785e-05	1.2383e-05	6.1148945047e-04
			MEXP	3. 3211e-04	3. 3212e-04	3. 3214e-04	—	—	—	6.1092455937e-04
		C.E2	MGP	3.41672e-06	9.2842e-06	1.54922e-05	2.88245e-05	2.46784e-05	1.62382e-05	6.1147159169e-04
			MEXP	3. 3208e-04	3. 83205e-04	3. 3210e-04	—	—	—	6.1092455936e-04
		PC.E2	MGP	3.66507e-05	7.96789e-05	1.36975e-04	4.02712748	3.21713262	3.43673675	0.00734053198846
			MEXP	23. 2664649	23.92664651	23. 2664655	—	—	—	0.00734011813462
	100	C.E1	MGP	2. 1840e-06	7. 36275e-06	1. 1357e-05	1. 395e-05	1. 1937e-05	1. 4550e-05	1.1480558004e-04
			MEXP	2. 5078e-04	2. 95079e-04	2. 5080e-04	—	—	—	1.1442571309e-04
		C.E2	MGP	2. 1843e-06	7. 6259e -06	2. 5078e-04	1. 4394e-05	1.31939e-05	1. 4548e-05	1.1479465307e-04

model	n	M.E	Dist.	MSE(λ_1)	MSE(λ_2)	MSE(λ_3)	MSE(ε_1)	MSE(ε_2)	MSE(ε_3)	MSE(pdf)
			MEXP	2. 5081e-04	2. 5084e-04	2. 5077e-04	—	—	—	1.1442551308e-04
		PC.E2	MGP	2.27399e-05	5.07458e-05	1.88915e-04	5.60940397	5.16980927	4.00655992	0.00138005475897
			MEXP	27. 5102558	27. 5102560	27. 5102565	—	—	—	0.00137994237739

Table (4). MSE estimation of the reliability function for tow distribution (MEXP, MGP) (III) shown in Table (1)

n	M.E	distribution	R	Mse(R)	R-all	Mse(R-all)
25	C.E1	MGP	0.3068752850893	0.529211811080	0.2911067565465	0.3600656131927
		MEXP	0.02185591526352	4. 0320063e-04	0. 824962444148	7. 81086e-04
	C.E2	MGP	0.5349465621443	0.3822411504724	0.436964751147	0.4503649058751
		MEXP	0.0218559152642	4. 03200625e-04	0. 8249624441477	7. 810850370e-04
	PC.E2	MGP	4.70072395095e-04	7.667694558013e-04	0.02769060230116	0.02168115299275
		MEXP	0.00207416390676	4.302155912110e-06	0.00116243230396	1.35124886129e-06
50	C.E1	MGP	0.4362019194860	0.28894890901084	0.3763144424883	0.2770167782416
		MEXP	0.00610426009126	3. 6199126185e-05	0.00608352313473	3.70092537309e-05
	C.E2	MGP	0.57955871421413	0.092839978347659	0.59287260797191	0.11375432398157
		MEXP	0. 104260091249	3. 99126184357e-05	0. 083523134729	3. 5373070503e-05
	PC.E2	MGP	0.00810575407948	6.57032491970e-05	0.00820000661251	6.72401084452e-05
		MEXP	2.11985550675e-04	4.493787369518e-08	1.11217229002e-04	1.23692720270e-08
75	C.E1	MGP	0.3143887653182	0.2005410076470	0.3691013049279	0.0320529295782
		MEXP	0.052322751196	0.00273767029274	0.05289486532944	0.002797866778
	C.E2	MGP	0.40521758192737	0.127675619534939	0.36430620723533	0.16015874598577
		MEXP	0.05232275119026	0.002737670292536	0.05289486532932	0.0027978667771
	PC.E2	MGP	0.01668622056619	2.784299567835e-04	0.00747837813070	5.59261394658e-05
		MEXP	0.00146491656588	2.145980545001e-06	7.54641873459e-04	5.69484357178e-07
100	C.E1	MGP	0.4467893235347	0.1507762190955	0.37699780397555	0.108747020408
		MEXP	0.00134396451004	1.80624060424e-06	0.0013933808545	1.94151020585e-06
	C.E2	MGP	0.53993551325403	0.0570705133386	0.51698215431585	0.11224415617658
		MEXP	0.00134396451001	1.8062 4060412e-06	0.00139338085326	1.94151020565e-06
	PC.E2	MGP	0.00253981356591	6.450652949580e-06	0.00294343506207	8.66380996467e-06
		MEXP	2.48217897687e-05	6.161212473216e-10	1.40067490451e-05	1.96189018814e-10

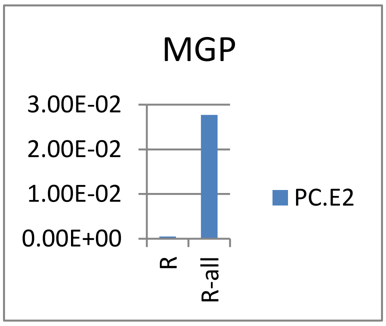

Figure 1. (PC.E1) at (n = 25)

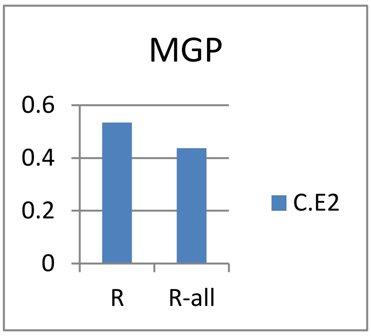

Figure 2. (C.E2) at (n = 25)

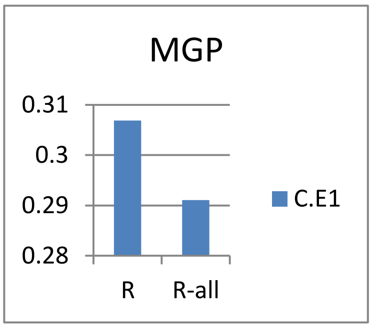

Figure 3. (C.E1) at (n = 25)

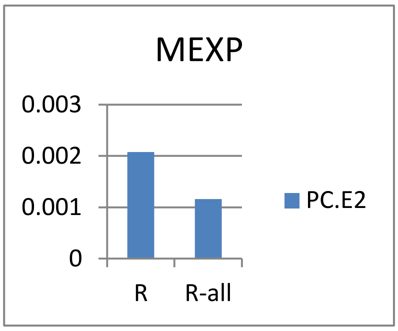

Figure 4. (PC.E1) at (n = 25)

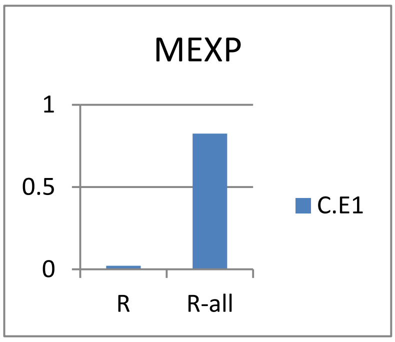

Figure 5. (C.E2) at (n = 25)

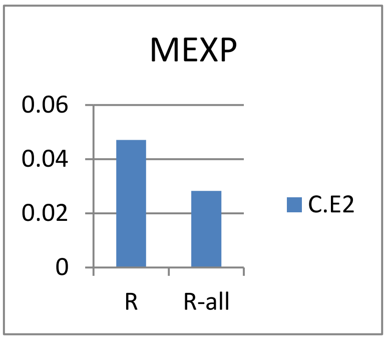

Figure 6. (C.E1) at (n = 25)

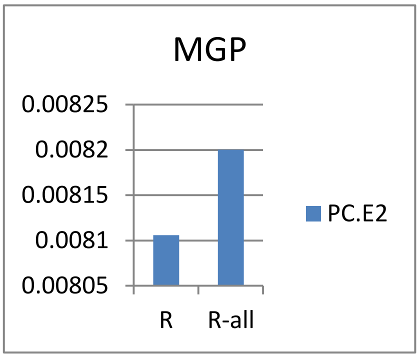

Figure 7. (PC.E1) at (n = 50)

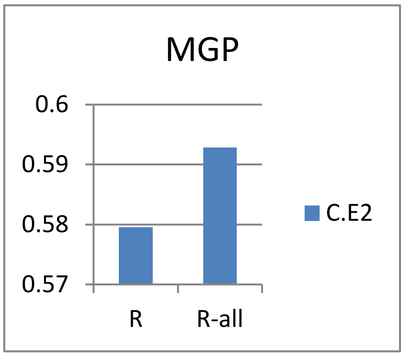

Figure 8. (C.E2) at (n = 50)

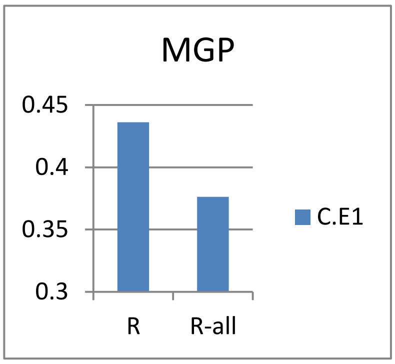

Figure 9. (C.E1) at (n = 50)

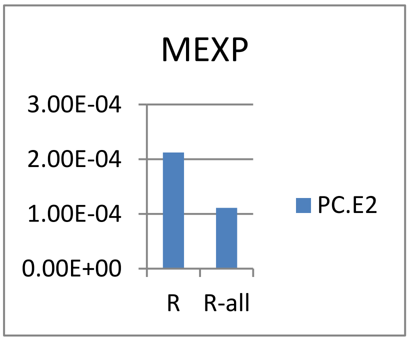

Figure10. (PC.E1) at (n = 50)

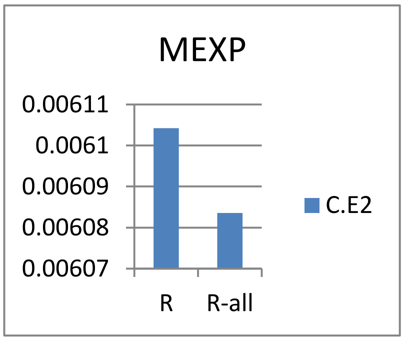

Figure 11. (C.E2) at (n = 50)

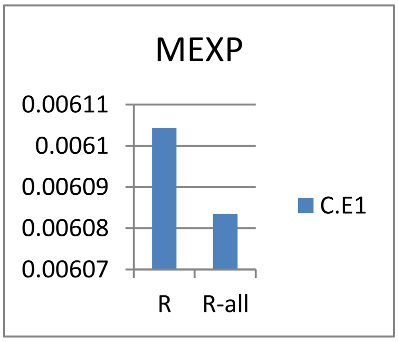

Figure 12. (C.E1) at (n = 50)

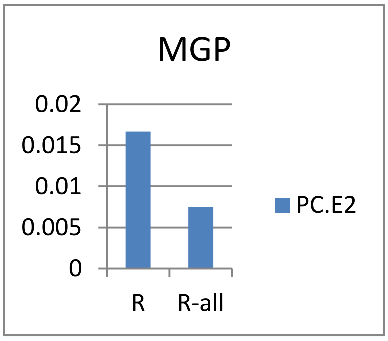

Figure13. (PC.E1) at (n = 75)

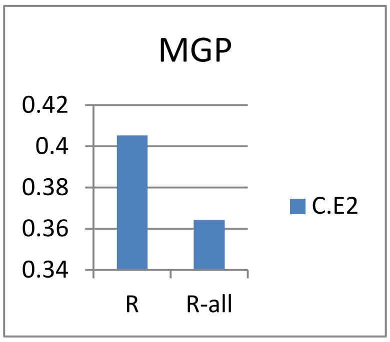

Figure 14. (C.E2) at (n = 75)

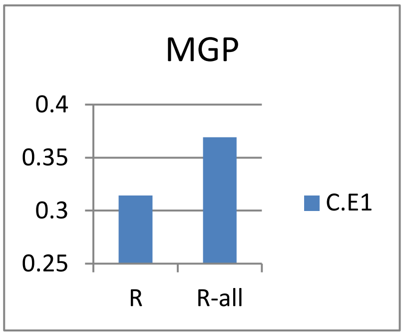

Figure 15. (C.E1) at (n = 75)

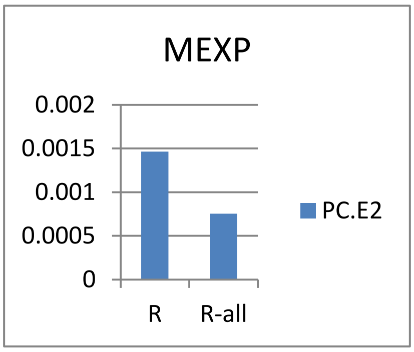

Figure 16. (PC.E1) at (n = 75)

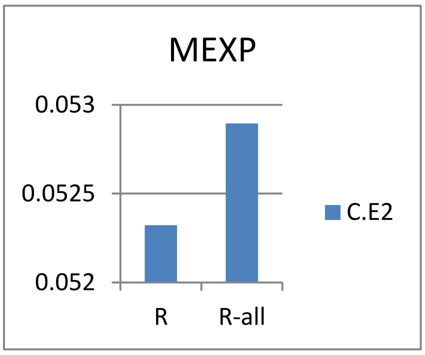

Figure 17. (C.E2) at (n = 75)

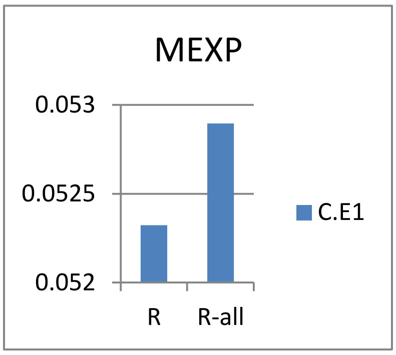

Figure 18. (C.E1) at (n = 75)

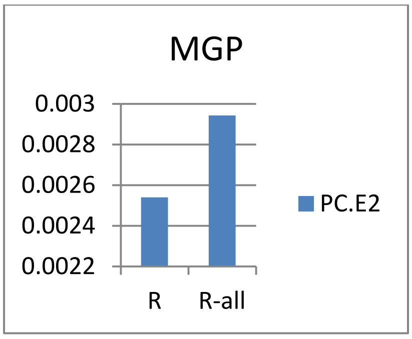

Figure 19. (PC.E1) at (n = 100)

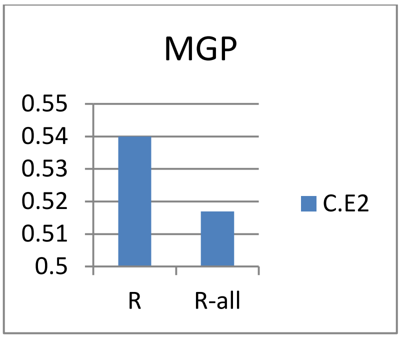

Figure 20. (C.E2) at (n = 100)

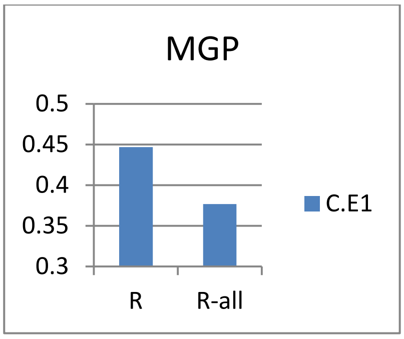

Figure 21. (C.E1) at (n = 100)

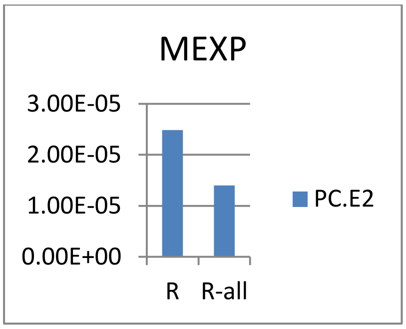

Figure 22. (PC.E1) at (n = 100)

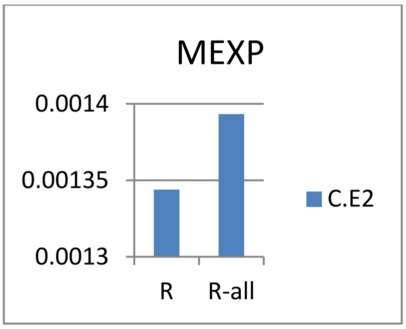

Figure 23. (C.E2) at (n = 100)

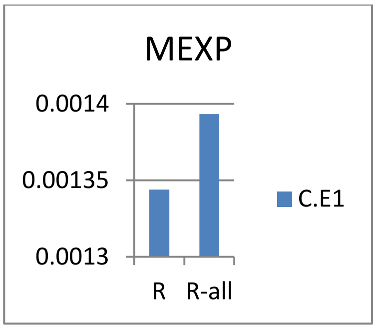

Figure 24. (C.E1) at (n = 100)

9. Analysis of the simulation experiment results

In Table (2), the simulation experiment has shown the following:

1. The first model shows that the distribution of MGP and the estimation method C.E1 is the best in the sample (n = 25) while the MEXP distribution is the best in the samples (n = 50, n = 75, n = 100), the estimator method (C.E1,C.E2) are the best in the sample (n = 50) and C.E2 the best in the samples (n = 75, n = 100) based on the value of the MSE for the probability density function of the two distributions.
2. The second model shows that the distribution of MGP and the estimation method C.E1 is the best in both samples (n = 25, n = 100). The MEXP distribution is the best in the samples (n = 50, n = 75, n = 100)) and the estimator method (C.E2) is the best in both the samples (n = 50, n = 75) based on the value of the MSE for the probability density function of the two distributions.

In Table (3), the simulation experiment has shown the following:
1. The third model shows that the distribution of MGP and the estimation method C.E1 are the best in sample (n = 25). The MEXP distribution is the best in samples (n = 50, n = 75, n = 100) and the estimator method (C.E2) is the best in samples (n = 50, n = 75, n = 100)) based on the value of the MSE for the probability density function of the distribution.
2. The fourth model shows that the distribution of MGP and the estimation method C.E1 are the best in the samples (n = 25). The MEXP distribution is the best in the samples (n = 50, n = 75, n = 100) and the estimator method (C.E2) is the best in samples (n = 50, n = 75, n = 100), based on the value of the MSE for the probability density function of the distribution.
In Table (4) and the Figures(1-24), the simulation experiment approximated the reliability function extracted for the complete sample R-all from the reliability function of the sample divided into two equal parts R.
1. The estimation methods (C.E1, C.E2) showed that the R-all reliability function of the MGP distribution is better than the R, whereas the reliability function for the MEXP distribution of R is less than R-all. The PC.E2 method with R is better than R -all distribution of MGP and R-all is better than R for MEXP distribution based on MSE for reliability function at sample size (n = 25).
2. The estimation methods (C.E1) showed that the R-all reliability function of the MGP distribution was better than R. For the same method and (PC.E2, C.E2), the R is better than R-all. Based on reliability function of the MEXP distribution (C.E1, C.E2), R is less than R-all. The R-all and the method PC.E2 is better than R based on the MSE for the reliability function at sample size (n = 50).
3. The estimation method (C.E1) showed that the R-all reliability function of the MGP distribution was better than R. For the same distribution (PC.E2, C.E2), R is better than R-all. For the reliability function for the MEXP distribution methods (C.E.1, C.E2, PC.E2), R-all is better than R and (PC.E2) according to MSE for reliability function at sample size (n = 50).
4. The estimation method (C.E1) has revealed that the R-all reliability function of the MGP distribution was better than R. For the same distribution (PC.E2, C.E2), the R is better than R-all. Based on the reliability function for the MEXP distribution methods (C.E1, C.E2), R is less than R-all. For (PC.E2), R-all is better than R, based on the MSE for the reliability function at sample size (n = 75).
5. The estimation methods (C.E1, C.E2) has exposed that the R-all reliability function of the MGP distribution is better than the R . The reliability function for the MEXP distribution R is less than R-all. For PC.E2 method, R-all is better than R distribution of MGP, and R-all is better than R for MEXP distribution based on MSE for reliability function at sample size (n = 100).

10. Conclusions

1. The POT sample has shown significant improvement in data and its superiority, especially in simple exponential distribution.
2. We conclude that the distribution of MEXP outweighs the distribution of MGP through the MSE comparison criterion of the probabilistic density function of-distributors.
3. We also conclude that C.E1 and C.E2 are the two preferred methods (PC.E2), but the C.E2 method is superior to C.E1.
4. Finally, the last conclusion is that the reliability function of the complete sample R-all and the reliability function of the divided sample R show good and convergent results, which is possible to use one as a substitute for the other.

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